# Fourier Series

Periodic Functions

# The Mathematic Formulation

- Any function that satisfies

$$f(t) = f(t + T)$$

where *T* is a constant and is called the *period* of the function.

# Example:

$$f(t) = \cos\frac{t}{3} + \cos\frac{t}{4}$$ Find its period.

$$f(t) = f(t+T) \Rightarrow \cos\frac{t}{3} + \cos\frac{t}{4} = \cos\frac{1}{3}(t+T) + \cos\frac{1}{4}(t+T)$$

Fact: $\cos\theta = \cos(\theta + 2m\pi)$

$$\frac{T}{3} = 2m\pi \qquad \frac{T}{4} = 2n\pi$$

$\Rightarrow$

$$T = 6m\pi \qquad T = 8n\pi$$

$\Rightarrow$

$T = 24\pi$ smallest $T$

# Example:

$f(t) = \cos\omega_1 t + \cos\omega_2 t$ Find its period.

$f(t) = f(t+T)$ ➡ $\cos\omega_1 t + \cos\omega_2 t = \cos\omega_1(t+T) + \cos\omega_2(t+T)$

$\omega_1 T = 2m\pi$

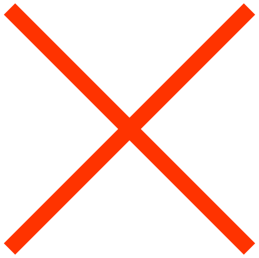
$\omega_2 T = 2n\pi$

➡ $\frac{\omega_1}{\omega_2} = \frac{m}{n}$ ➡ $\frac{\omega_1}{\omega_2}$ must be a rational number

# Example:

$$f(t) = \cos 10t + \cos(10+\pi)t$$

Is this function a periodic one?



$$\frac{\omega_1}{\omega_2} = \frac{10}{10+\pi}$$

not a rational number

# Orthogonal set of Sinusoidal Functions

$$\left\{\begin{array}{l} 1, \\ \cos\omega_0 t, \cos 2\omega_0 t, \cos 3\omega_0 t, \cdots \\ \sin\omega_0 t, \sin 2\omega_0 t, \sin 3\omega_0 t, \cdots \end{array}\right\}$$

an orthogonal set.

Define $\omega_0 = 2\pi / T$.

$$\int_{-T/2}^{T/2} \cos(m\omega_0 t)\,dt = 0, \qquad m \neq 0$$

$$\int_{-T/2}^{T/2} \sin(m\omega_0 t)\,dt = 0, \qquad m \neq 0$$

$$\int_{-T/2}^{T/2} \cos(m\omega_0 t)\cos(n\omega_0 t)\,dt = \begin{cases} 0 & m \neq n \\ T/2 & m = n \end{cases}$$

$$\int_{-T/2}^{T/2} \sin(m\omega_0 t)\sin(n\omega_0 t)\,dt = \begin{cases} 0 & m \neq n \\ T/2 & m = n \end{cases}$$

$$\int_{-T/2}^{T/2} \sin(m\omega_0 t)\cos(n\omega_0 t)\,dt = 0, \quad \text{for all } m \text{ and } n$$