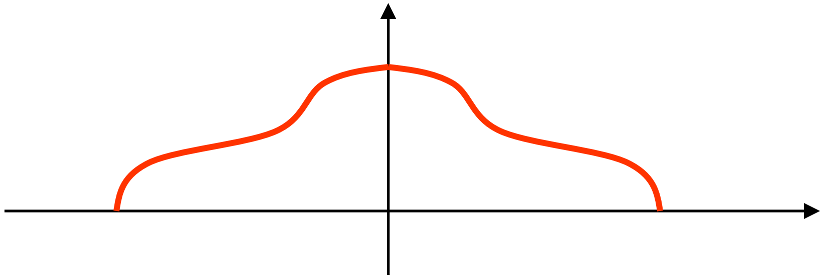
Fourier symmetry

Waveform Symmetry

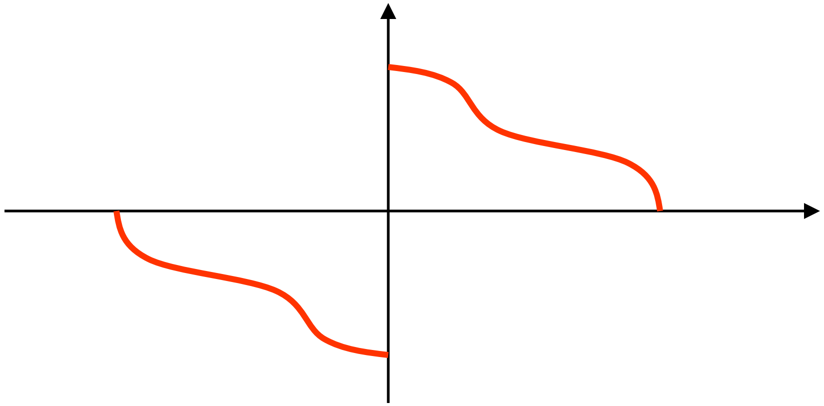
- Even Functions

$$f(t) = f(-t)$$



- Odd Functions

$$f(t) = -f(-t)$$



Decomposition

- Any function $f(t)$ can be expressed as the sum of an even function $f_e(t)$ and an odd function $f_o(t)$.

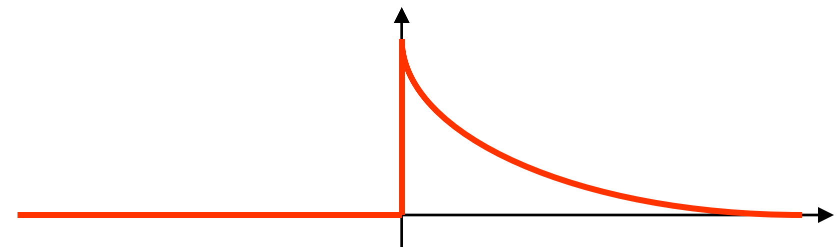
$$f(t) = f_e(t) + f_o(t)$$

$$f_e(t) = \tfrac{1}{2}[f(t) + f(-t)]$$ Even Part

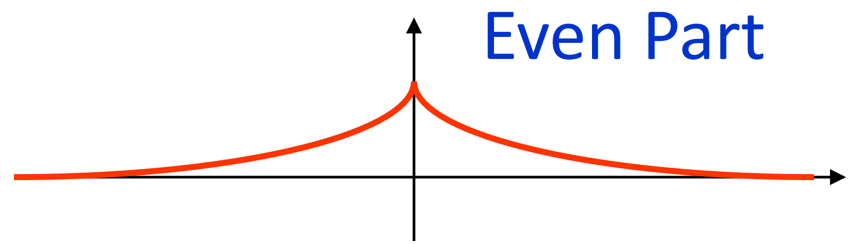
$$f_o(t) = \tfrac{1}{2}[f(t) - f(-t)]$$ Odd Part

Example

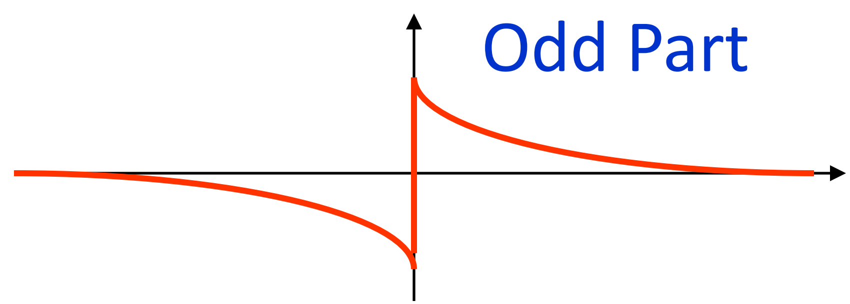
$$f(t) = \begin{cases} e^{-t} & t > 0 \\ 0 & t < 0 \end{cases}$$



$$f_e(t) = \begin{cases} \frac{1}{2} e^{-t} & t > 0 \\ \frac{1}{2} e^{t} & t < 0 \end{cases}$$

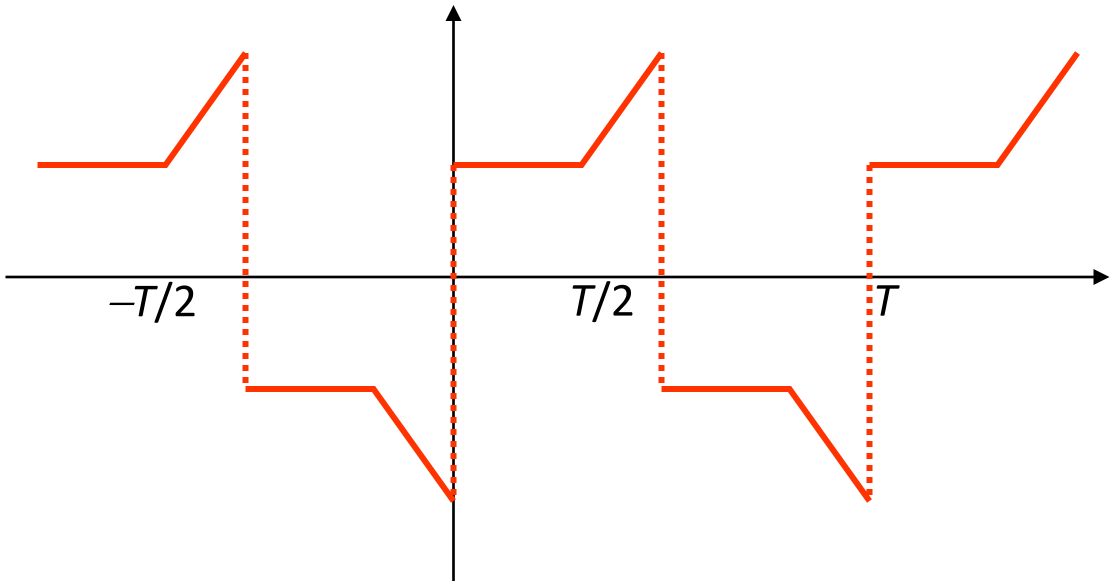


$$f_o(t) = \begin{cases} \frac{1}{2} e^{-t} & t > 0 \\ -\frac{1}{2} e^{t} & t < 0 \end{cases}$$



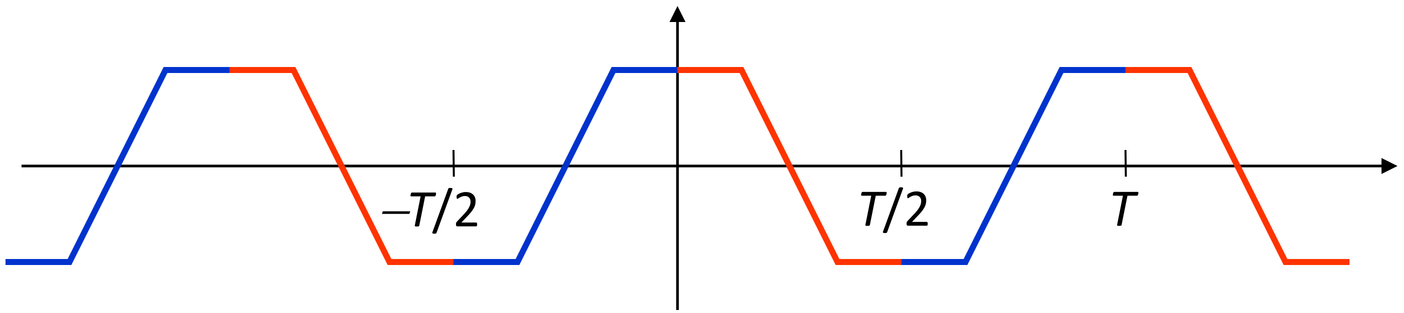
Half-Wave Symmetry

$$f(t) = f(t+T) \quad \text{and} \quad f(t) = -f\left(t+T/2\right)$$

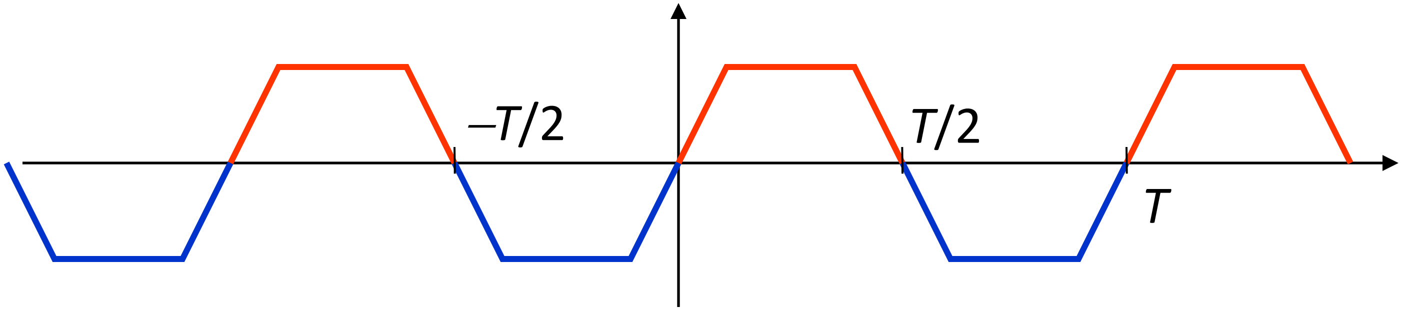


Quarter-Wave Symmetry

Even Quarter-Wave Symmetry

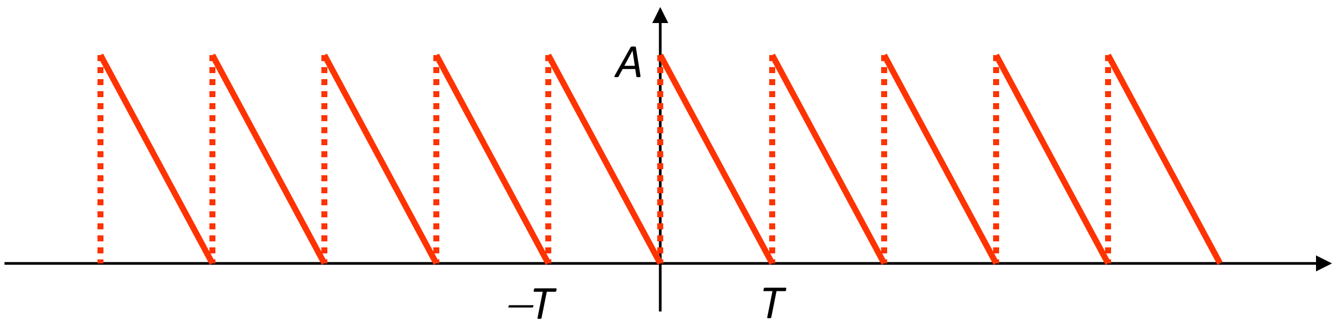


Odd Quarter-Wave Symmetry

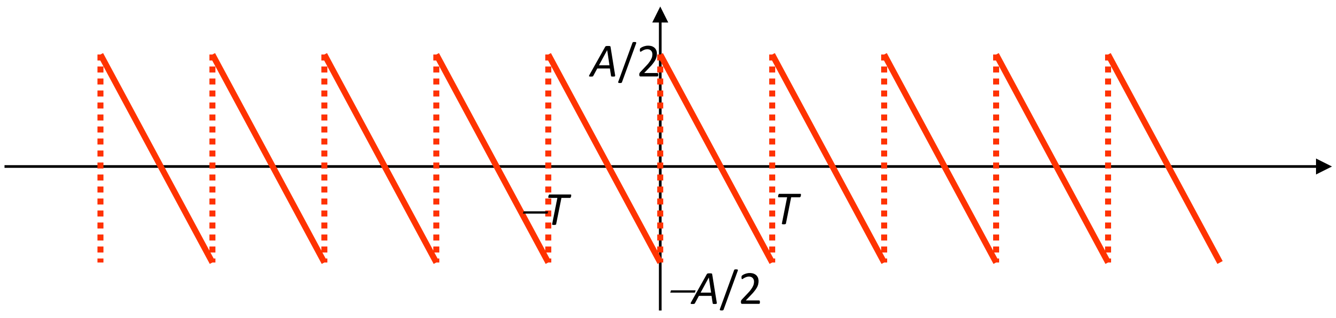


Hidden Symmetry

- The following is a asymmetry periodic function:



- Adding a constant to get symmetry property.

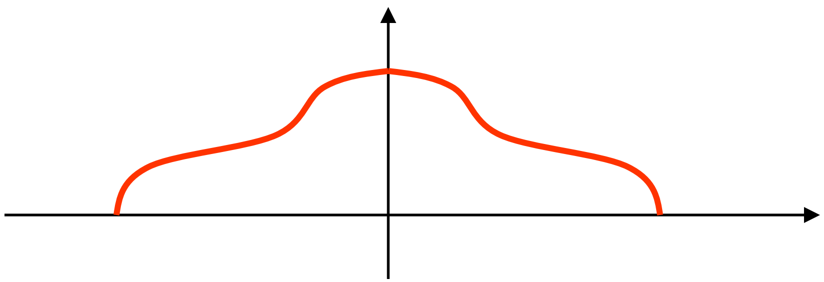


Fourier Coefficients of Symmetrical Waveforms

- The use of symmetry properties simplifies the calculation of Fourier coefficients.
  - Even Functions
  - Odd Functions
  - Half-Wave
  - Even Quarter-Wave
  - Odd Quarter-Wave
  - Hidden

Fourier Coefficients of Even Functions

$$f(t) = f(-t)$$

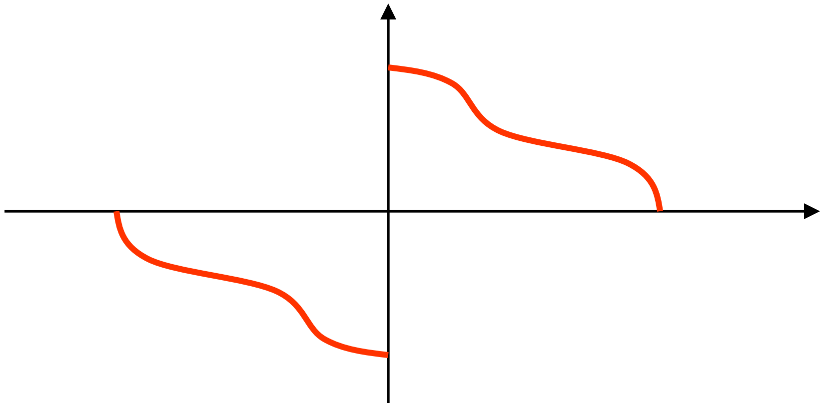


$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

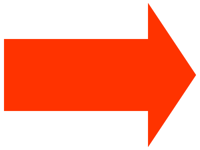
$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)dt$$

Fourier Coefficients of Even Functions

$$f(t) = -f(-t)$$



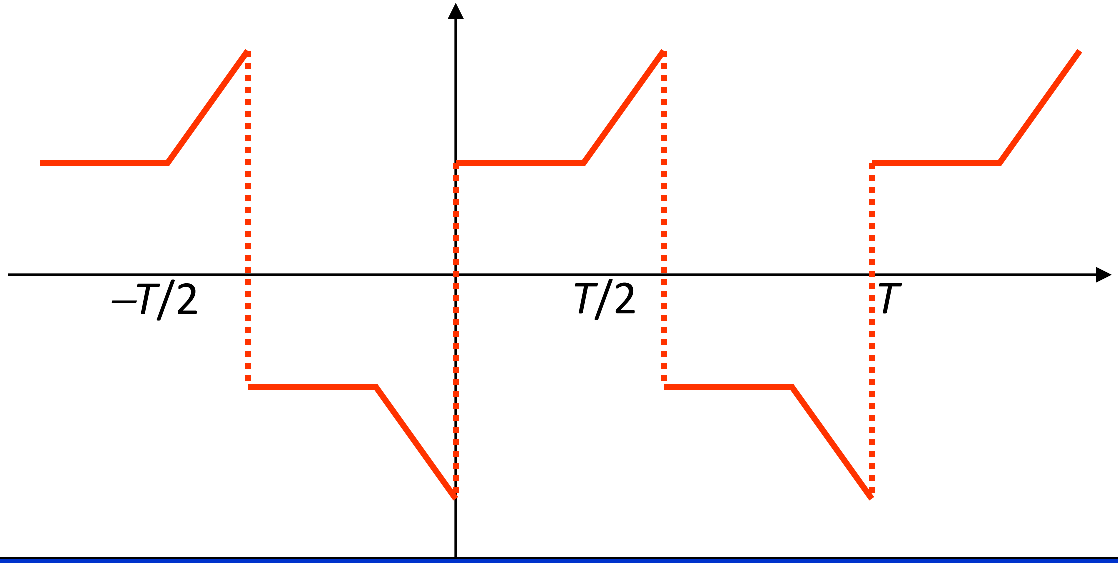
$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$



$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)dt$$

Fourier Coefficients for Half-Wave Symmetry

$$f(t) = f(t+T) \quad \text{and} \quad f(t) = -f\left(t + T/2\right)$$



The Fourier series contains only odd harmonics.

Fourier Coefficients for Half-Wave Symmetry

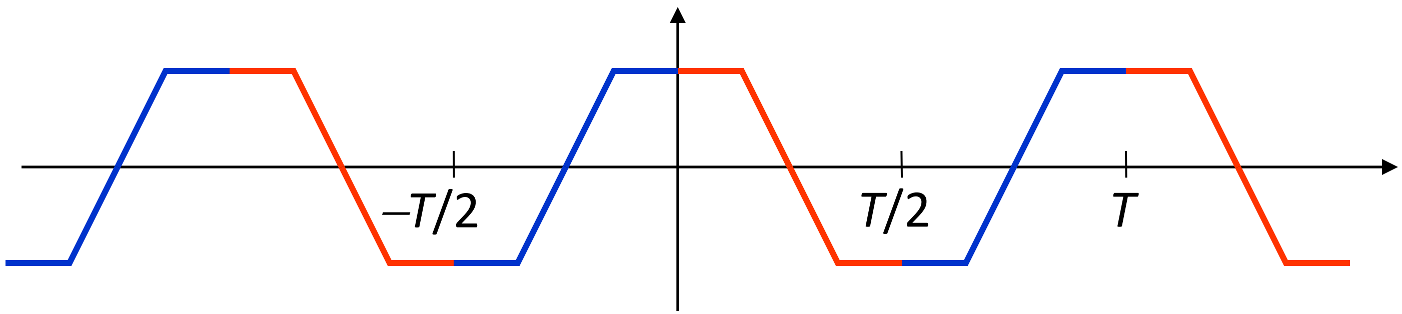
$$f(t) = f(t+T) \quad \text{and} \quad f(t) = -f\left(t + T/2\right)$$

$$\Rightarrow \quad f(t) = \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_n = \begin{cases} 0 & \text{for } n \text{ even} \\ \dfrac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt & \text{for } n \text{ odd} \end{cases}$$

$$b_n = \begin{cases} 0 & \text{for } n \text{ even} \\ \dfrac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)\,dt & \text{for } n \text{ odd} \end{cases}$$

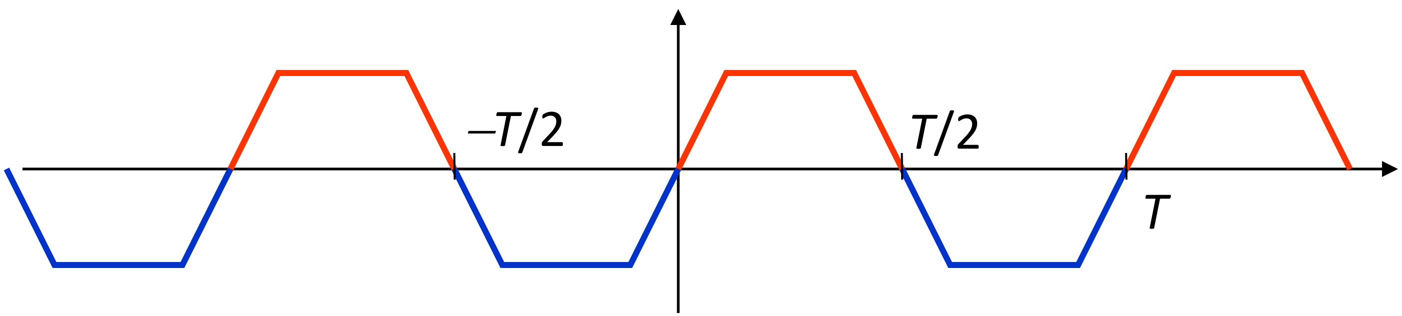
Fourier Coefficients for Even Quarter-Wave Symmetry



$$f(t) = \sum_{n=1}^{\infty} a_{2n-1} \cos[(2n-1)\omega_0 t]$$

$$\Rightarrow \quad a_{2n-1} = \frac{8}{T} \int_0^{T/4} f(t) \cos[(2n-1)\omega_0 t] dt$$

Fourier Coefficients for Odd Quarter-Wave Symmetry



$$f(t) = \sum_{n=1}^{\infty} b_{2n-1} \sin[(2n-1)\omega_0 t]$$

$$b_{2n-1} = \frac{8}{T}\int_0^{T/4} f(t)\sin[(2n-1)\omega_0 t]dt$$