Fourier Transform: Properties

Fourier Transform Definition

Represent the signal as an infinite weighted sum of an infinite number of sinusoids

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$$

Note: $e^{ik} = \cos k + i \sin k \qquad i = \sqrt{-1}$

Arbitrary function ⟶ Single Analytic Expression

Spatial Domain (*x*) ⟶ Frequency Domain (*u*)

(Frequency Spectrum *F(u)*)

Inverse Fourier Transform (IFT)

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{i2\pi ux} dx$$

Fourier Transform

- Also, defined as:

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-iux}dx$$

Note: $e^{ik}=\cos k+i\sin k \qquad i=\sqrt{-1}$

- Inverse Fourier Transform (IFT)

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)e^{iux}dx$$

Conditions of existence of FT

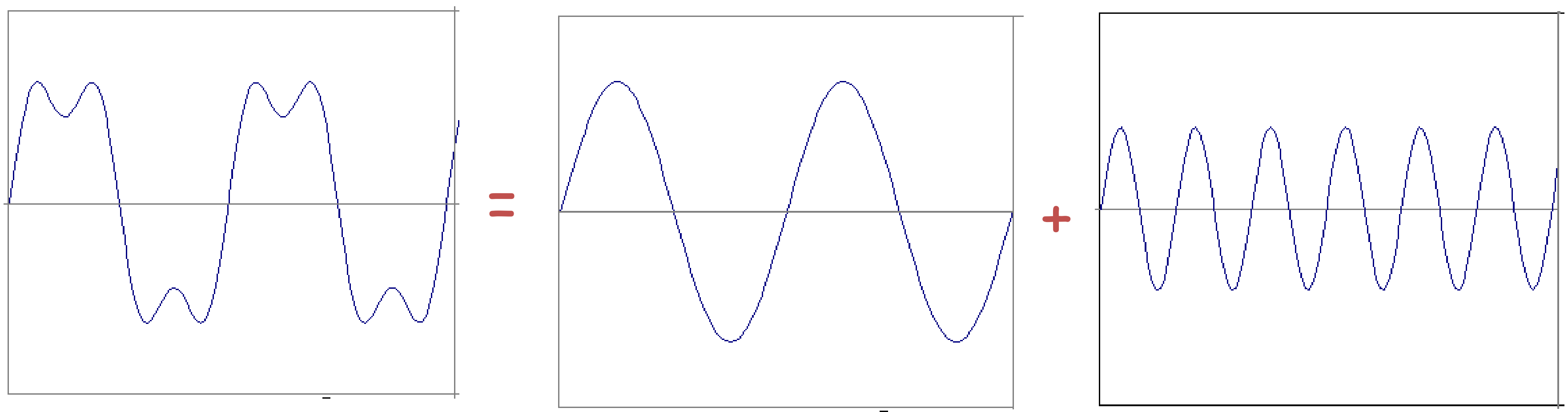
- Sufficient condition for the existence of a Fourier transform

$$\int_{-\infty}^{\infty} | f(t) | dt < \infty$$

- That is, $f(t)$ is absolutely integrable.
- However, the above condition is not the necessary one.

Phase Spectra

- example : $g(t) = \sin(2pi\, f\, t) + (1/3)\sin(2pi\, (3f)\, t)$



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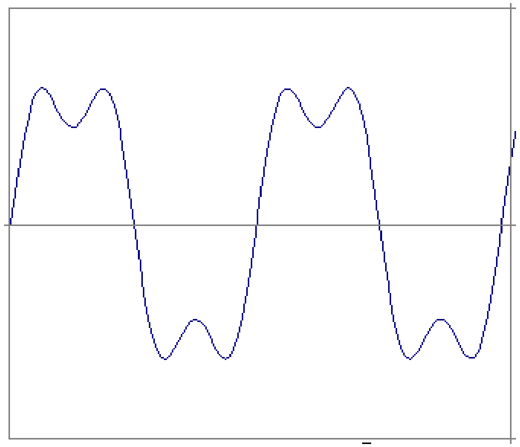

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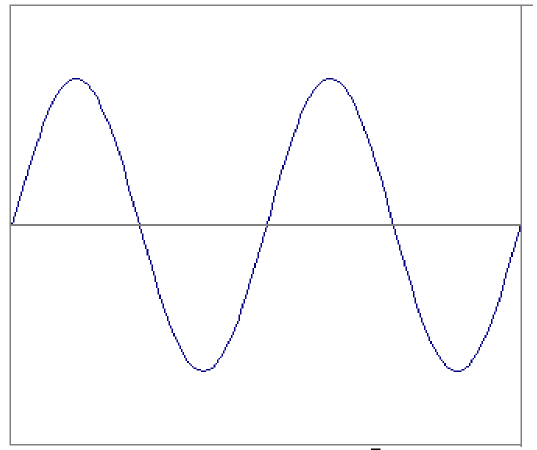

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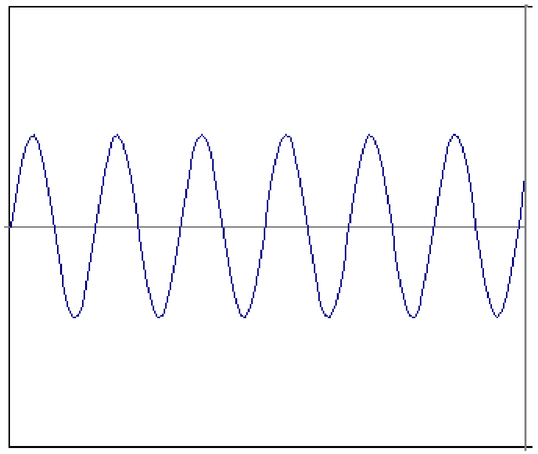
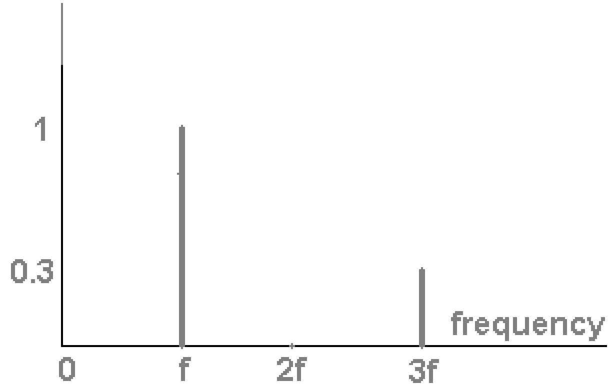
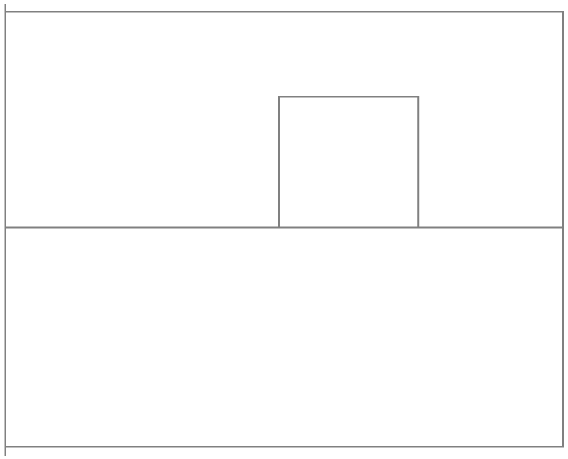


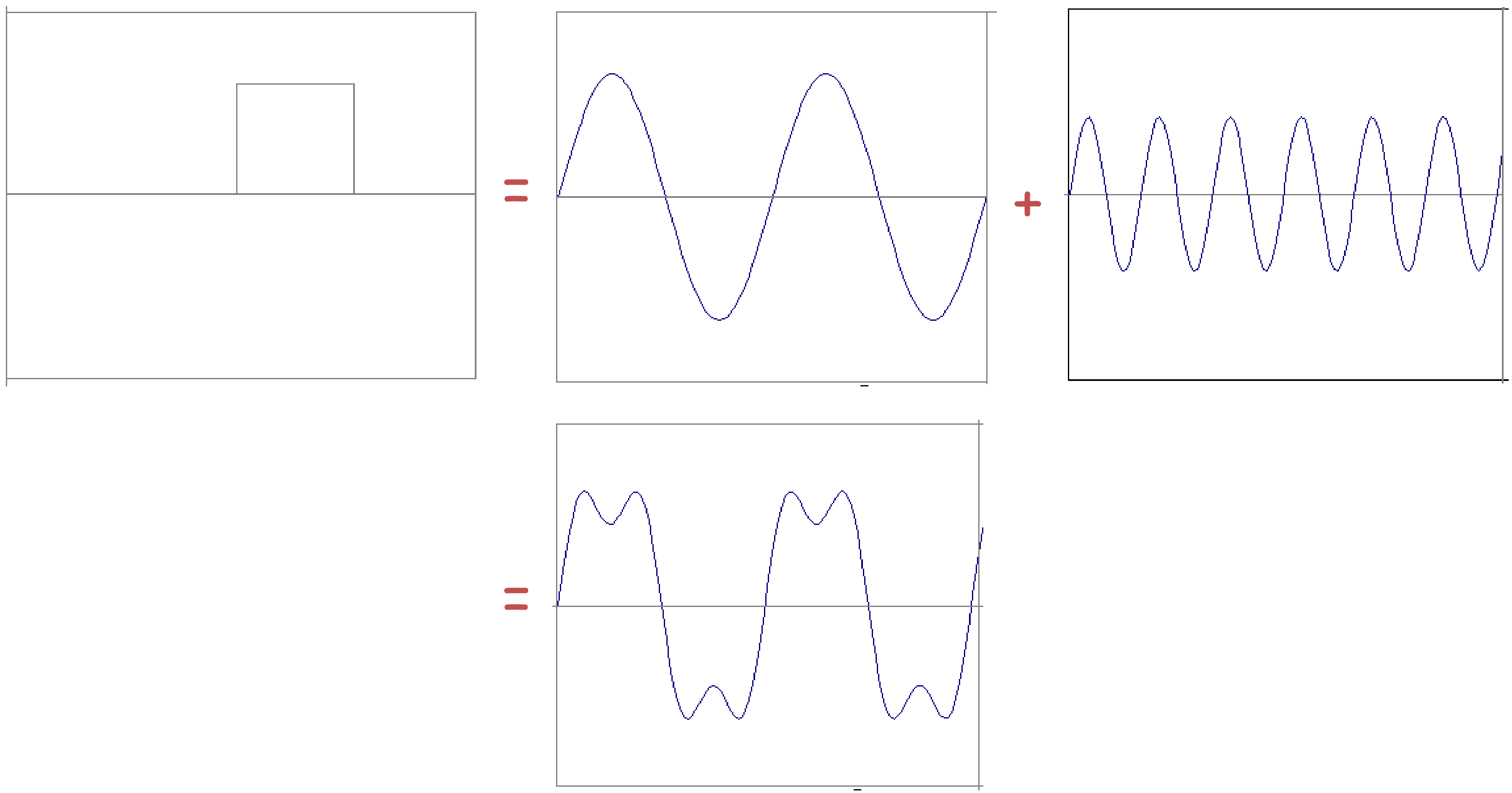


Phase Spectra

- Usually, frequency is more interesting than the phase



Phase Spectra



Frequency Spectra