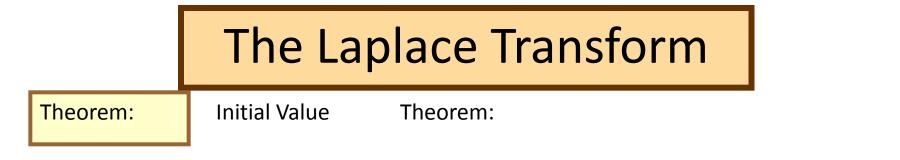
Initial and Final Value theorems



If the function f(t) and its first derivative are Laplace transformable and f(t) Has the Laplace transform F(s), and the $\lim_{s \to \infty} sF(s)$ exists, then

$$\lim_{s \to \infty} sF(s) = \lim_{t \to 0} f(t) = f(0)$$

Initial Value Theorem

The utility of this theorem lies in not having to take the inverse of F(s) in order to find out the initial condition in the time domain. This is particularly useful in circuits and systems.

The Laplace Transform

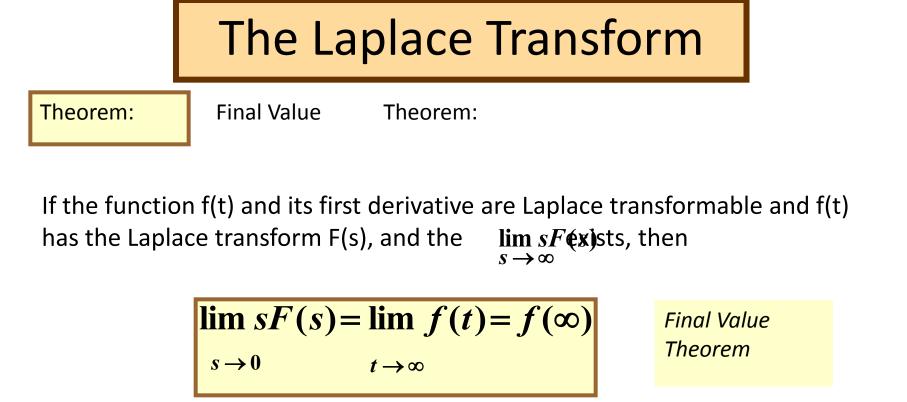
Example: Initial Value Theorem:

Given;

$$F(s) = \frac{(s+2)}{(s+1)^2 + 5^2}$$

Find f(0)

$$f(0) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} s \frac{(s+2)}{(s+1)^2 + 5^2} = \lim_{s \to \infty} \left[\frac{s^2 + 2s}{s^2 + 2s + 1 + 25} \right]$$
$$= \lim_{s \to \infty} \frac{\frac{s^2/s^2 + 2s}{s^2 + 2s}}{s^2 + 2s} = 1$$



Again, the utility of this theorem lies in not having to take the inverse of F(s) in order to find out the final value of f(t) in the time domain. This is particularly useful in circuits and systems.

The Laplace Transform

Example: Final Value Theorem:

Given:

$$F(s) = \frac{(s+2)^2 - 3^2}{[(s+2)^2 + 3^2]} \quad note \ F^{-1}(s) = te^{-2t} \cos 3t$$

Find $f(\infty)$.

$$f(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} s \frac{(s+2)^2 - 3^2}{[(s+2)^2 + 3^2]} = 0$$