Applications of State-Variable technique to the analysis of linear systems

Contd…

OBSERVABILITY:

All the poles of the closed-loop system can be placed arbitrarily in the complex plane if and only if the system is **observable** and **controllable**. Observability refers to the ability to estimate a state variable.

A system is completely observable if and only if there exists a finite time T such that the initial state x(0) can be determined from the observation history y(t) given the control u(t).

Consider the single-input, single-output system

$$\dot{x} = Ax + Bu \quad \text{and} \quad y = Cx$$

where C is a 1xn row vector, and x is an nx1 column vector. This system is completely observable when the determinant of the **observability matrix P_0** is nonzero.

The observability matrix, which is an nxn matrix, is written as

$$P_O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Example:

Consider the previously given system

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Dorf and Bishop, Modern Control Systems

$$CA = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad , \quad CA^2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Thus, we obtain

$$P_O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The det P_0=1, and the system is completely observable. Note that determination of observability does not utility the B and C matrices.

Example: Consider the system given by

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

We can check the system controllability and observability using the P_c and P_0 matrices.

From the system definition, we obtain

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad AB = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Therefore, the controllability matrix is determined to be

$$P_c = [B \quad AB] = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

det P_c=0 and rank(P_c)=1. Thus, the system is not controllable.

From the system definition, we obtain

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \text{and} \quad CA = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Therefore, the observability matrix is determined to be

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

det P_O=0 and rank(P_O)=1. Thus, the system is not observable.

If we look again at the state model, we note that

$$y = x_1 + x_2$$

However,

$$\dot{x}_1 + \dot{x}_2 = 2x_1 + (x_2 - x_1) + u - u = x_1 + x_2$$

Thus, the system state variables do not depend on u, and the system is not controllable. Similarly, the output (x_1+x_2) depends on $x_1(0)$ plus $x_2(0)$ and does not allow us to determine $x_1(0)$ and $x_2(0)$ independently. Consequently, the system is not observable.

The observability matrix P_O can be constructed in Matlab by using **obsv** command.

From two-mass system,

```
clc
clear
A=[2 0;-1 1];
C=[1 1];
Po=obsv(A,C)
rank_Po=rank(Po)
det_Po=det(Po)
```

```
Po =

   1   1
   1   1


rank_Po =

   1


det_Po =

   0
```

The system is not observable.