Solution of State Equations for homogeneous and non-homogeneous systems

This note examines the response of linear, time-invariant models expressed in the standard state -equation form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

that is, as a set of coupled, first-order differential equations. The solution proceeds in two steps; first the state-variable response **x(*t*)** ***is found by solving the set of first-order state*** equations

The state-variable response of a system described by Eq. (1) with zero input, **u(*t*) ≡ *0*,** and an arbitrary set of initial conditions **x(0) is the solution of the set of *n homogeneous*** first-order differential equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

- To derive the homogeneous response **x*h(t)*, *we begin by considering the response of a first order*** (scalar) system with state equation

$$\dot{x} = ax + bu$$

- With initial condition *x(0). In this case the homogeneous response xh(t) has an exponential* form defined by the system time constant *τ = −1/a, or*

$$x(t) = e^\wedge(\mathrm{at})x(0)$$