Solution of difference equations using one-sided ZT

Properties of z-Transform

(1) Time Shifting $x[n-n_0] \longleftrightarrow z^{-n_0}X(z),$

The rationality of $X(z)$ unchanged, ROC unchanged except for the possible addition or deletion of the origin or infinity

$n_o > 0 \Rightarrow$ ROC $z \neq 0$ (maybe)

$n_o < 0 \Rightarrow$ ROC $z \neq \infty$ (maybe)

(2) z-Domain Differentiation $nx[n] \longleftrightarrow -z\frac{dX(z)}{dz},$

same ROC

(3) Linearity : $ax[n]+by[n] \longleftrightarrow aX(z)+bY(z)$

Properties of z-Transform

(4) Z-scale Property: $a^n x[n] \longleftrightarrow X\left(\dfrac{z}{a}\right)$

(5) Initial Value : $x[0] = \lim_{z\to\infty} X(z)$

(6) Final Value : $x[\infty] = \lim_{z\to 1} (z-1)X(z)$

(Applicable only if the ROC of $(z-1)X(z)$ includes the unit circle, i.e., all the poles are inside the unit circle)

(7) Convolution : $h[n]*x[n] \longleftrightarrow H(z)X(z)$

Rational *z*-Transform

For most practical signals, the *z*-transform can be expressed as a ratio of two polynomials

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_0(z - z_1)(z - z_2)\cdots(z - z_M)}{a_0(z - p_1)(z - p_2)\cdots(z - p_N)}$$

where $z_1, z_2, \cdots, z_M$ are the *zeroes* of *X(z)*, i.e., the roots of the numerator polynomial

and $p_1, p_2, \cdots, p_N$ are the *poles* of *X(z)*, i.e., the roots of the denominator polynomial.

Rational z-Transform

It is customary to normalize the denominator polynomial to make its leading coefficients one, i.e.,

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_0(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_N)}$$

$$= \frac{b_0 z^M + b_1 z^{M-1} + \cdots + b_M}{z^N + a_1 z^{N-1} + \cdots + a_N}$$

Also, it $x[n]$ is a causal signal, then $X(z)$ will be a proper rational polynomial with $M \le N$, i.e., # of zeroes $\le$ # of poles.