# Dronacharya Group of Institutions, Greater Noida
## Electrical & Electronics Engineering Department
**Question Bank**

**Subject: Engineering Mathematics-III (NAS-301)** **Branch: EEE 3rd Semester**

## COMPLEX ANALYSIS

1. State and prove necessary and sufficient condition for $f(z)$ to be analytic (C-R equations).
2. Show that the complex variable function $f(z)=|z|^2$ is differentiable only at the origin.
3. Show that the real and imaginary parts of the function $w = log\ z$ satisfy the Cauchy – Riemann equations when z is not zero.
4. Show that the function $z|z|$ is not analytic anywhere
5. Show that the function $f(z)=e^{-z^{-4}}, z \neq 0$ and $f(0)=0$ is not analytic at $z=0$ although Cauchy-Riemann equations are satisfied at the point.
6. Show that the function $f(z)$ defined by $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0$ and $f(0)=0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin. Yet f'(0) does not exist.
7. Show that the function defined by $f(z)=\sqrt{|xy|}$ satisfies Cauchy – Riemann equation at the origin but is not analytic at that point.
8. Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic. Find its harmonic conjugate.
9. Show that the function $x^2-y^2+2y$ which is harmonic remains harmonic under the transformation $z=w^3$.
10. If $\emptyset$ and $\varphi$ are functions of x and y satisfying Laplace's equation, show that $\boldsymbol{s}+\boldsymbol{it}$ is analytic, where $\boldsymbol{s}=\frac{d\emptyset}{dy}-\frac{d\varphi}{dx}$ and t$=\frac{d\emptyset}{dx}+\frac{d\varphi}{dy}$.
11. Prove that an analytic function with constant modulus is also constant.
12. If $f(z)=u+iv$ is an analytic function of z and $u-v=\frac{\cos x+\sin x-e^{-y}}{2\cos x-2\cosh y}$ ; Prove that $f(z)=\frac{1}{2}\left[1-\cot\frac{z}{2}\right]$ when $f\left(\frac{\pi}{2}\right)=0$.
13. If $f(z)$ is regular function of z, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.
14. Evaluate the line integral $\int_c z^2 dz$ where C is the boundary of z triangle with the vertices $0, 1+i, -1+i$ clockwise.
15. State and derive Cauchy integral theorem and derive Cauchy integral formula.

16. Evaluate the integral $\int_c \frac{3z^2+7z+1}{z+1} dz$, where C is the circle $|z| = \frac{1}{2}$ clockwise.

17. Use Cauchy integral formula to evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$.

18. State & prove Taylor Series and Laurent Series.

19. Expand $\frac{1}{z^2-3z+2}$ in the region (a) $|z|<1$ (b) $|z|>2$ (c) $1<|z|<2$.

20. Find out the zeros and discuss the nature of the singularities of $f(z) = \frac{z-2}{z^2} \sin\left(\frac{1}{z-1}\right)$.

21. Evaluate $\oint_c \left[\frac{3z^2+z+1}{(z-1)(z+3)}\right] dz \quad c = |z| = 2$, using Cauchy's residue theorem.

22. Show that: $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$.

23. Apply calculus of residue to prove that: $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta = \frac{2\pi a^2}{1-a^2} \; (a^2<1)$.

24. Using the complex variable techniques, evaluate the integral

(a) $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$

(b) $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)} dx$

25. Evaluate $\int_0^{\infty} \frac{\cos mx}{(x^2+1)} dx$

# NUMERICAL TECHNIQUES

1. Discuss the Rate of convergence for the Newton-Raphson method.
2. Use Newton-Raphson method to find the real root (correct to four decimal places) of the following equations:

   (a) $x^4 - x - 10 = 0$ Near $x = 2$ (b) $3x - \cos x - 1 = 0$
3. Use by Newton-Raphon formula to find $\sqrt[3]{18}$ correct to two decimals, assuming 2.5 as the initial approximation.
4. Discuss the Rate of convergence for the Newton Raphson method and Regula-Falsi method.
5. Use Regula-Falsi method to find the real root (correct to four decimal places) of the following equations: (a) $x^3 - 5x - 7 = 0, (2,3)$ (b) $xe^x = \cos x, (0,1)$.
6. Use Newton iterative method to find the real root (correct to four decimal places) of the following equations: (a) $x \log_{10} x = 1.2$ (b) $(48)^{1/3}$.
7. Prove that the following two sequences, both has convergence of the second order with the same limit $\sqrt{a}$, $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right) and\ x_{n+1} = \frac{1}{2} x_n \left(3 - \frac{a}{x_n^2}\right)$.
8. Define the shift operator, forward and backward difference operators, the central difference operator and the average operator. Establish

   $$\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = 2\left(1 + \frac{1}{2}\Delta\right)(1+\Delta)^{\frac{1}{2}}$$

   $$\Delta(1+\Delta)^{-\frac{1}{2}} = \nabla(1-\nabla)^{-\frac{1}{2}}$$ where, all the above notations have usual meanings.

   $$\mu = \sqrt{1 + \frac{1}{4}\delta^2}$$
9. Prove that

   (a) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

   (b) $\Delta^2 = h^2 D^2 - h^3 D^3 + \frac{7}{12} h^4 D^4 + ..........$

   (c) $hD = \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} ..........$ , provided $|\Delta| < 1$.

10. Prove that $\left(\frac{\Delta^2}{E}\right) e^x . \frac{Ee^x}{\Delta^2 e^x} = e^x$, the difference of interval being $h$.
11. Evaluate $\Delta\left(\frac{2^x}{(x+1)!}\right)$.
12. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46.

| Age | 45 | 50 | 55 | 60 | 65 |
|---|---|---|---|---|---|
| Premium(Rs.) | 114.84 | 96.16 | 83.32 | 74.48 | 68.48 |

13. Estimate the population for 1964 and 1966 from the following data:

| $x$ | 1961 | 1962 | 1963 | 1964 | 1965 |
|---|---|---|---|---|---|
| $y$ | 200 | …… | 260 | …… | 350 |

14. The Population of a city was as given. Estimate the population for the year 1925.

| Year | 1891 | 1901 | 1911 | 1921 | 1931 |
|---|---|---|---|---|---|
| Population (in thousand) | 46 | 66 | 81 | 93 | 101 |

15. Develop the divided-difference table from the data given below and obtain the interpolation polynomial $f(x)$:

| $x$ | 1 | 3 | 5 | 7 | 11 |
|---|---|---|---|---|---|
| $f(x)$ | 5 | 11 | 17 | 23 | 29 |

Also, find the value of $f(19.5)$.

16. Apply Gauss-Seidal method to find the solution of

$(i)\, 20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$

$(ii)\, 4x+2y+13z=24, 3x+9y-2z=11, 4x-4y+3z=-8$

17. Use the Crout's method to solve the following system:

$(i)\, x+y+z=6, x+2y+3z=14, x-2y+3z=6$

$(ii)\, x+y+z=1, 3x+y-3z=5, x-2y-5z=10$

18. The table given below reveals the velocity '$v$' of a body during the time t specified. Find its acceleration at t=1.1 :

| $t$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|---|
| $v$ | 43.1 | 47.7 | 52.1 | 56.4 | 60.8 |

19. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by taking $h=\frac{1}{6}$, using (i) Simpson's $\frac{1}{3}$ rule (ii) Trapezoidal rule and compare with the exact result.

20. A motorbike starts from rest , its velocity $v$ in km/hour is given in time $t$ as :

| $t$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $v$ | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

Estimate the approximate distance covered by motorbike in 20 minutes.

21. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and given in the following table. Using Simpson's $\frac{1}{3}$ rule finds the velocity of the rocket at $t = 80$ seconds.

| $t$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| $f(cm/\sec^2)$ | 30 | 31.63 | 33.34 | 35.47 | 37.75 | 40.33 | 43.25 | 46.69 | 50.67 |

22. A river is 80m wide. The depth $d$ of the river at a distance $x$ from one bank is given by the table:

| $x$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| $d$ | 0 | 4 | 7 | 9 | 12 | 15 | 14 | 8 | 3 |

Find approximately the area of cross-section of the river using Simpson's $\frac{1}{3}$ rule.

23. Using Picard's method of successive approximation $\frac{dy}{dx} = \frac{x^2}{y^2+1}, \quad y(0) = 0$. Obtain $y(0.25), y(0.5) \,\&\, y(1)$ correct to three decimal places.

24. Apply Picard's method to find the third approximation of the solution of $\frac{dy}{dx} = x + y^2, \quad y(0) = 1$

25. Use R-K method of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y},\ y(0.4) = 0.41$. Assume the step length (0.2).

# LINEAR ALGEBRA

1. Show that the vectors $X_1$ =(1,2,3), $X_2$=(3,-1,4) and $X_3$=(4,1,7) are linearly dependent.
2. Examine the following vectors for linear dependence and find the relation if it exists.
   $X_1$=(1,2,4) , $X_2$= (2,-1,3), $X_3$=(0,1,2), $X_4$=(-3,7,2).
3. Examine for linear dependence and find the relation if possible.
   X1= (1,0,2,1), X2= (3,1,2,1), X3=(4,6,2,-4) and (-6,0,-3,-4).
4. Find whether the vectors are linearly dependent or independent.
   $V_1$=(1,2,1), $V_2$=(3,1,5), $V_3$=(3,-4,7).
5. Prove that the set (1,x,1+x+$x^2$) is linearly independent set of the vectors in the vector space of all polynomials over the real number field.
6. Solve if the vector (2,-5,3) in the subspace of R3 spanned by the vectors (1,-3,2), (2,-4,1), (1,-5,7).
7. Let R be the field of real numbers. Which of the following are the subspaces of $V_{3(}$R) ?
   (I) $W_1 = \{ (x, 2y, 3z) : x,y,z \in R\}$
   (II) $W_2 = \{ (x, x, x) : x \in R\}$
   (III) $W_3 = \{ (x, y, z) : x,y,z \in R\}$
8. If V is a set of all (n x n) matrices over any field F, then a set w of all (n xn ) symmetric matrices forms a vector subspace of V(F).
9. Let V be a vector space of all real valued funtions over R. Then show that the solution set W of the differential equation where y =f(x) is a subspace of V.

$$2\frac{d^2y1}{dx^2} - 9\frac{dy}{dx} + 2y = 0$$

10. Prove that the intersection of two subspaces of a vector space is a subspace of the same but union of two subspaces may not be a subspace.
11. Show that the vectors (1,0,0), (1,1,0) and (1,1,1) form a basis for $R^3$.
12. Show that the set S= {(1,0,0), (1,1,0), (1,1,1), (0,1,0)} is not a basis set.
13. Show that the vectors form a basis of $V_3$(F) : { (1,2,1). (2,1,0), (1,-1,2)}
14. Prove that the vectors $X_1$=(1,0,-1), $X_2$= (1,2,1) , $X_3$ = (0,-3,2) forms a basis of $V_3$(R).
15. Show that the following vectors form a basis of $R_3$. Express each of the standard basis vectors $e_i$ , i =1,2,3 as linear combination of the above basis vectors.
    S= {(1,2,1), (2,1,0), (1,-1,2)}.
16. Find the coordinate vector V= (3,-5,2) relative to the basis of $e_1$ =(1,1,1), $e_2$= (0,2,3), $e_3$= (0,2,-1).
17. If $V_3$(R) is a vector space then show that S={ (0,1,-1), (1,1,0), (1,0,2)} is a basis of $V_3$ and hence find the coordinates of the vector (1,0,-1) with respect to the basis.
18. Determine the null space for the following matrix : $\begin{bmatrix} -3 & 0 \\ 2 & -4 \end{bmatrix}$.
19. Determine the basis and null space for the following matrix :
    $$\begin{bmatrix} 1 & 2 & -3 & 2 & -3 \\ 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 2 & 5 & -6 & -3 \end{bmatrix}.$$

20. Determine a basis for the null space , row space, column space and rank of the matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 3 & 4 \\ 2 & 3 & 2 & 6 & 8 \\ 4 & 7 & 4 & 12 & 16 \\ 5 & 11 & 6 & 15 & 20 \end{bmatrix}.$$

21. Define inner product spaces, orthogonal and orthonormal vectors.
22. Let $X_1$ = (1,2,1), $X_2$= (2,1,4), $X_3$= (3,-2,-1) in R3 then,
    (i) Show that they form an orthogonal set under the standard Euclidean inner product for $R^3$ but not orthonormal set.
    (ii) Convert them into the set of vectors that will form an orthonormal set of vectors under the standard Euclidean inner product for $R^3$.
23. State and prove Schwarz Inequality.
24. Construct orthonormal set of vectors form the set :
    $X_1$ =(1,2,1), $X_2$=(2,1,4), $X_3$= (4,5,6).
25. Let $R^3$ have the Euclidean inner product . Use the Gram Schmidt process to transform the basis vector $u_1$= (1,1,1), $u_2$ =(-1,1,0), $u_3$= (1,2,1) into orthogonal basis $\{v_1,v_2,v_3\}$.
26. Obtain the orthogonal basis for $P_2$, the space of all real polynomials of degree atmost 2, the inner product being defined by (P1,P2)= $\int_0^1 P1(t)P2(t)dt$.
27. Orthonormalise the set of linearly independent vectors X1 =(1,0,1,1), X2= (-1,0,-1,1), X3= (0,-1,1,1) of $R^4$ with the standard inner product.
28. If p =p(x) =$p_0$ +$p_1$x +$p_2x^2$ and q=q(x) =$q_0$ +$q_1$x+$q_2x^2$ , the inner product is defined by (p,q) =$p_0q_0+p_1q_1+p_2q_2$ for the vectors
    $X_1$= 1+2x+3$x^2$, $X_2$ =3+5x+5$x^2$, $X_3$=2+x+8$x^2$.
    Find the orthonormal vectors.