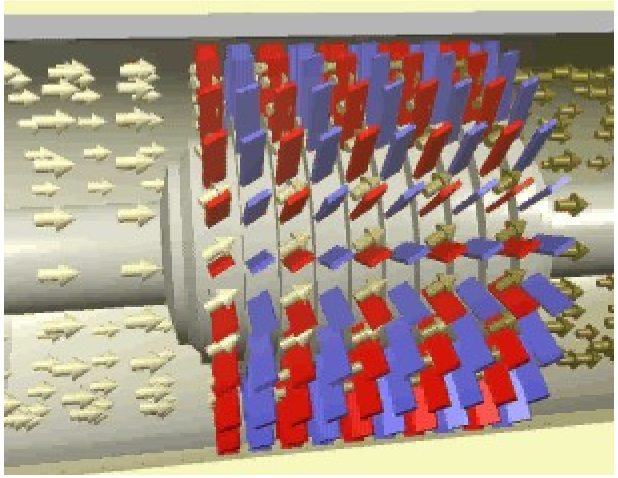
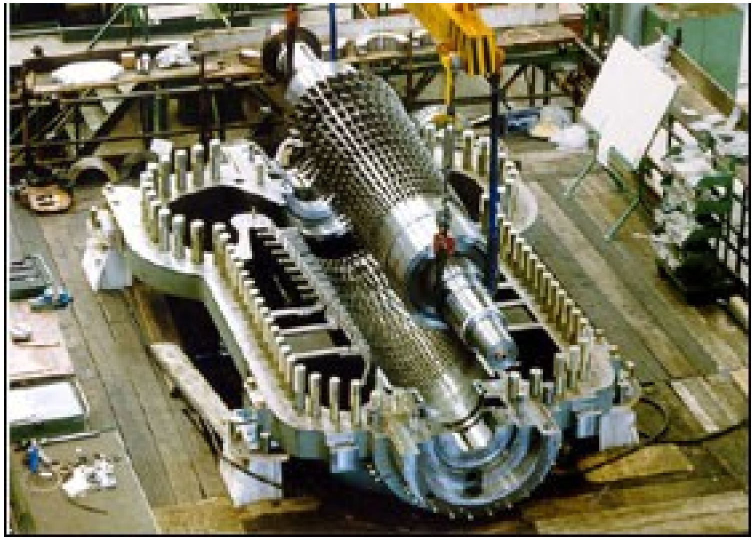
THERMAL AND HYDRAULIC MACHINES

UNIT 3



Compressors

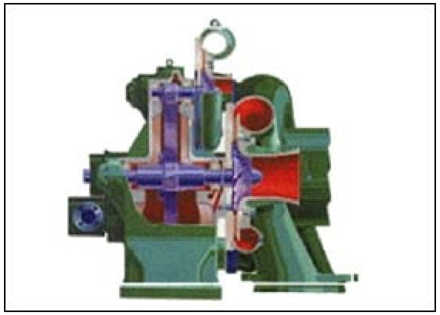


Types of Compressors

Gasses can be compressed in the following ways:

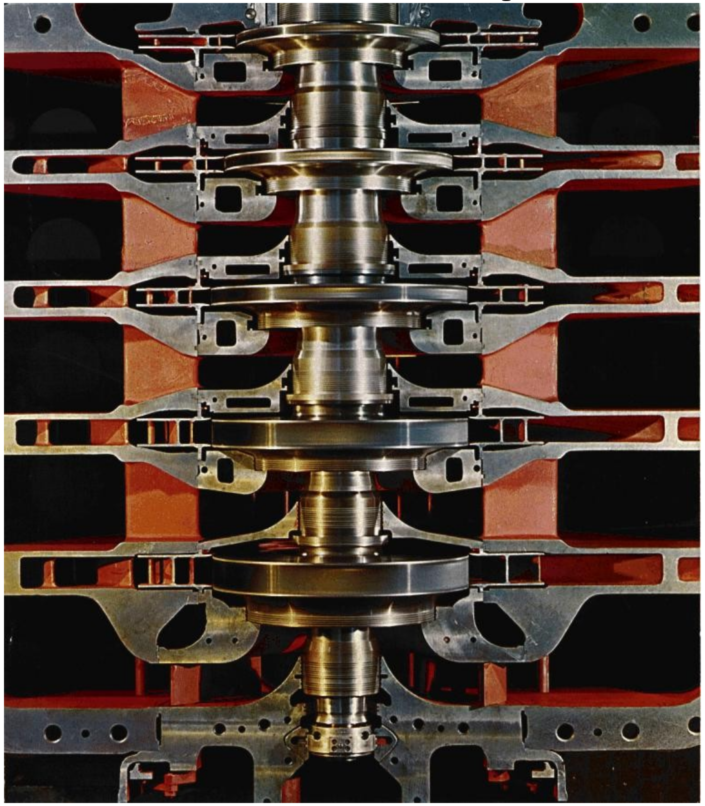
- Reciprocating piston compressors
    - Low flow rates
    - High compression ratios
- Rotating centrifugal compressors
    - High flow rates
    - Low compression ratios
    - Several centrifugal stages may be used to obtain higher compression ratios

Types of Compressors



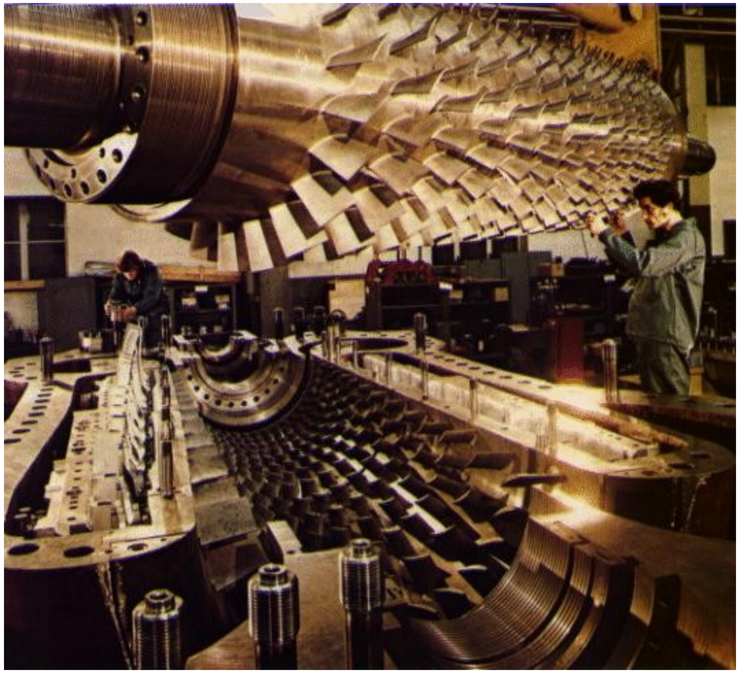
Centrifugal Compressors

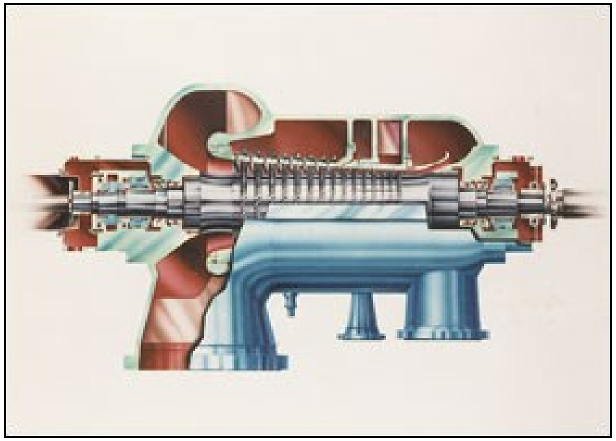
Types of Compressors



Radial Flow Compressors (multi-stage)

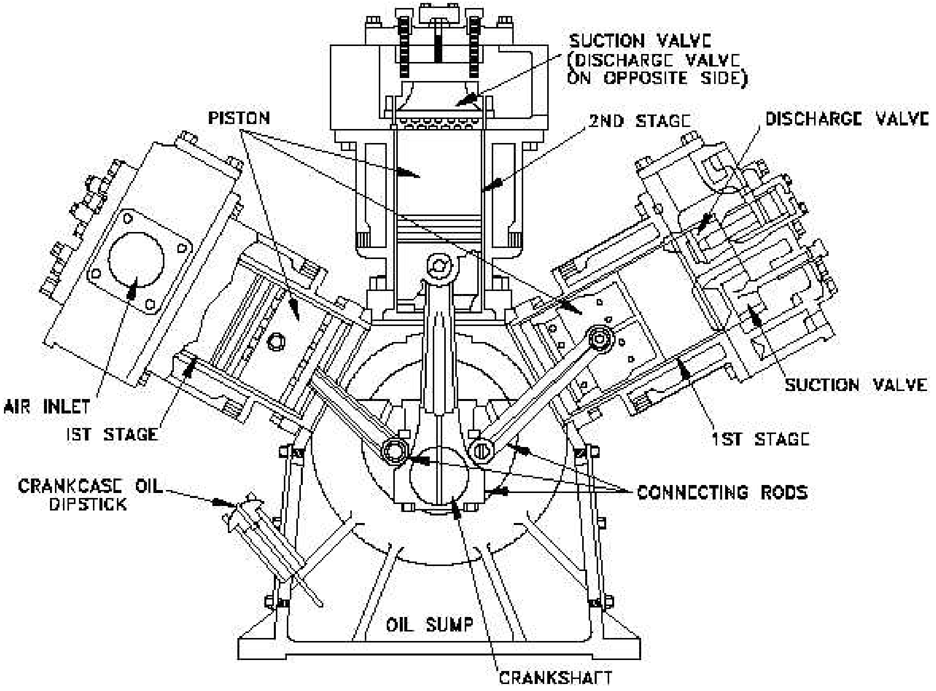
Types of Compressors





Axial Compressor

Types of Compressors



Reciprocating Compressor

Typical Compressor Installation

Compressor Design Equations

Mechanical Energy Balance

$$\hat{W} = \Delta\left(\frac{\alpha u^2}{2} + gZ\right) + \int_{p_1}^{p_2}\left(\frac{dp}{\rho}\right) + h_f$$

What is the unexpected assumption?

Viscous dissipation is negligible!

h_f will be accounted for via the efficiency of the compressor

Mechanical Energy Balance

$$\hat{W} = \int_{p_1}^{p_2} \left(\frac{dp}{\rho} \right)$$

$\hat{W}$ is the work done on the fluid by the compressor

ρ must remain inside integral since ρ changes with p.

Total Energy Balance

$$\Delta\left(\frac{\alpha u^2}{2} + gZ + H\right) = \frac{Q}{\dot{m}} + \hat{W}_c$$

adiabatic compression

$$\hat{W}_c = \Delta H = C_p\left(T_2 - T_1\right)$$

Note that $\hat{W}_c$ in TEB includes efficiency while $\hat{W}$ in MEB does not include efficiency

Isentropic Work of Compression

As a first approximation, a compressor without any internal cooling can be assumed to be adiabatic. If the process is also assumed to be reversible, it will be isentropic.

$$p/\rho^{\gamma} = p_1/\rho_1^{\gamma} = const.$$

Solve for ρ, substitute into MEB, and integrate

Isentropic Work of Compression

Upon Integration

$$\hat{W}_{\Delta S=0} = \frac{p_1 \gamma}{\rho_1 (\gamma - 1)} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$

This is the isentropic (adiabatic) work of compression.

The quantity p_2/p_1 is the compression ratio.

Compressor Work

Compression is not reversible, however. Deviations from ideal behavior must be accounted for by introducing an isentropic compressor efficiency such that the true work of compression is given by.

$$\hat{W}_c = \frac{\hat{W}_{\Delta S=0}}{\eta_{ad}}$$

How can η_{ad} be found?

$$\hat{W}_c = C_p\left(T_2 - T_1\right)$$

Isothermal Compression

If sufficient cooling is provided to make the compression process isothermal, the work of compression is simply:

$$\hat{W}_{\Delta T=0} = \frac{RT_1}{M} \ln \frac{p_2}{p_1}$$

For a given compression ratio and suction condition, the work requirement in isothermal compression is less than that for adiabatic compression. This is one reason that cooling is useful in compressors.

Polytropic Compression

In actuality the $\Delta S = 0$ path assumed in writing the expression $p/\rho^{\gamma} = $ const. is not the true thermodynamic path followed by gases in most large compressors and the compression is neither adiabatic not isothermal. A polytropic path is a better representation for which:

$$p/\rho^{n} = p_1/\rho_1^{n} = const.$$

Here n depends on the nature of the gas and details of the compression process.

Polytropic Compression

$$\hat{W}_p = \frac{p_1 n}{\rho_1 (n-1)} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

where $\hat{W}_p$ is the work for polytropic compression

Again the actual work of compression is larger than the calculated work and:

$$\hat{W}_c = \frac{\hat{W}_p}{\eta_p}$$

Polytropic Compression

The polytropic efficiency η_p is often the efficiency quoted by manufacturers. From this efficiency useful relations can be stated to convert from polytropic to adiabatic results:

To get n the polytropic exponent:

$$n = \frac{\eta_p \gamma}{1 + \eta_p \gamma - \gamma} \quad \text{or} \quad \frac{n}{n-1} = \frac{\gamma}{\gamma - 1} \eta_p$$

To get relationships between T or ρ and compression ratio simply replace γ with n.

$$e.g. \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

A “REAL” Impact of Efficiency

Multistage Compression

Consider a two stage compression process p1→p2→p3 with perfect intercooling (temperature reduced to T_1 after each compression)

$$\hat{W}_{\Delta S=0} = \frac{\gamma R T_1}{\gamma - 1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + \frac{\gamma R T_1}{\gamma - 1}\left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Now find p_2 which will minimize work, differentiate wrt p_2

$$p_2^{opt} = \sqrt{p_1 p_3}$$

Multistage Compression

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \left(\frac{p_3}{p_1}\right)^{1/2}$$

So the compression ratio that minimizes total work is such that each stage has an identical ratio.

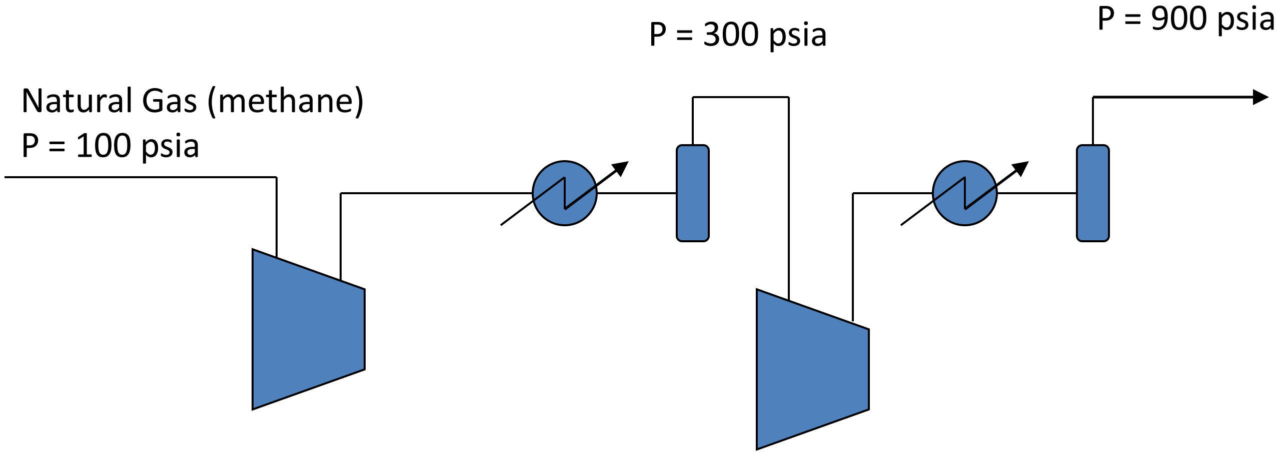
This can be generalized for n stages as:

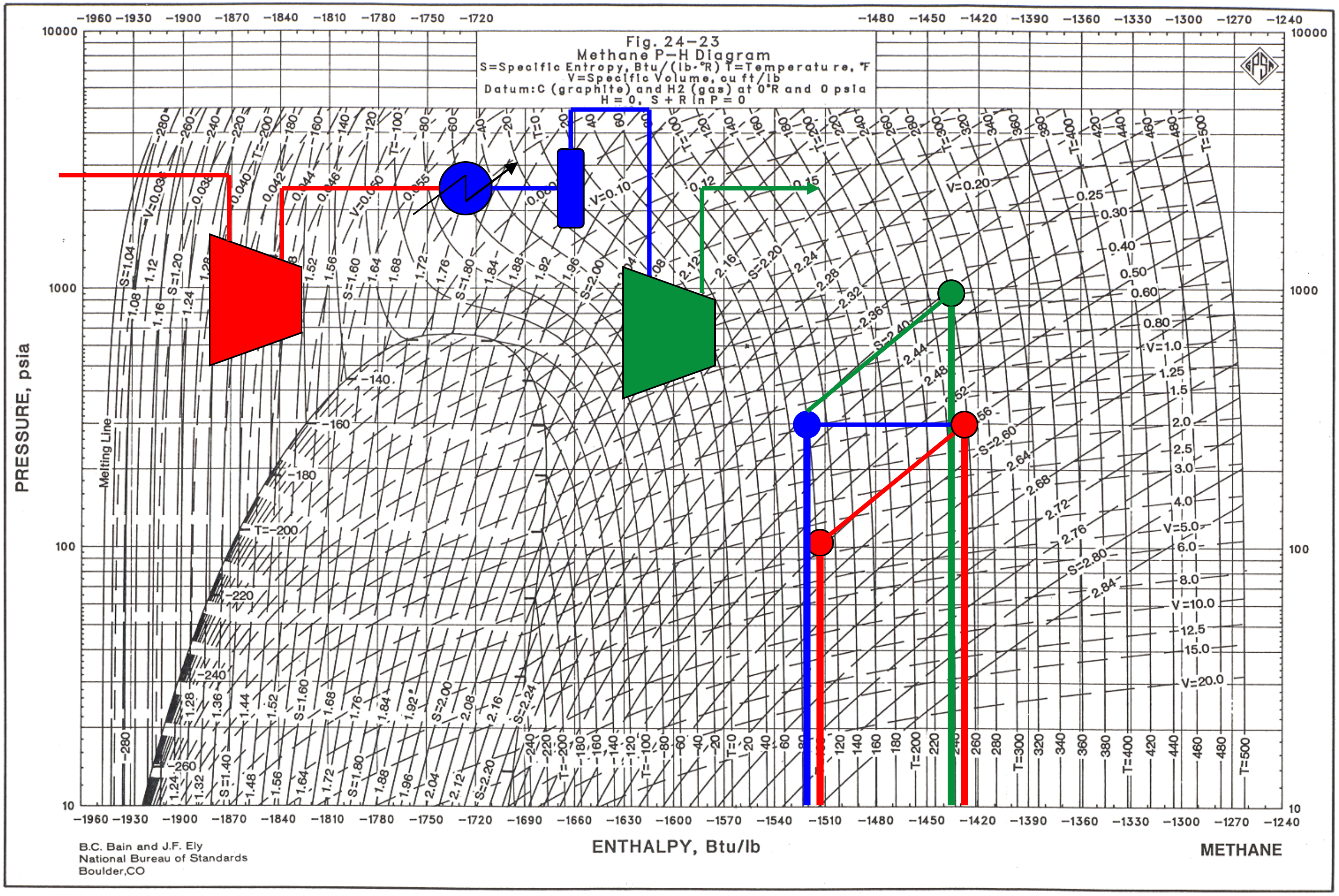
$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \cdots = \left(\frac{p_{n+1}}{p_1}\right)^{1/n}$$

When T is not cooled to T_1: $$T_i \left(\frac{p_{i+1}}{p_i}\right)^{\frac{\gamma-1}{\gamma}} = const. \qquad T_i \uparrow \quad \frac{p_{i+1}}{p_i} \downarrow$$

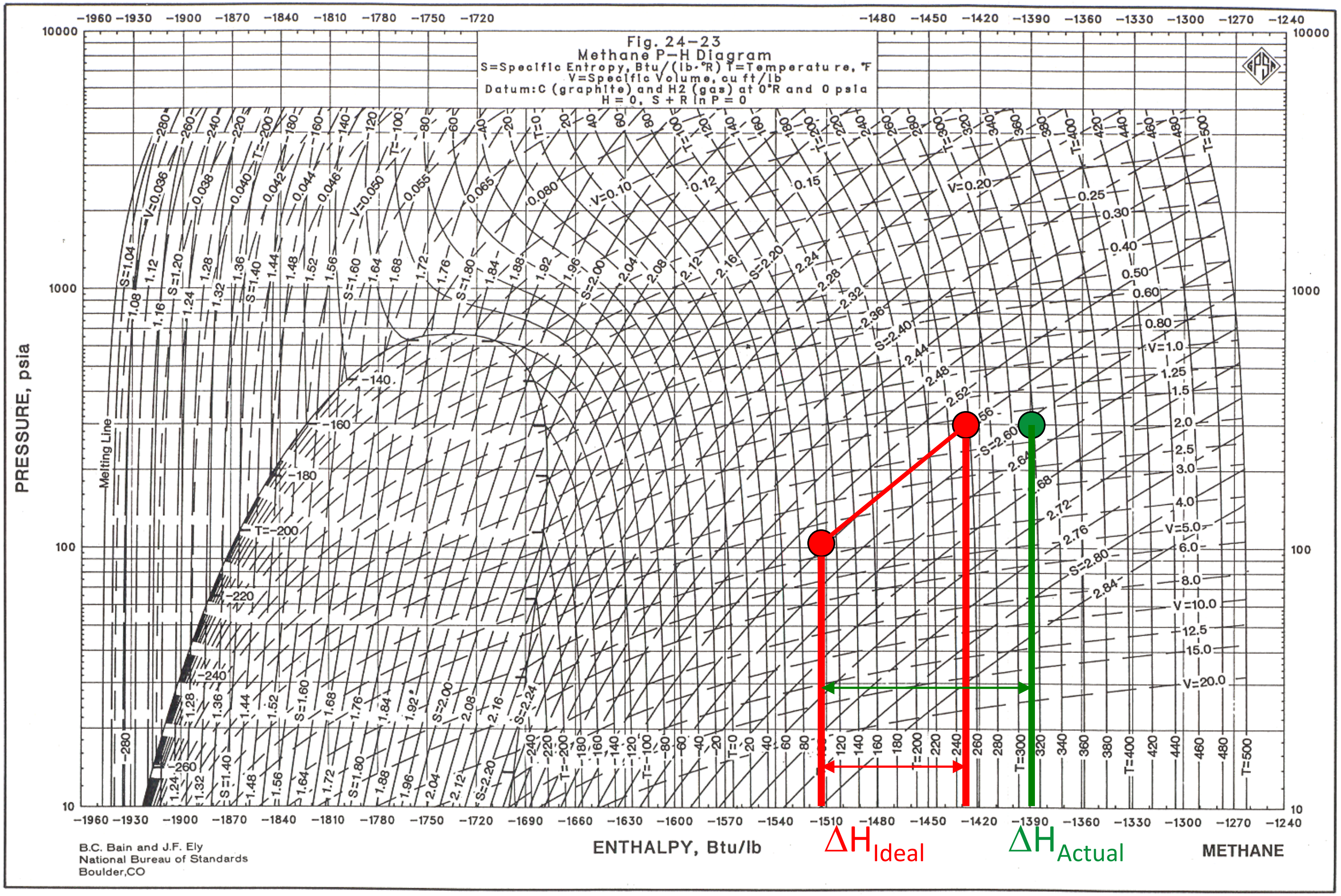
Multistage Compression

P-H Chart Method Example



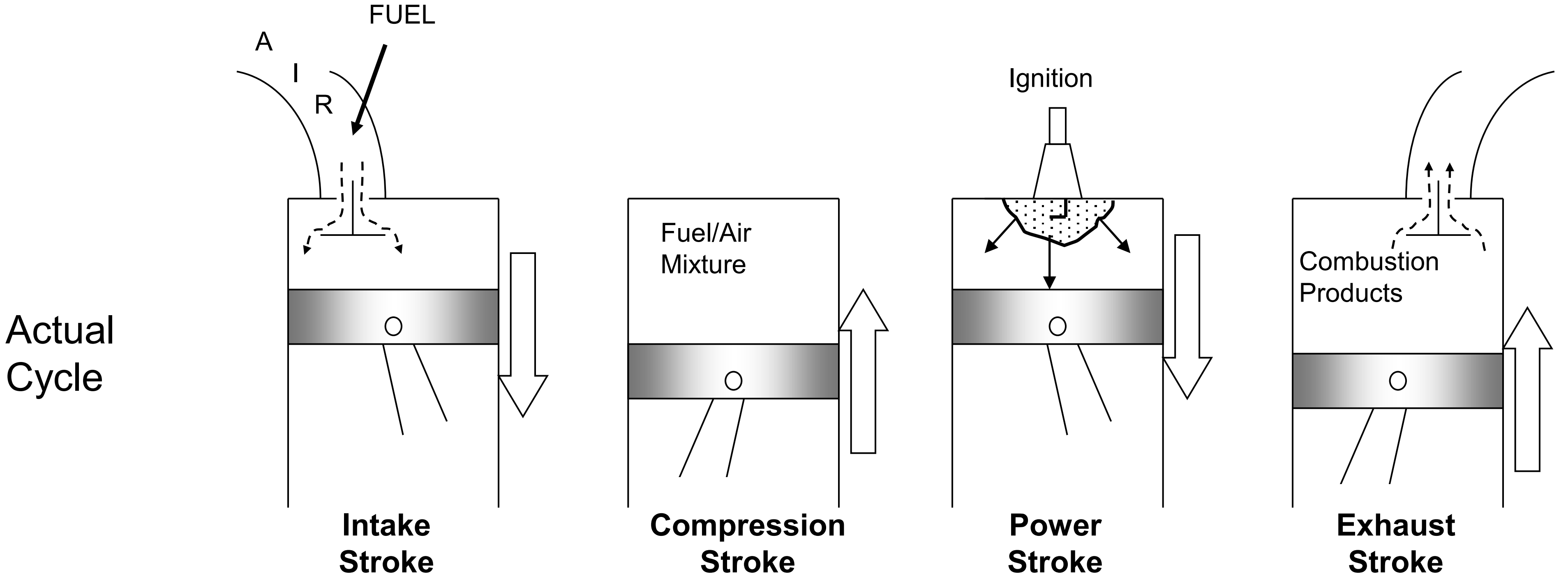


Isentropic Compression



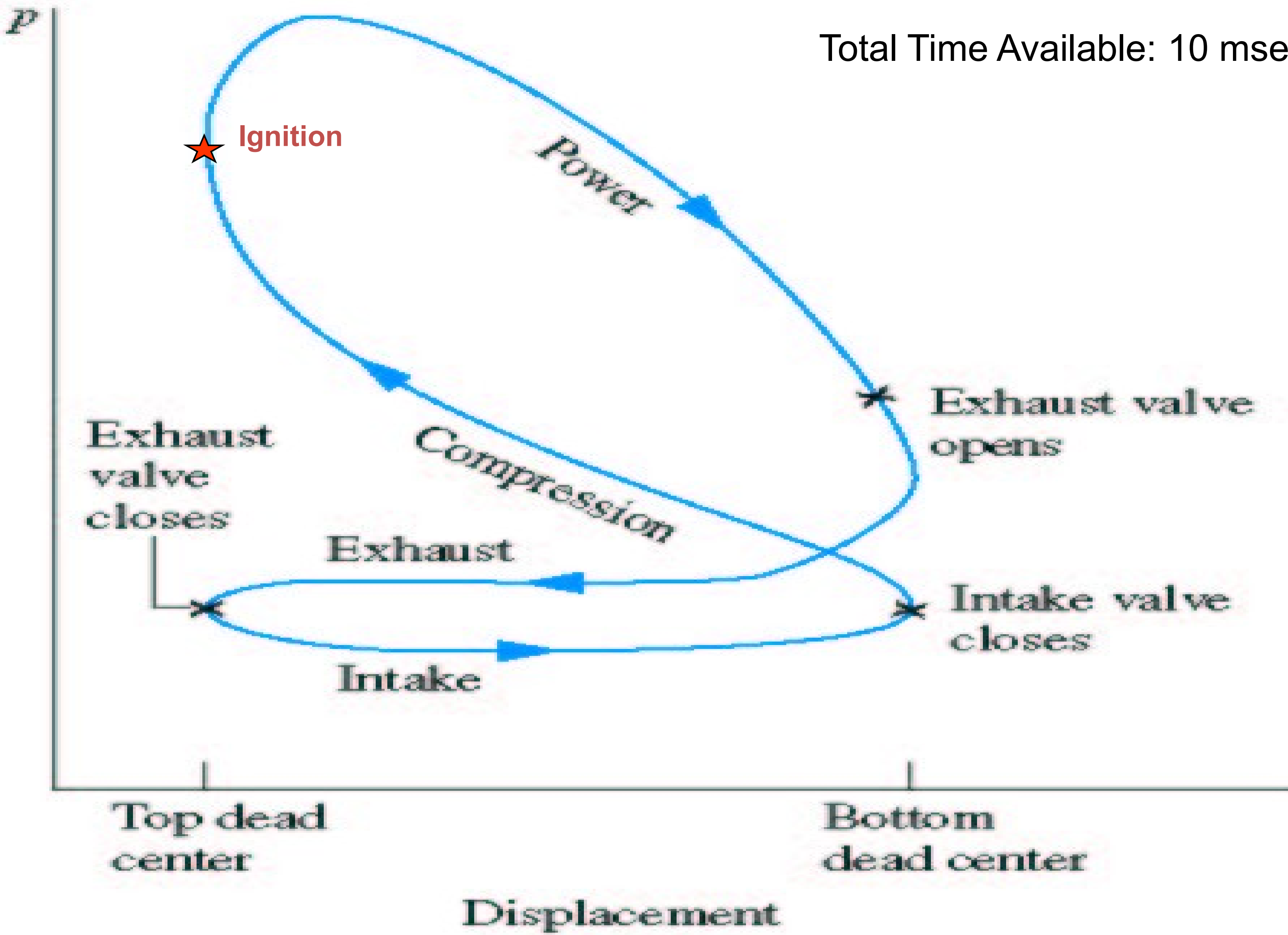
“Real” Compression

SI Engine Cycle

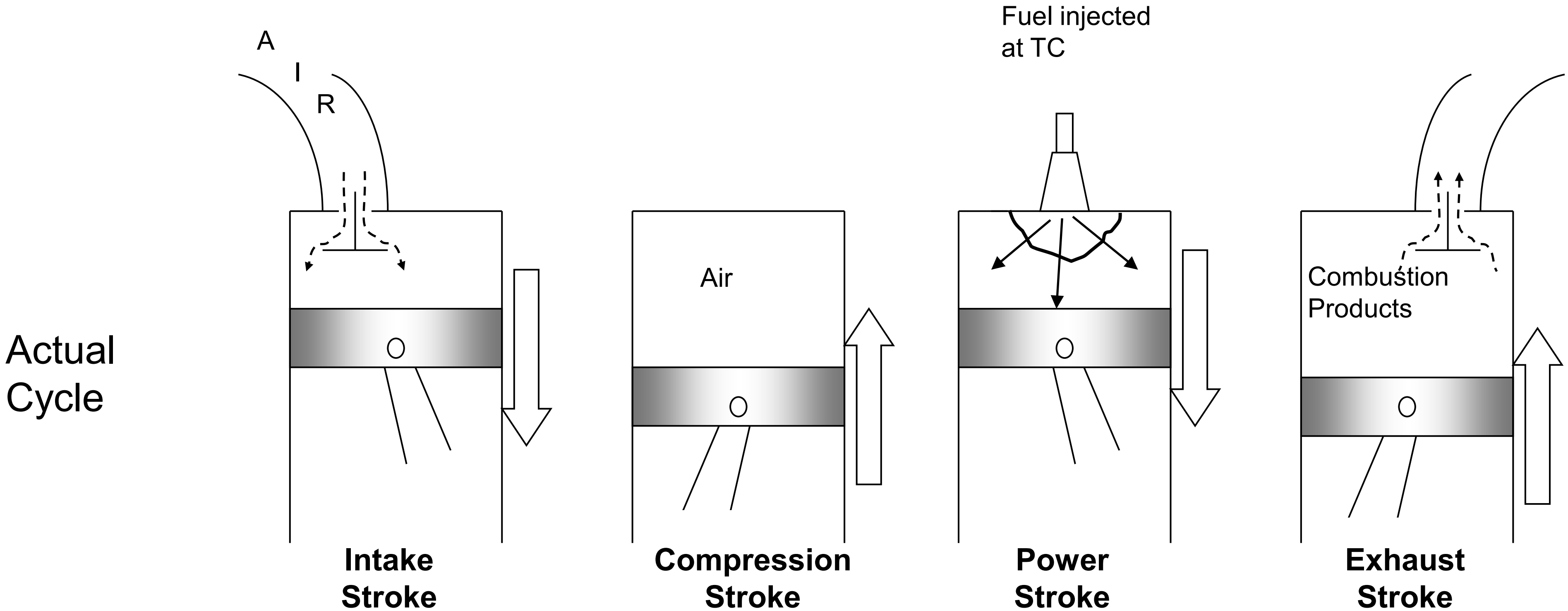


Actual SI Engine cycle

Total Time Available: 10 msec

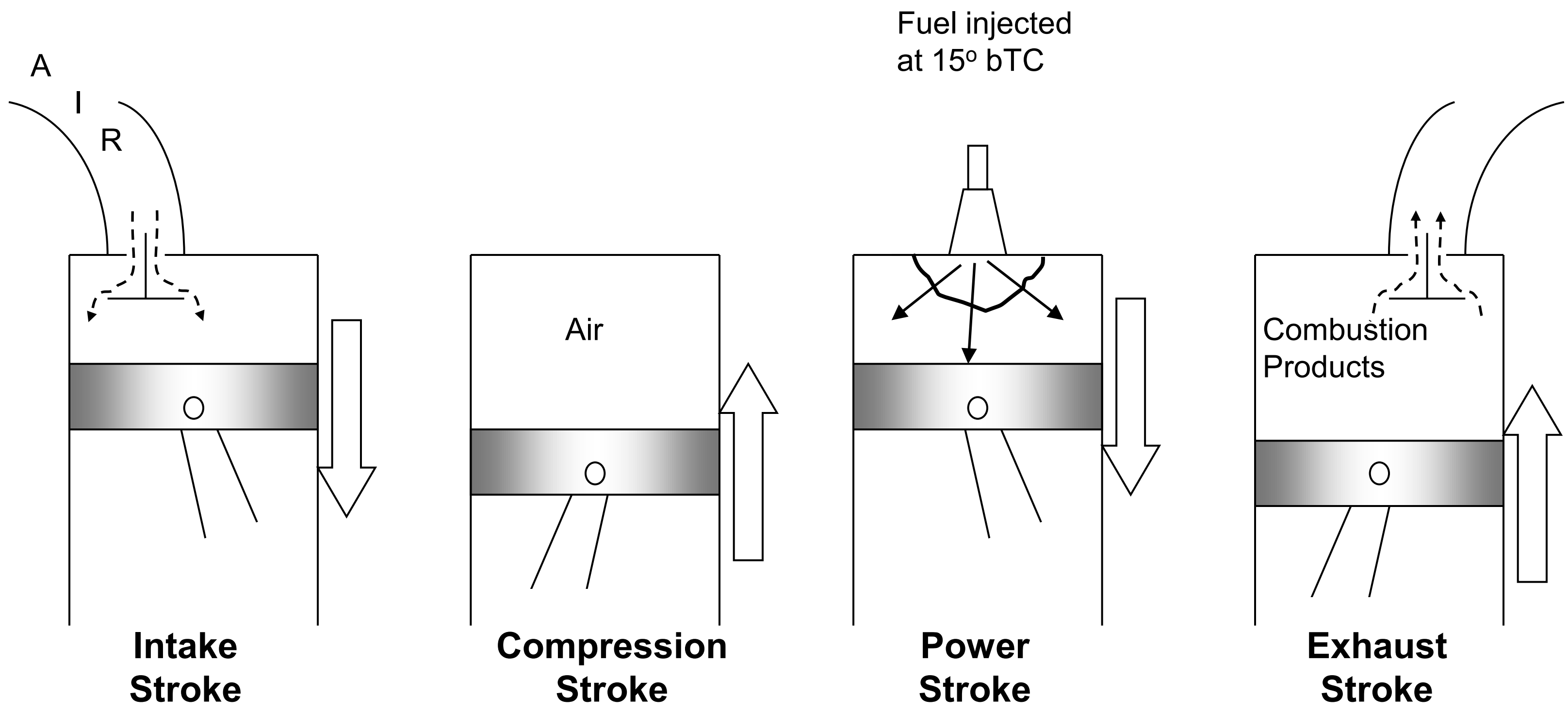


Early CI Engine Cycle



Modern CI Engine Cycle

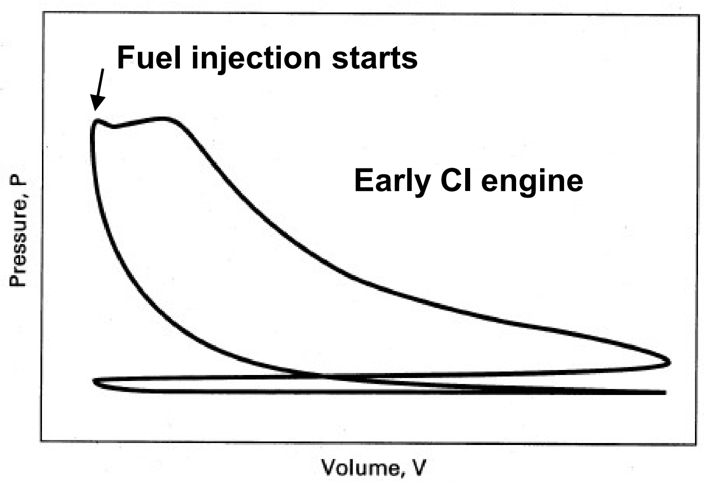
Actual Cycle

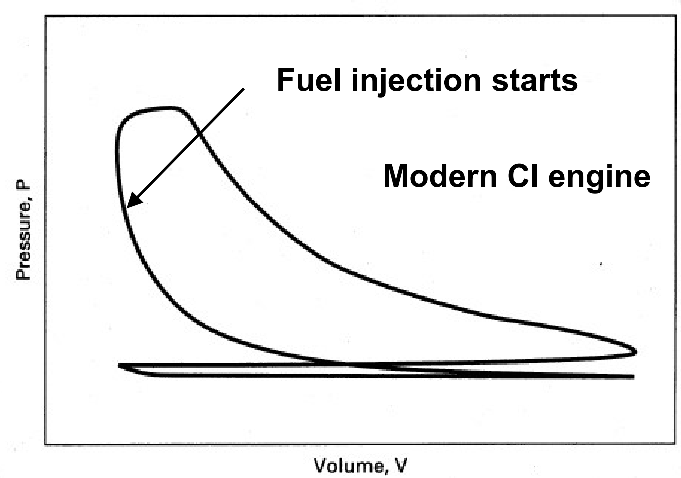


Thermodynamic Cycles for CI engines

• In early CI engines the fuel was injected when the piston reached TC and thus combustion lasted well into the expansion stroke.

• In modern engines the fuel is injected before TC (about 15^o)





• The combustion process in the early CI engines is best approximated by a constant pressure heat addition process → **Diesel Cycle**

• The combustion process in the modern CI engines is best approximated by a combination of constant volume and constant pressure → **Dual Cycle**

Thermodynamic Modeling

- The thermal operation of an IC engine is a transient cyclic process.
- Even at constant load and speed, the value of thermodynamic parameters at any location vary with time.
- Each event may get repeated again and again.
- So, an IC engine operation is a transient process which gets completed in a known or required Cycle time.
- Higher the speed of the engine, lower will be the Cycle time.
- Modeling of IC engine process can be carried out in many ways.
- Multidimensional, Transient Flow and heat transfer Model.
- Thermodynamic Transient Model USUF.
- Fuel-air Thermodynamic Mode.
- Air standard Thermodynamic Model.

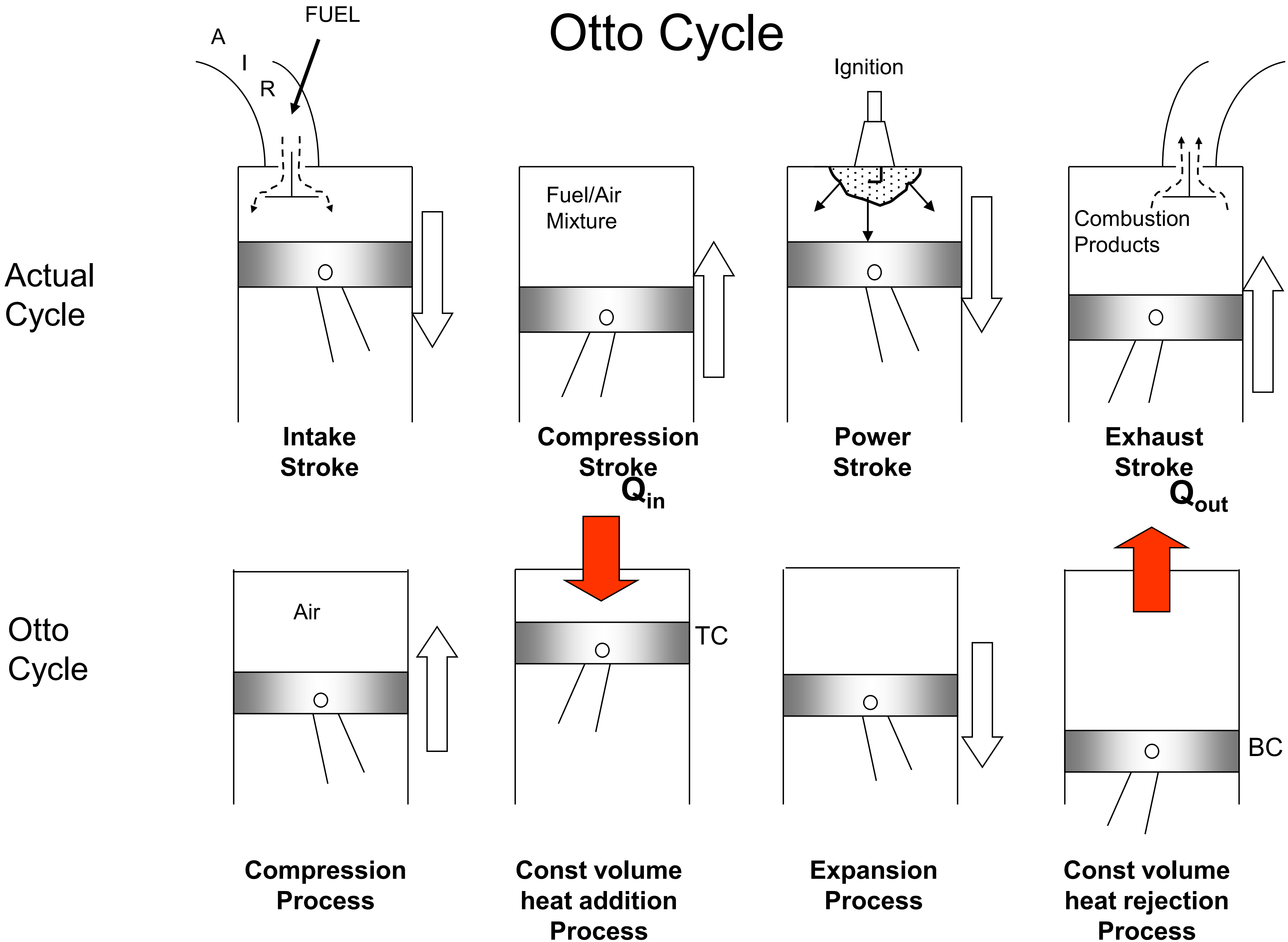
Ideal Thermodynamic Cycles

- **Air-standard analysis** is used to perform elementary analyses of IC engine cycles.

- Simplifications to the real cycle include:
  1) Fixed amount of air (ideal gas) for working fluid
  2) Combustion process not considered
  3) Intake and exhaust processes not considered
  4) Engine friction and heat losses not considered
  5) Specific heats independent of temperature

- The two types of reciprocating engine cycles analyzed are:
  1) Spark ignition – Otto cycle
  2) Compression ignition – Diesel cycle

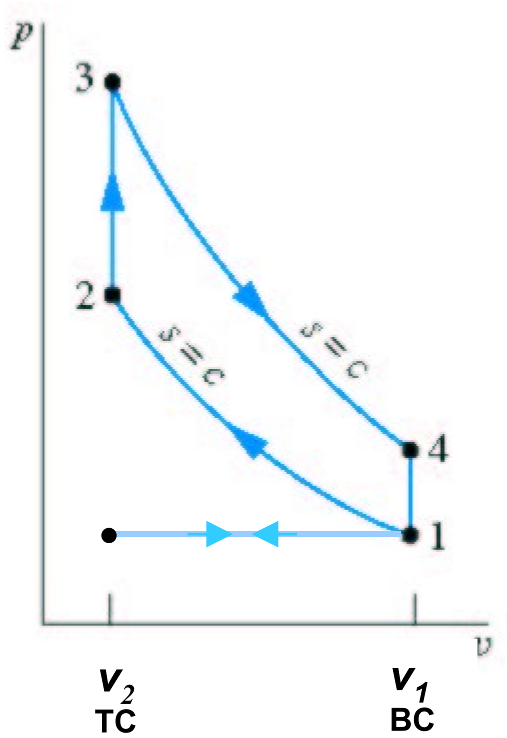
Otto Cycle

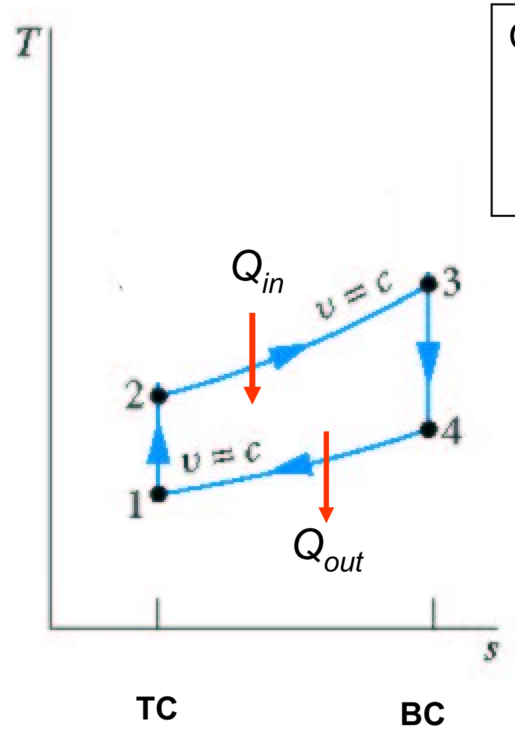


FUEL
A
I
R
Ignition
Actual Cycle
Fuel/Air Mixture
Combustion Products
Intake Stroke
Compression Stroke
Power Stroke
Exhaust Stroke
Q_{in}
Q_{out}
Otto Cycle
Air
TC
BC
Compression Process
Const volume heat addition Process
Expansion Process
Const volume heat rejection Process

Air-Standard Otto cycle

Process 1→ 2 Isentropic compression
Process 2 → 3 Constant volume heat addition
Process 3 → 4 Isentropic expansion
Process 4 → 1 Constant volume heat rejection



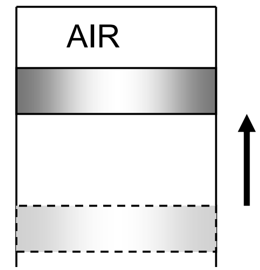


Compression ratio:

$$r = \frac{v_1}{v_2} = \frac{v_4}{v_3}$$

First Law Analysis of Otto Cycle

1→2 Isentropic Compression



$$(u_2 - u_1) = \frac{\cancel{Q}}{m} - (-\frac{W_{in}}{m})$$

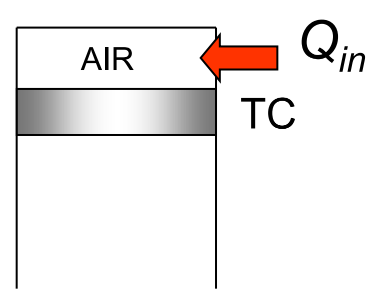
$$\frac{W_{in}}{m} = (u_2 - u_1) = c_v(T_2 - T_1)$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = r^{k-1}$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \cdot \frac{v_1}{v_2}$$

2→3 Constant Volume Heat Addition

$$(u_3 - u_2) = (+\frac{Q_{in}}{m}) - \frac{\cancel{W}}{m}$$

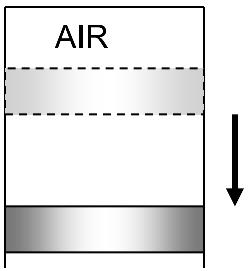


$$\frac{Q_{in}}{m} = (u_3 - u_2) = c_v(T_3 - T_2)$$

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$

3 → 4 Isentropic Expansion

$$(u_4 - u_3) = \frac{\cancel{Q}}{m} - (+\frac{W_{out}}{m})$$



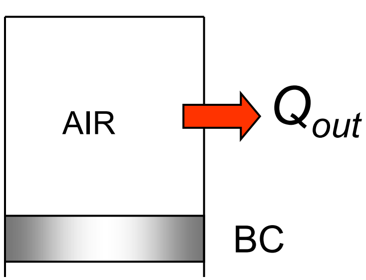
$$\boxed{\frac{W_{out}}{m} = (u_3 - u_4) = c_v (T_3 - T_4)}$$

$$\boxed{\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{1}{r^{k-1}}}$$

$$\boxed{\frac{P_4}{P_3} = \frac{T_4}{T_3} \cdot \frac{v_3}{v_4}}$$

4 → 1 Constant Volume Heat Removal

$$(u_1 - u_4) = (-\frac{Q_{out}}{m}) - \frac{\cancel{W}}{m}$$



$$\boxed{\frac{Q_{out}}{m} = (u_4 - u_1) = c_v (T_4 - T_1)}$$

$$\boxed{\frac{P_4}{T_4} = \frac{P_1}{T_1}}$$

First Law Analysis Parameters

Net cycle work:

$$W_{cycle} = W_{out} - W_{in} = m(u_3 - u_4) - m(u_2 - u_1)$$

Cycle thermal efficiency:

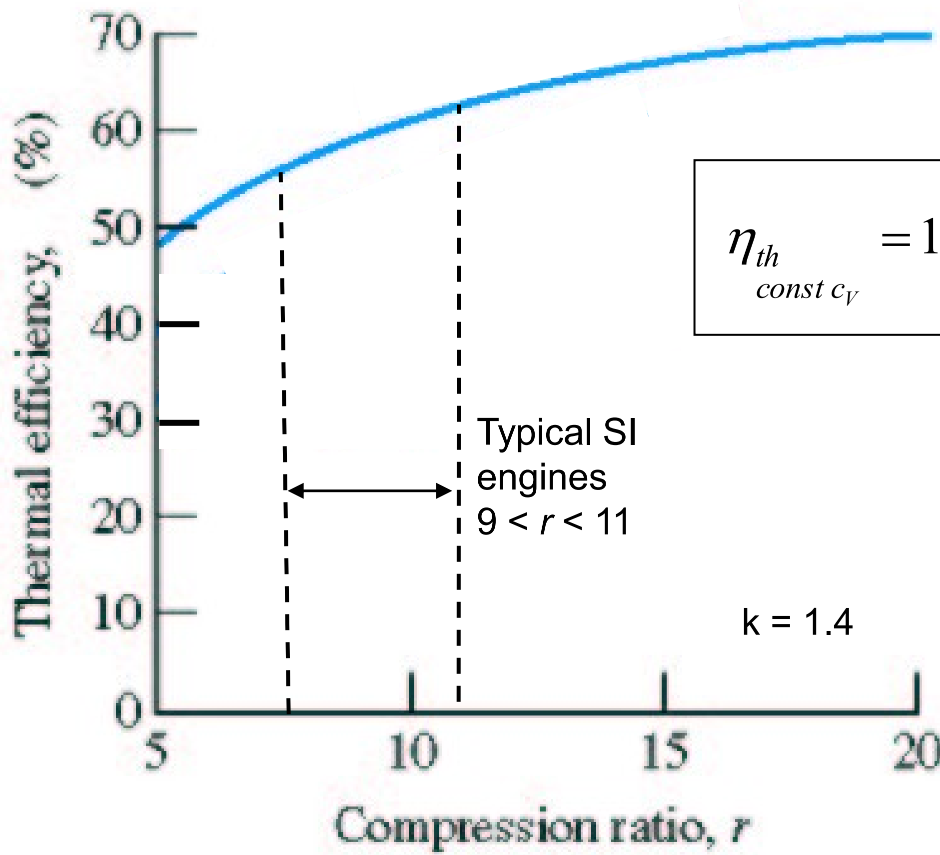
$$\eta_{th} = \frac{W_{cycle}}{Q_{in}} = \frac{(u_3 - u_4) - (u_2 - u_1)}{(u_3 - u_2)} = \frac{(u_3 - u_2) - (u_4 - u_1)}{u_3 - u_2} = 1 - \frac{u_4 - u_1}{u_3 - u_2}$$

$$= 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{T_1}{T_2} = \boxed{1 - \frac{1}{r^{k-1}}}$$

Indicated mean effective pressure is:

$$imep = \frac{W_{cycle}}{V_1 - V_2} \rightarrow \frac{imep}{P_1} = \frac{Q_{in}}{P_1 V_1}\left(\frac{r}{r-1}\right)\eta_{th} = \boxed{\frac{1}{k-1}\left(\frac{Q_{in}/m}{u_1}\right)\left(\frac{r}{r-1}\right)\eta_{th}}$$

Effect of Compression Ratio on Thermal Efficiency



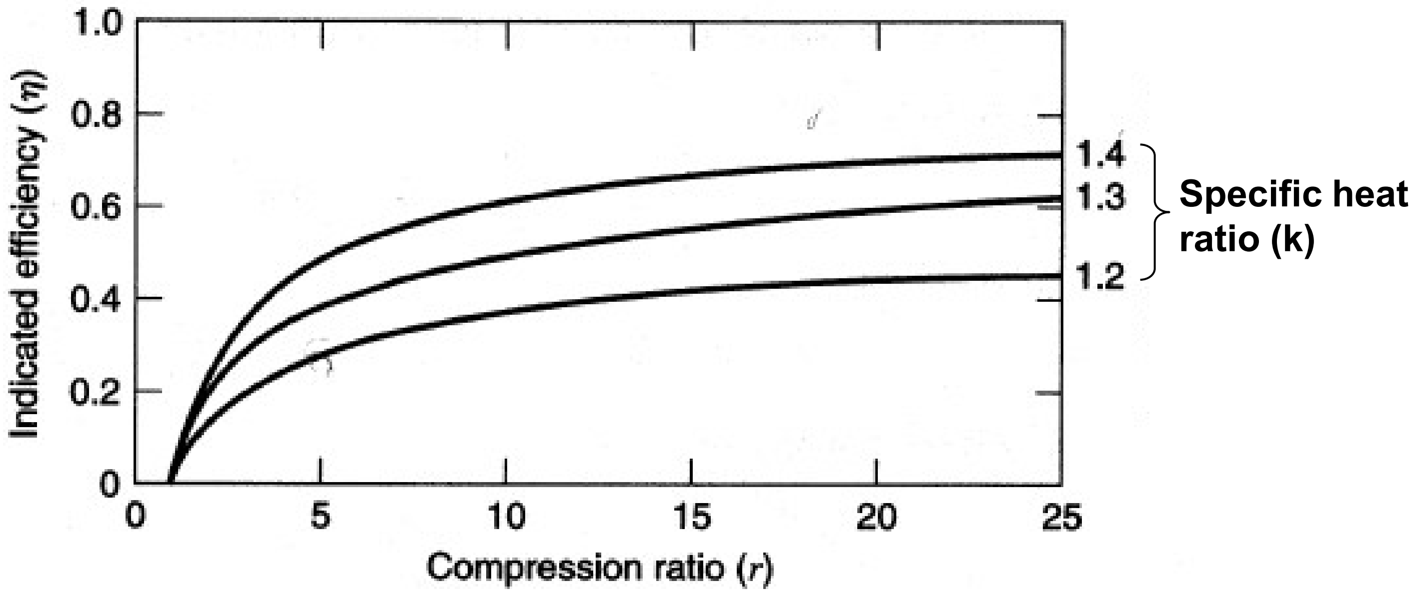
$$\eta_{th}\big|_{const\ c_V} = 1 - \frac{1}{r^{k-1}}$$

• Spark ignition engine compression ratio limited by T_3 (autoignition) and P_3 (material strength), both ~r^k

• For *r* = 8 the efficiency is 56% which is twice the actual indicated value

Effect of Specific Heat Ratio on Thermal Efficiency

$$\eta_{th_{const\, c_V}} = 1 - \frac{1}{r^{k-1}}$$



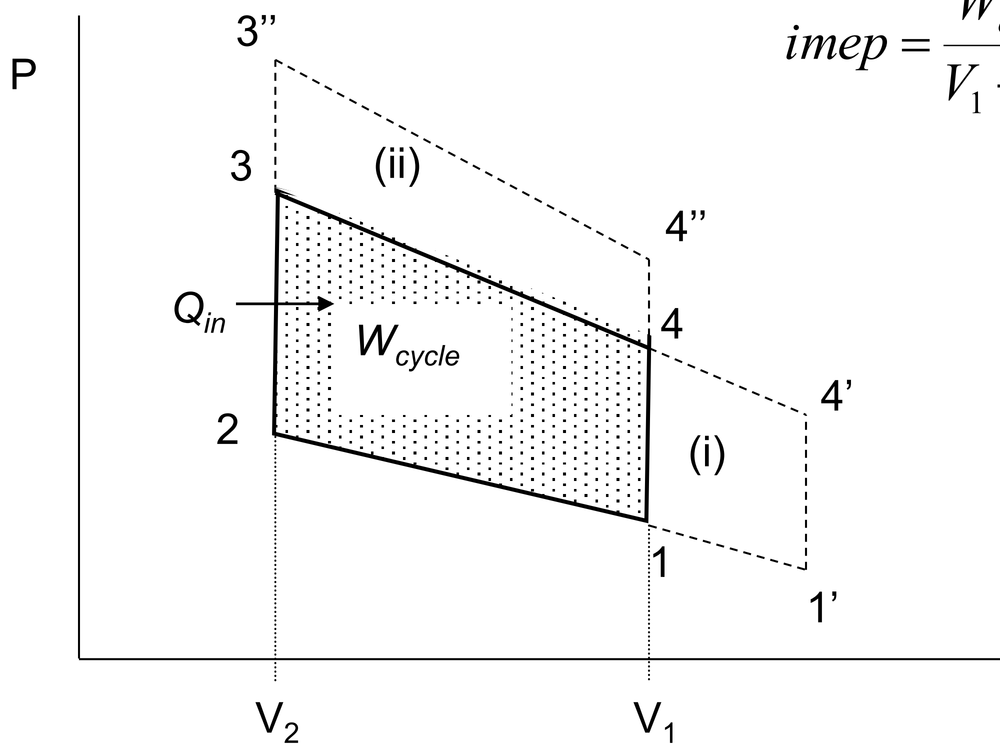
Cylinder temperatures vary between 20K and 2000K so 1.2 < k < 1.4
k = 1.3 most representative

Factors Affecting Work per Cycle

The net cycle work of an engine can be increased by either:

i) Increasing the r (1'→2)

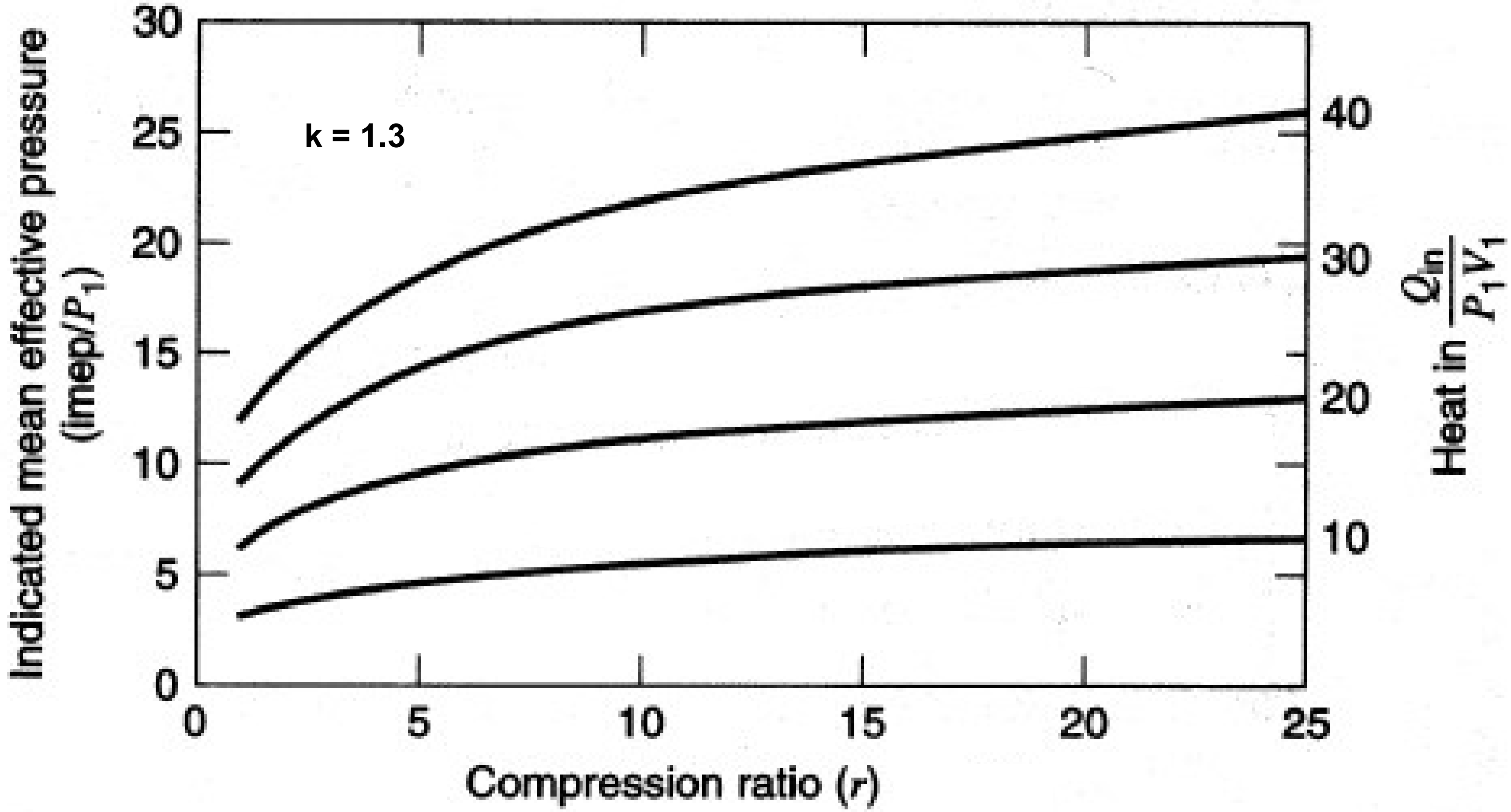
ii) Increase Q_{in} (2→3")



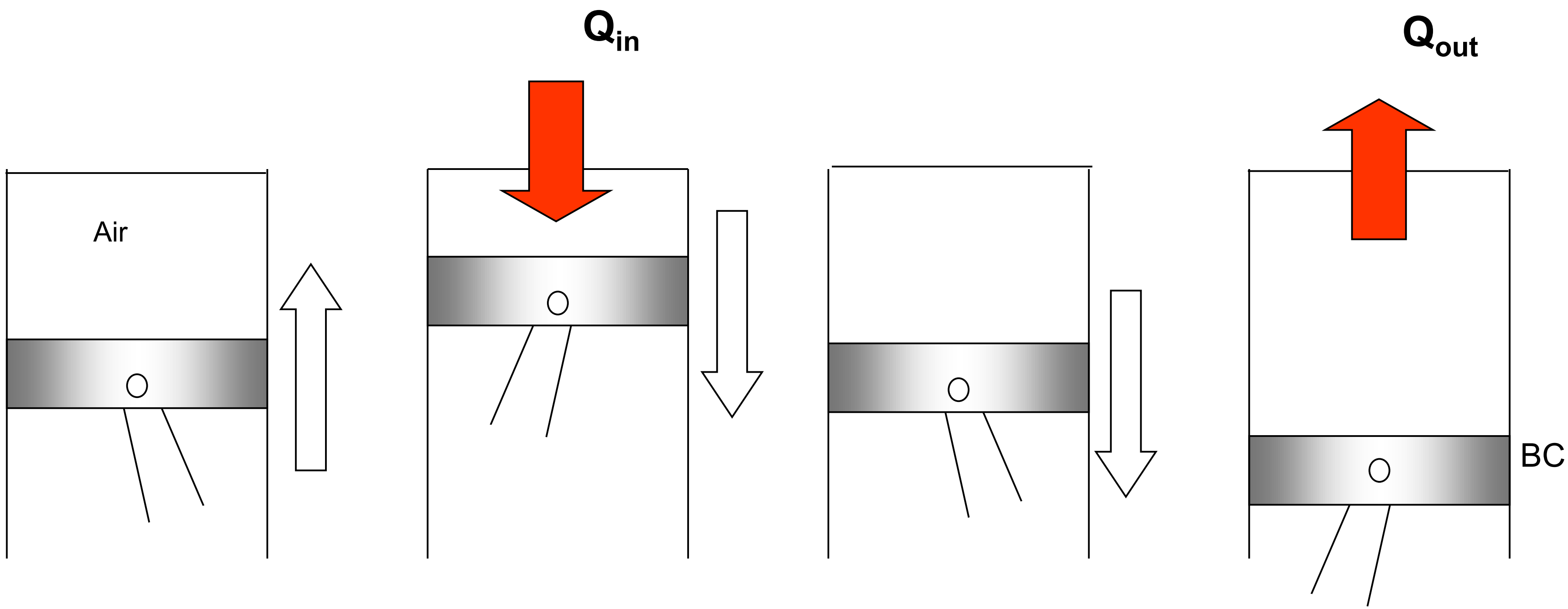
$$imep = \frac{W_{cycle}}{V_1 - V_2} = \frac{Q_{in}}{V_1}\left(\frac{r}{r-1}\right)\eta_{th}$$

Effect of Compression Ratio on Thermal Efficiency and MEP

$$\frac{imep}{P_1} = \frac{Q_{in}}{P_1 V_1}\left(\frac{r}{r-1}\right)\left(1-\frac{1}{r^k}\right)$$



Ideal Diesel Cycle



Compression Process

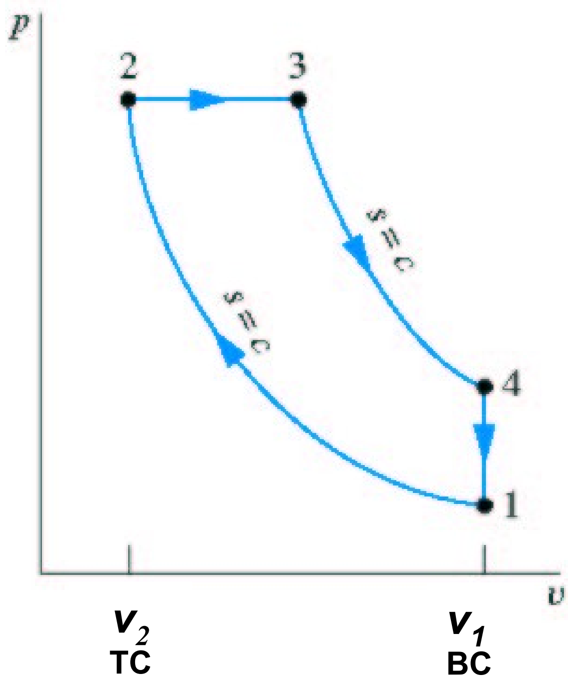
Const pressure heat addition Process

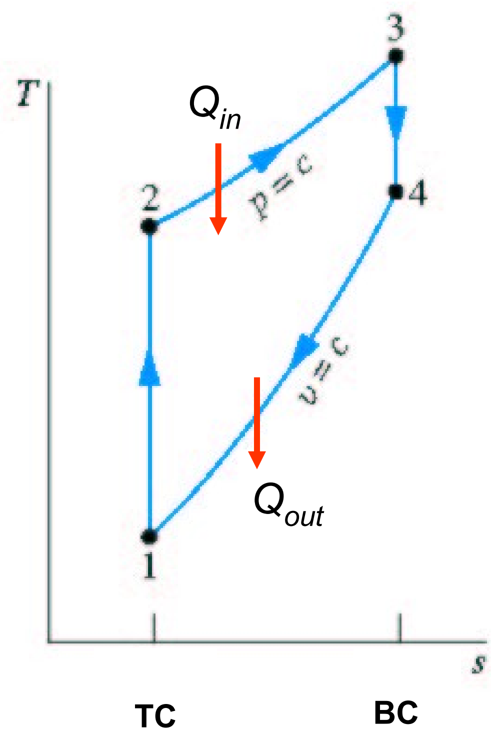
Expansion Process

Const volume heat rejection Process

Air-Standard Diesel cycle

Process 1→ 2 Isentropic compression
Process 2 → 3 Constant pressure heat addition
Process 3 → 4 Isentropic expansion
Process 4 → 1 Constant volume heat rejection





Cut-off ratio:

$$r_c = \frac{v_3}{v_2}$$

Thermal Efficiency

$$\eta_{\substack{Diesel \\ cycle}} = 1 - \frac{Q_{out}/m}{Q_{in}/m} = 1 - \frac{u_4 - u_1}{h_3 - h_2}$$

For cold air-standard the above reduces to:

$$\boxed{\eta_{\substack{Diesel \\ const\ c_V}} = 1 - \frac{1}{r^{k-1}}\left[\frac{1}{k} \cdot \frac{\left(r_c^k - 1\right)}{\left(r_c - 1\right)}\right]}$$

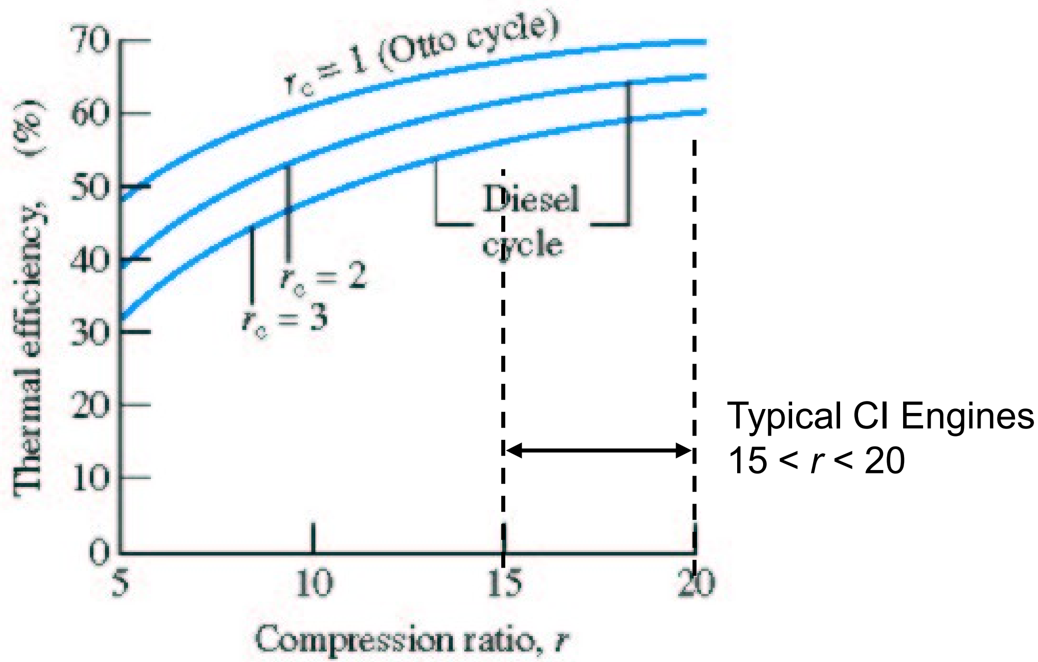
recall,

$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}}$$

Note the term in the square bracket is always larger than one so for the same compression ratio, *r*, the Diesel cycle has a *lower* thermal efficiency than the Otto cycle

Note: CI needs higher *r* compared to SI to ignite fuel

Thermal Efficiency

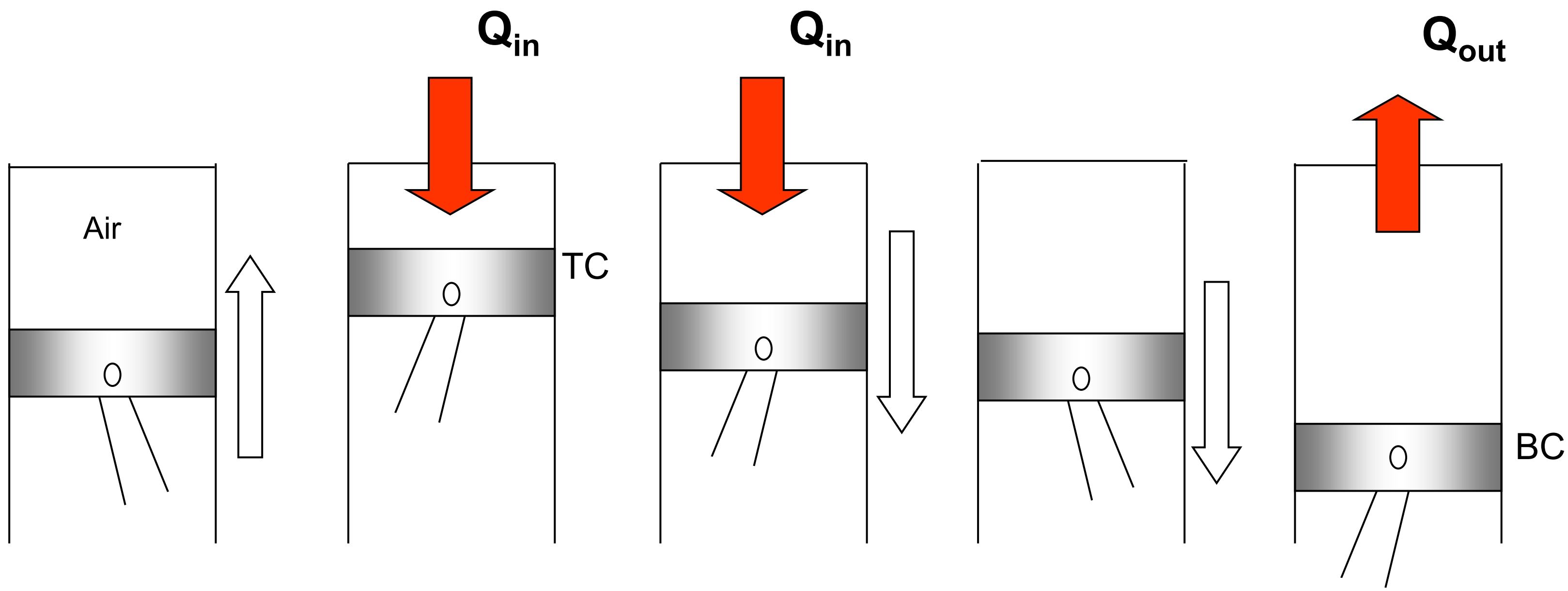


When r_c (= v_3/v_2)→1 the Diesel cycle efficiency approaches the efficiency of the Otto cycle

Higher efficiency is obtained by adding less heat per cycle, Q_{in},
→ run engine at higher speed to get the same power.

Thermodynamic Dual Cycle

Dual Cycle



Compression Process

Const volume heat addition Process

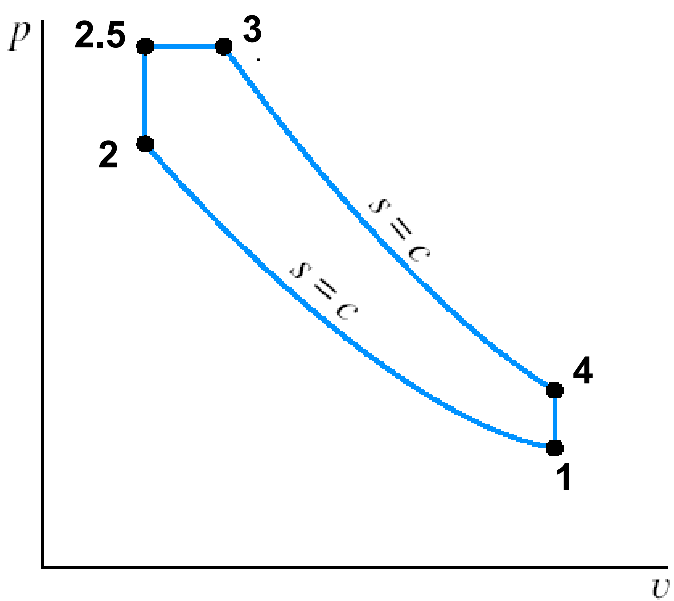
Const pressure heat addition Process

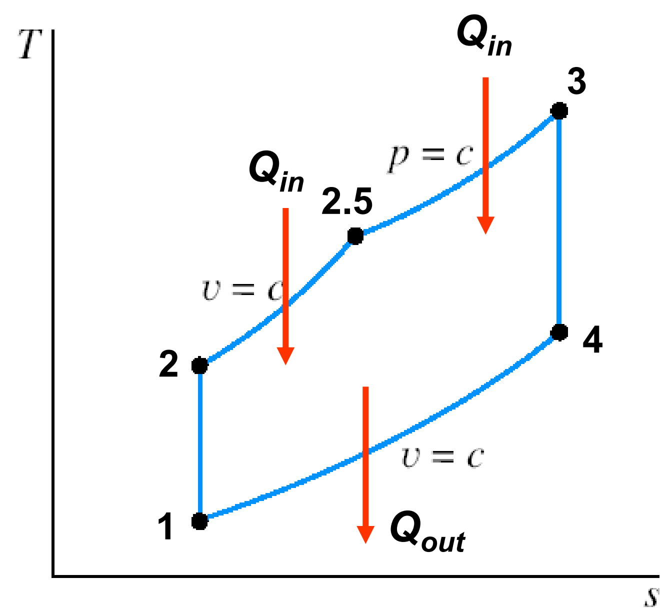
Expansion Process

Const volume heat rejection Process

Dual Cycle

Process 1 → 2 Isentropic compression
Process 2 → 2.5 Constant volume heat addition
Process 2.5 → 3 Constant pressure heat addition
Process 3 → 4 Isentropic expansion
Process 4 → 1 Constant volume heat rejection





$$\frac{Q_{in}}{m} = (u_{2.5} - u_2) + (h_3 - h_{2.5}) = c_v (T_{2.5} - T_2) + c_p (T_3 - T_{2.5})$$

Thermal Efficiency

$$\eta_{\substack{Dual\\cycle}} = 1 - \frac{Q_{out}/m}{Q_{in}/m} = 1 - \frac{u_4 - u_1}{(u_{2.5} - u_2) + (h_3 - h_{2.5})}$$

$$\boxed{\eta_{\substack{Dual\\const\ c_v}} = 1 - \frac{1}{r^{k-1}}\left[\frac{\alpha r_c^k - 1}{(\alpha - 1) + \alpha k(r_c - 1)}\right]}$$

$$\text{where } r_c = v_3 / v_{2.5} \text{ and } \alpha = P_3 / P_2$$

Note, the Otto cycle (r_c=1) and the Diesel cycle (α=1) are special cases:

$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}} \qquad \eta_{\substack{Diesel\\const\ c_V}} = 1 - \frac{1}{r^{k-1}}\left[\frac{1}{k} \cdot \frac{(r_c^k - 1)}{(r_c - 1)}\right]$$

The use of the Dual cycle requires information about either:

i) the fractions of constant volume and constant pressure heat addition (common assumption is to *equally* split the heat addition), or

ii) maximum pressure P_3.

Transformation of r_c and α into more natural variables yields

$$r_c = 1 - \frac{k-1}{\alpha k}\left[\left(\frac{Q_{in}}{P_1 V_1}\right)\frac{1}{r^{k-1}} - \frac{\alpha - 1}{k-1}\right] \qquad \alpha = \frac{1}{r^k}\frac{P_3}{P_1}$$

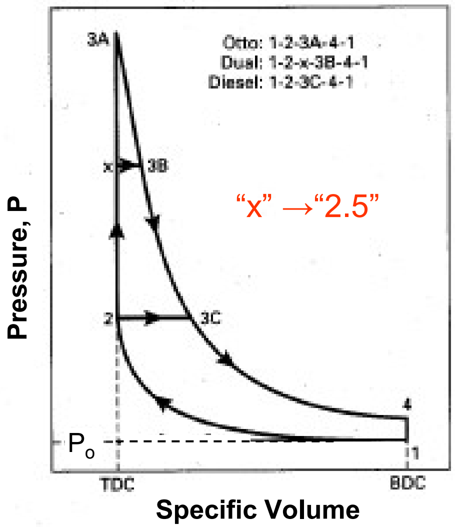
For the same inlet conditions P_1, V_1 and the same compression ratio:

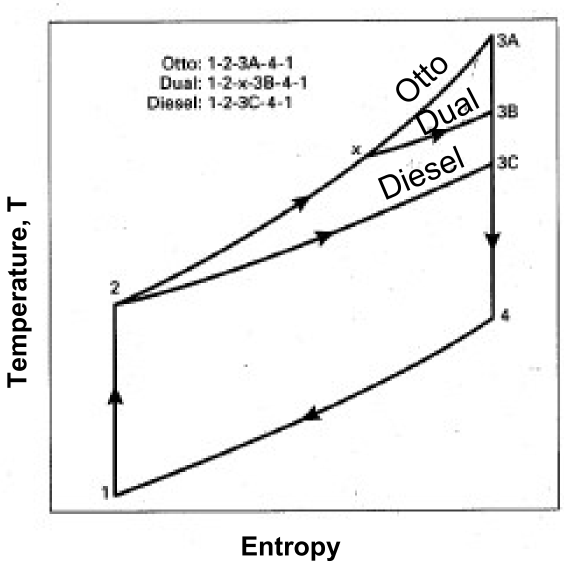
$$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

For the same inlet conditions P_1, V_1 and the same peak pressure P_3 (actual design limitation in engines):

$$\eta_{Diesel} > \eta_{Dual} > \eta_{otto}$$

For the same inlet conditions P_1, V_1 and the *same compression ratio* P_2/P_1:





$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{\int_4^1 T ds}{\int_2^3 T ds}$$

For the same inlet conditions P_1, V_1 and the *same peak pressure* P_3: