Ch 12- Control Charts for Attributes

- p chart – fraction defective
- np chart – number defective

- c, u charts – number of defects

Defect vs. Defective

- 'Defect' – a single nonconforming quality characteristic.

- 'Defective' – items having one or more defects.

Legal Concerns with Term 'Defect'

- Often called 'nonconformity'.

- Possible Legal Dialog
  - *Does your company make a lot of 'defects'?*
  - *Enough to track them on a chart ?*
  - *If they are not 'bad', why do you call them 'defects', sounds bad to me.*
  - *So you knowingly track and ship products with 'defects'?*

p, np - Chart

- P is fraction nonconforming.
- np is total nonconforming.

- Charts based on Binomial distribution.
- Sample size must be large enough *(example p=2%)*
- Definition of a nonconformity.
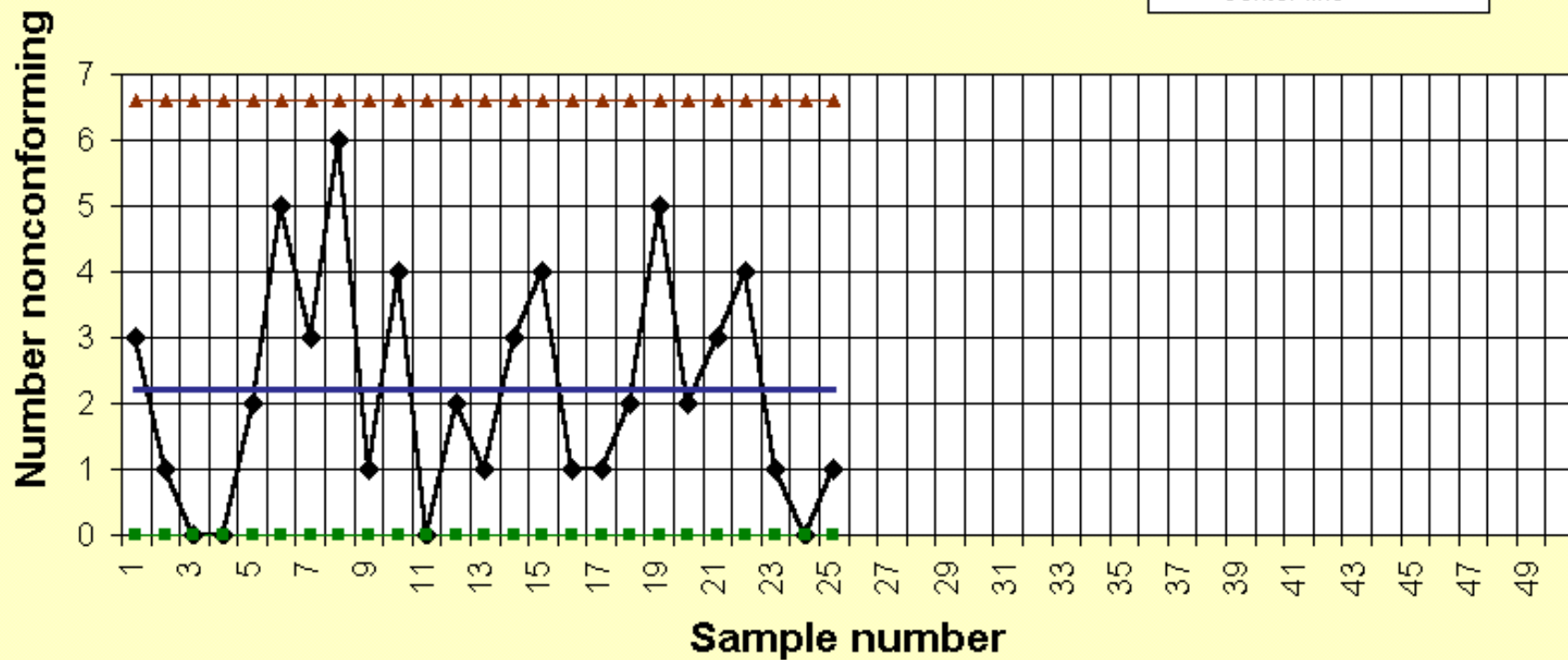- Probability the same from item to item.

c, u - Charts

- c and u charts deal with nonconformities.
  - c Chart – total number of nonconformities.
  - u Chart – nonconformities per unit.

- Charts based on Poisson distribution.
- Sample size, constant probabilities.

	A	B	C	D	E	F	G	H	I
1	**Number Nonconforming (np) Chart**								
2	This spreadsheet is designed for up to 50 samples. Enter data ONLY in yellow-shaded cells.								
3	Each sample must have a constant sample size; enter this in cell C6.								
4	Click on the sheet tab to display the control chart (some rescaling may be needed).								
5									
6	**Sample size**		100						
7									
8	**Average (np-bar)**		2.2						
9	**Standard deviation**		1.466833324						
10									
11		**Number**							
12	**Sample**	**Nonconforming**	**LCLnp**	**CL**	**UCLnp**				
13	1	3	0	2.2	6.6005				
14	2	1	0	2.2	6.6005				
15	3	0	0	2.2	6.6005				
16	4	0	0	2.2	6.6005				
17	5	2	0	2.2	6.6005				
18	6	5	0	2.2	6.6005				
19	7	3	0	2.2	6.6005				
20	8	6	0	2.2	6.6005				
21	9	1	0	2.2	6.6005				
22	10	4	0	2.2	6.6005				
23	11	0	0	2.2	6.6005				
24	12	2	0	2.2	6.6005				
25	13	1	0	2.2	6.6005				
26	14	3	0	2.2	6.6005				
27	15	4	0	2.2	6.6005				
28	16	1	0	2.2	6.6005				
29	17	1	0	2.2	6.6005				
30	18	2	0	2.2	6.6005				
31	19	5	0	2.2	6.6005				
32	20	2	0	2.2	6.6005				
33	21	3	0	2.2	6.6005				
34	22	4	0	2.2	6.6005				
35	23	1	0	2.2	6.6005				
36	24	0	0	2.2	6.6005				
37	25	1	0	2.2	6.6005				

Average Number of Defects (c) Chart

This spreadsheet is designed for up to 50 samples. Enter data ONLY in yellow-shaded cells.

Click on the sheet tab to display the control chart (some rescaling may be needed).

Average (c-bar)	1.8
Standard deviation	1.341640786

Sample	Number of Defects	LCLc	CL	UCLc
1	2	0	1.8	5.824922
2	3	0	1.8	5.824922
3	0	0	1.8	5.824922
4	1	0	1.8	5.824922
5	3	0	1.8	5.824922
6	5	0	1.8	5.824922
7	3	0	1.8	5.824922
8	1	0	1.8	5.824922
9	2	0	1.8	5.824922
10	2	0	1.8	5.824922
11	0	0	1.8	5.824922
12	1	0	1.8	5.824922
13	0	0	1.8	5.824922
14	2	0	1.8	5.824922
15	4	0	1.8	5.824922
16	1	0	1.8	5.824922
17	2	0	1.8	5.824922
18	0	0	1.8	5.824922
19	3	0	1.8	5.824922
20	2	0	1.8	5.824922
21	1	0	1.8	5.824922
22	4	0	1.8	5.824922
23	0	0	1.8	5.824922
24	0	0	1.8	5.824922
25	3	0	1.8	5.824922

# How to Interpret Attribute Charts

- Points beyond limits- primary test.
  - *Below lower limits means process has improved.*
- Zone rules do not apply.
- Rules for trends, shifts do apply.

Only get One Chart !!

Examples of When to Use

- p,np charts–
  - Number of nonconforming cables is found for 20 samples of size 100.
  - Number of nonconforming floppy disks is found for samples of 200 for 25 trials.

- c,u charts-
  - Number of paint blemishes on auto body observed for 30 samples.
  - Number of imperfections in bond paper – by area inspected and number of imperfections.

Control Chart Selection
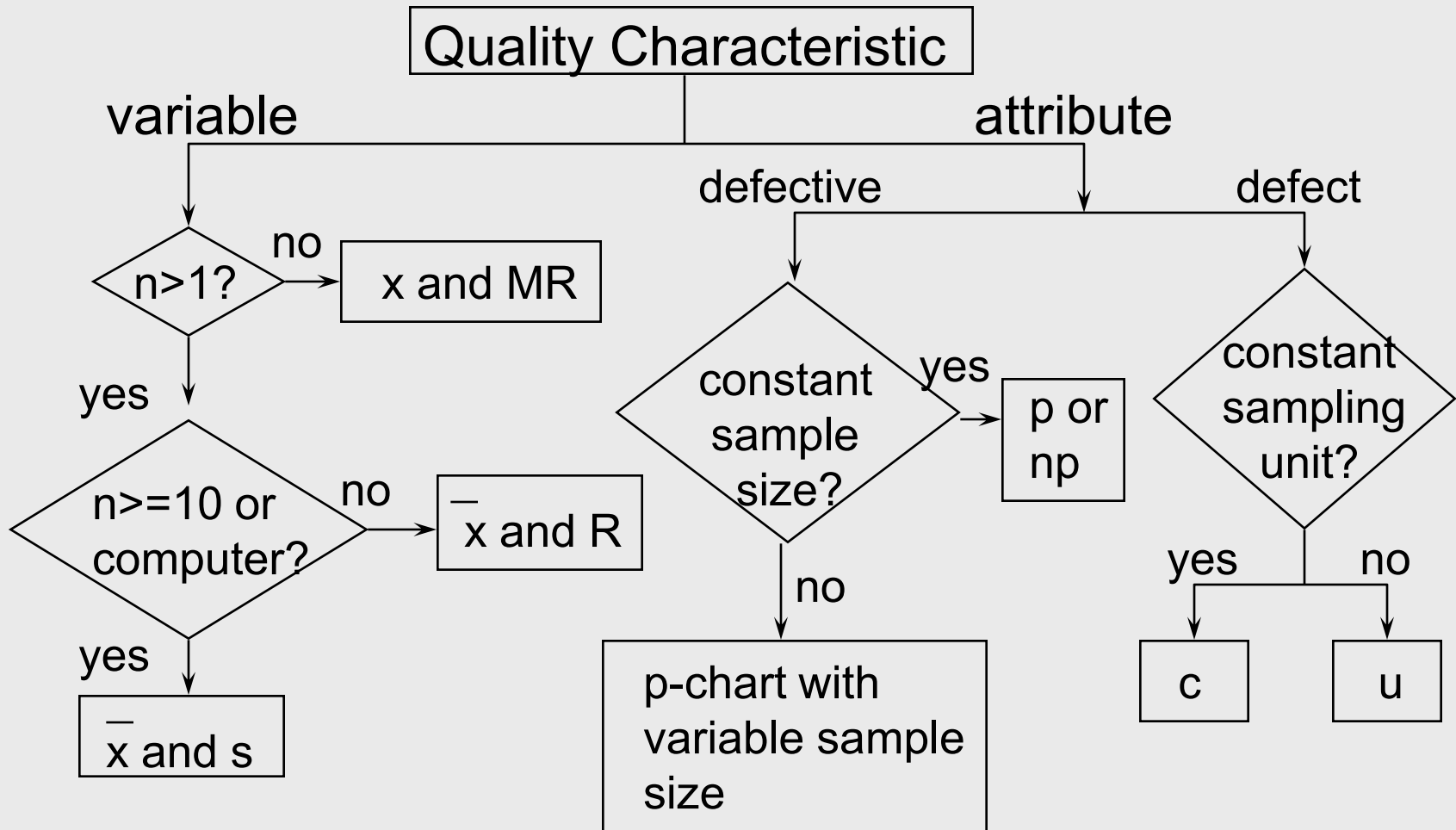
Comparison of Variables v. Attributes

- **<u>Variables</u>**
  - Fit certain cases.
  - Both mean and variation information.
  - More expensive?
  - Identify mean shifts sooner before large number nonconforming.

- **<u>Attributes</u>**
  - Fit certain cases – taste, color, etc.
  - Larger sample sizes.
  - Provides summary level performance.
  - Must define nonconformity.

When are Shifts Detected ?
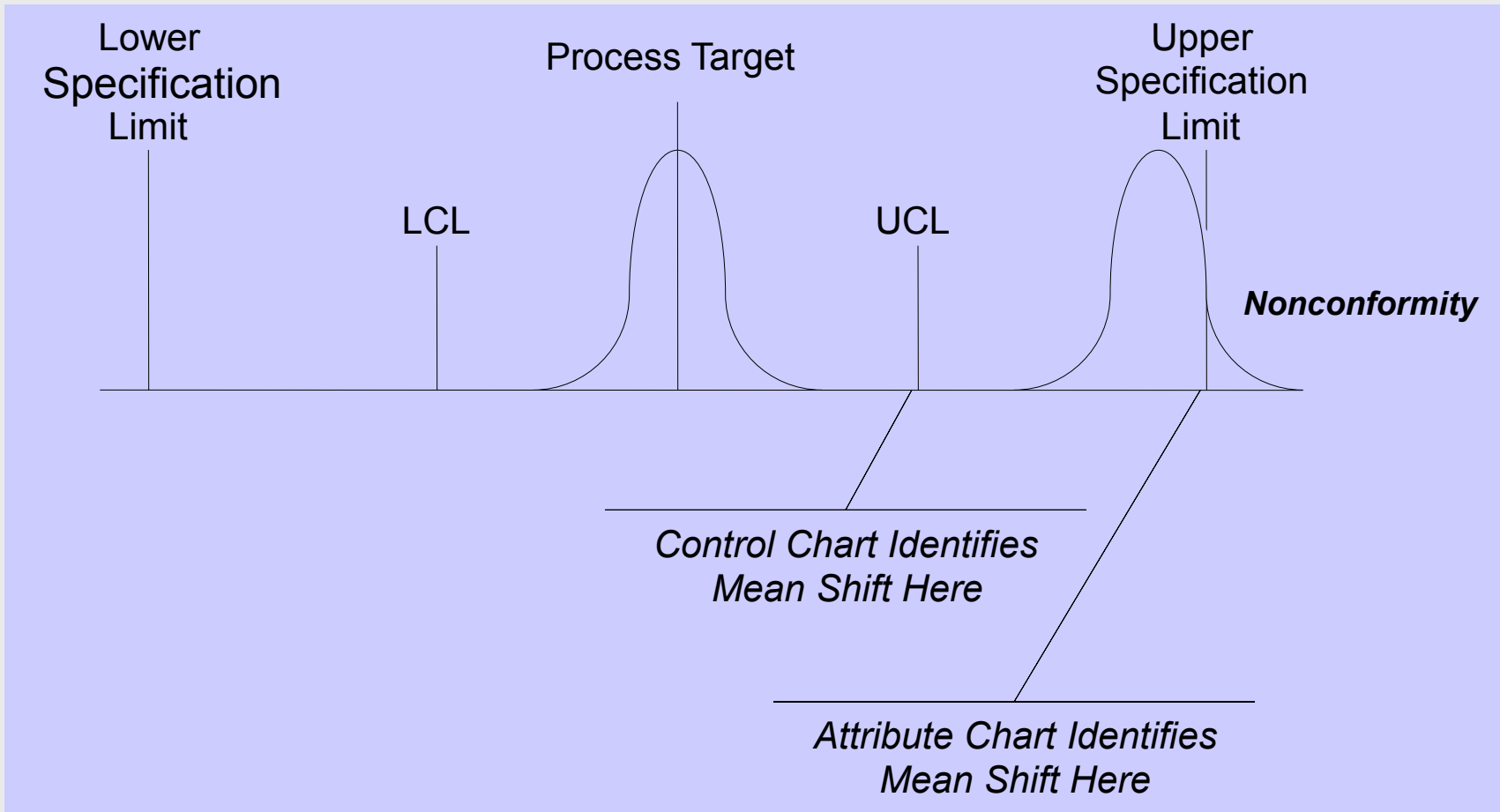
Variables v. Attributes

- Both have advantages.
- At High levels - Attribute charts, identify problem areas.

- At Lower levels – Variables charts, quantitative problem solving tools.

Intro to Acceptance Sampling

- **Acceptance Sampling** –

  a historically significant topic but less used today.

- Part of Ch. 11 on Inspection Methods.

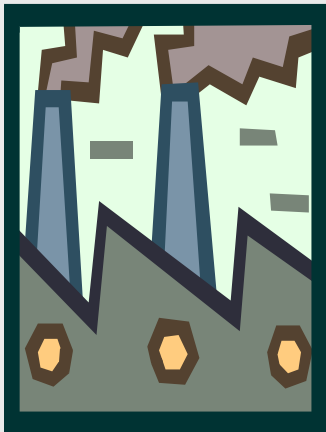- Still used in some applications today.

History and Status

- Used extensively in WW II.
- Many Mil-Spec plans developed (105-E, ANSI/ASQC Z1.4-1993).

- Still popular as a defense procurement tool.
  - Very large lots, screening tool.
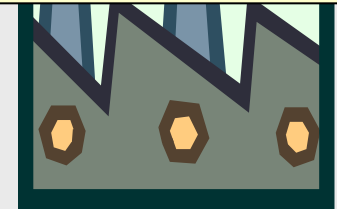 - Low bid suppliers – no history.

Acceptance Sampling Flow Chart
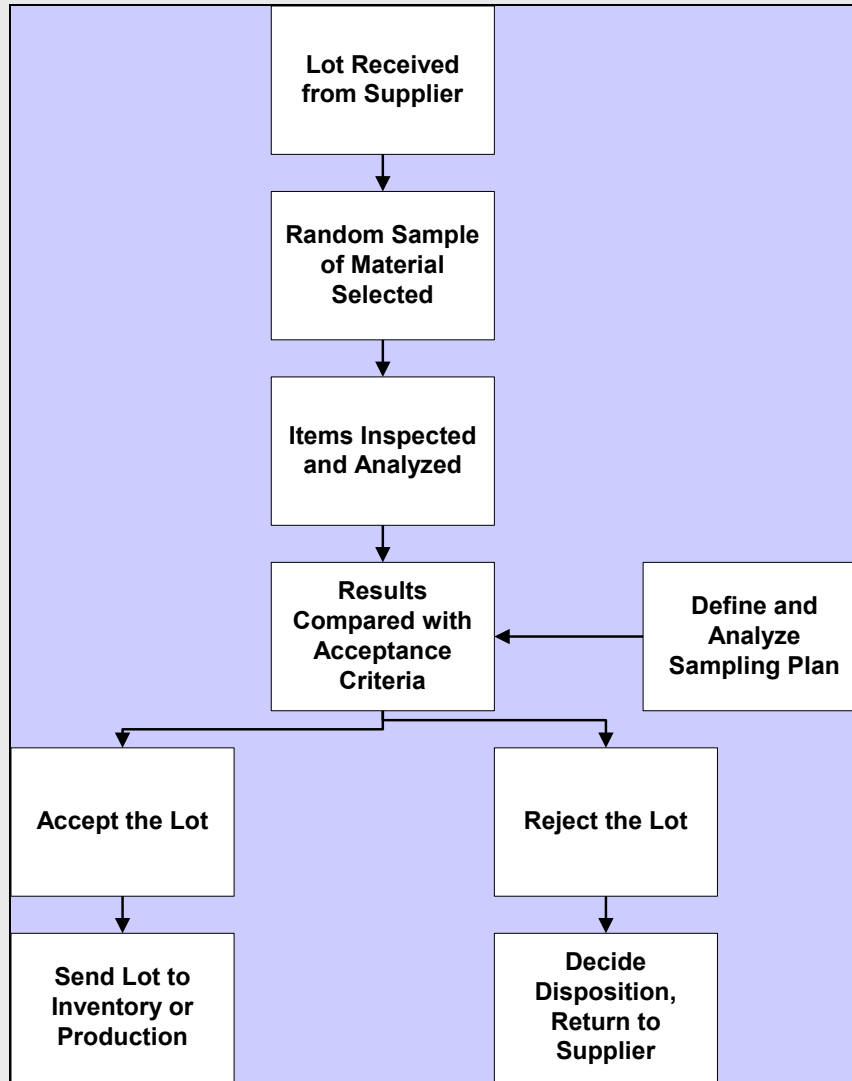
Role of Producer and Consumer

**Take a Sample
Size 'n',
Accept if 'c' or less.**

Producer

Risk is a 'good' lot will be rejected and sent back.

Consumer

Risk is a 'bad' lot will be accepted.

Terminology

- **Producer's risk** – risk associated with rejecting a lot of 'good' quality.

- **Acceptable Quality Level (AQL)** – Numerical definition of a 'good' lot, associated with the producer's risk.

- **Consumer's risk** – risk of accepting a 'poor' lot.

- **Limiting Quality Level** - Numerical definition of a 'poor' lot, associated with the consumer's risk.
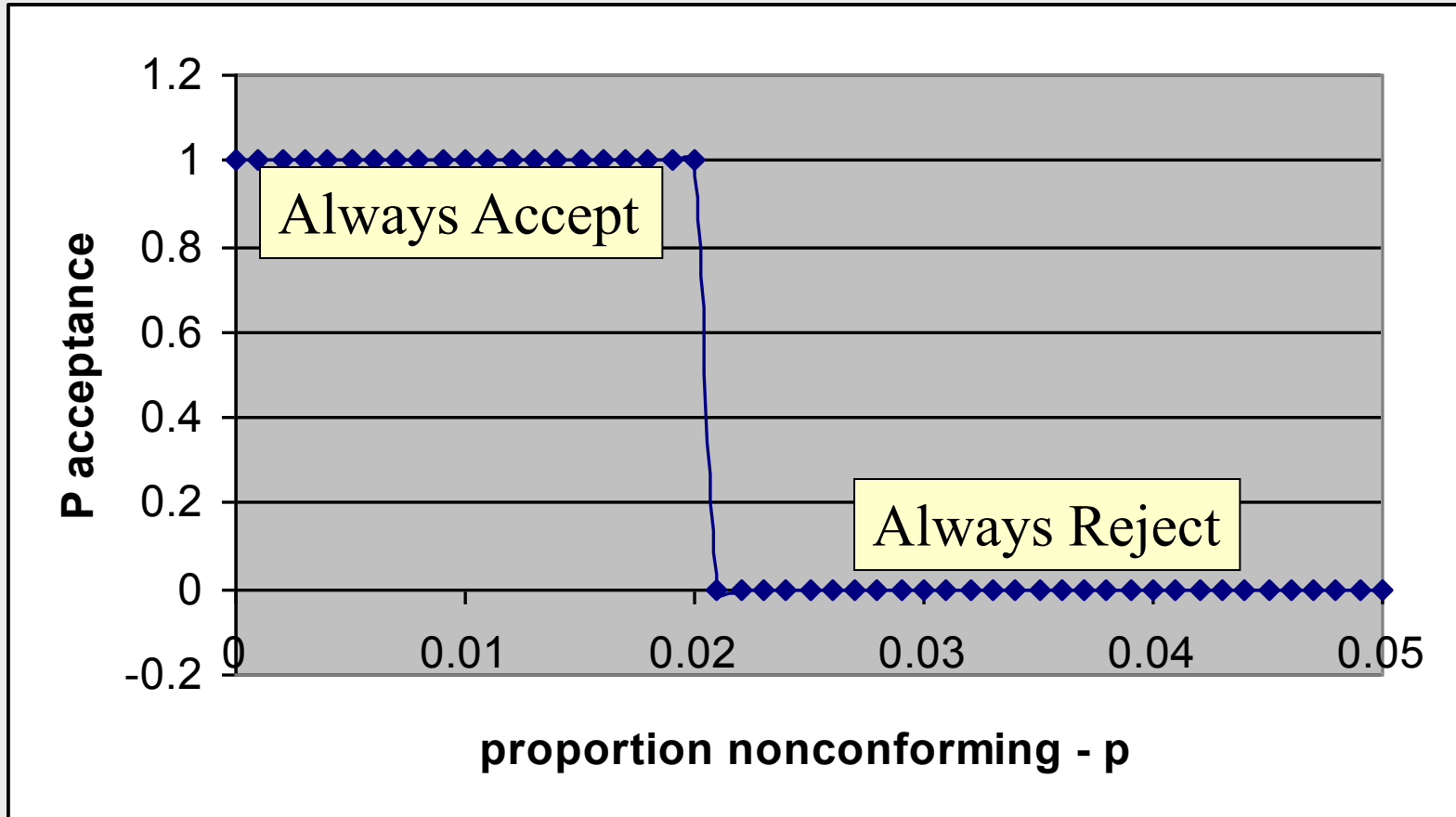
Examples

- Producer's risk is 5% for an AQL of 0.02.

  – Means batches that are 2% nonconforming are good and prefer to reject these no more than 5% of the time.

- Consumer's risk is 10% for an LQL of 0.08.

  – Means batches that are 8% nonconforming are bad and prefer to accept these only 10% of the time.

Operating Characteristic (OC) Curve

- Defines the performance of a sampling plan.

- Plots
  - probability of acceptance versus
  - proportion nonconforming (p).

Ideal OC Curve

Actual OC Curves

- Are determined by sample size [n] and acceptance number [c].

  – *Accept the lot if 'c' or fewer nonconforming are obtained, reject if more.*

- OK to assume Binomial distribution (if lot size is 10x sample size).

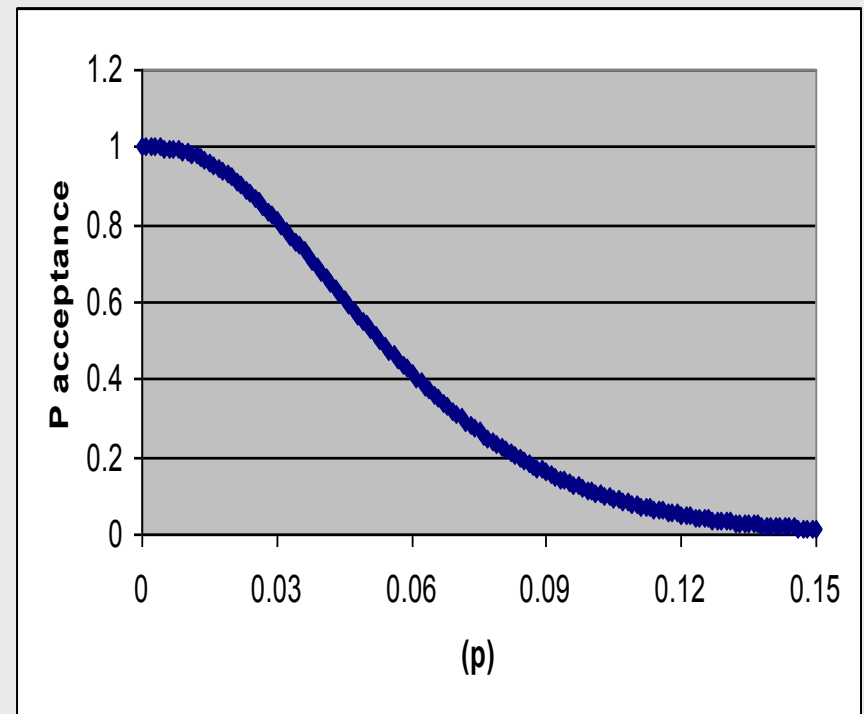- Calculate Paccept for range of incoming p levels.

Sample problem

- Given a lot size of N=2000, a sample size n=50, and an acceptance number c=2.

- Calculate the OC curve for this plan.

Create OC Curve

$$b(x) = \binom{n}{x} \; p^x \, (1-p)^{(n-x)} \qquad x = 0, 1, ..n$$

Probability of accepting is obtaining c=2 or less non-conforming items in samples of size n=50.

Vary p from 0 to 0.15 (what if p = ….)

Acceptance Sampling

- ### **Pros**

  1. Vary level of risk in decisions.
  2. Inexpensive, less work than 100% inspection.
  3. Flexibility – vary plan based on history.
 4. Lots rejected – pressure on supplier.

- ### **Cons**

  1. Plan to accept bad quality.
  2. Detects bad quality, not prevention or improvement.
  3. Deming views on inspection.
  4. Risk of rejecting 'good' lots.

Sample Calculations

- Binomial table only goes up to n=20.
- Approximate Binomial by Poisson, u=np.
- Calculate p(2 or less). This is Paccept.

- Example n=50, p=0.03, u=1.5. P(x≤2)=0.809.

- Vary p from 0 to 0.15.

Single sampling plan n = 50, c= 2
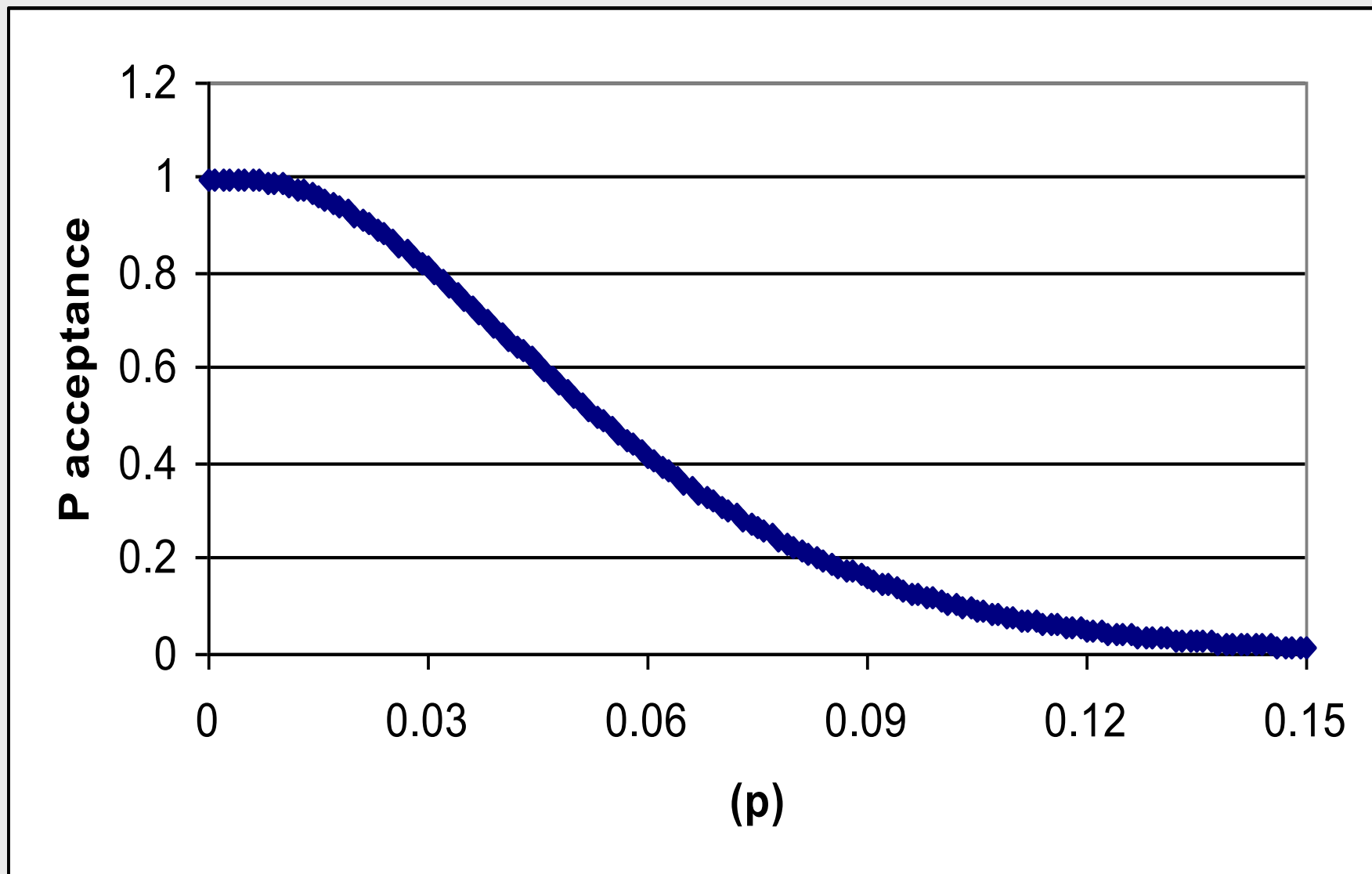
Producer and Consumer Risk

- Assume AQL(acceptable quality level) is 0.01. Then Paccept = .986.

- Producer's Risk is 1-0.986 = 0.014.

- Assume LQL(limiting quality level) is 0.11. Then Paccept = 0.076.
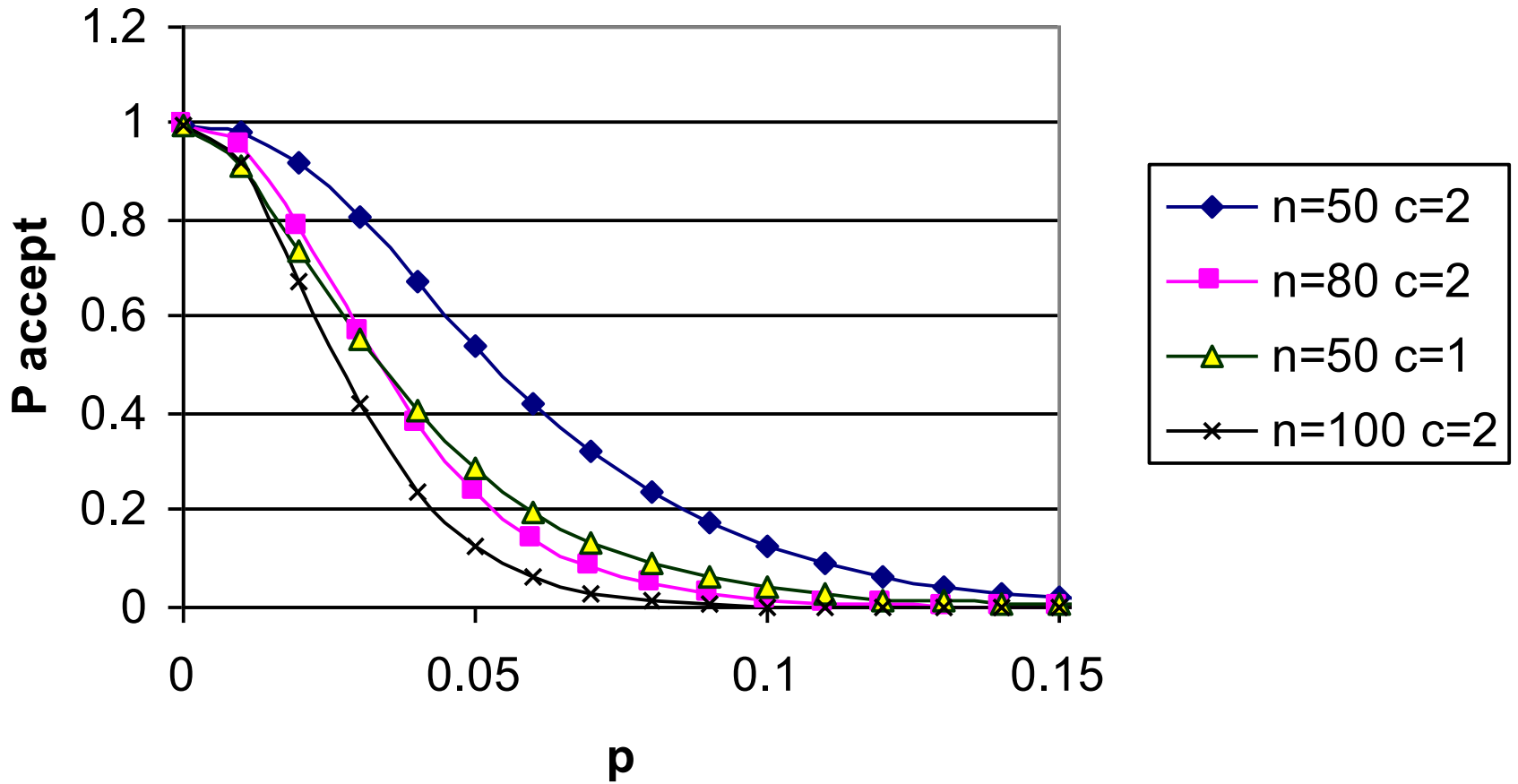
- Consumer's Risk is 0.076.

Designing Plan Performance

- Vary n and c to obtain different OC curves.

- Single and multiple sampling.

- Refer to standard published sampling plans.

Double Sampling Plan

- Application of double sampling requires that a first sample of size $n1$ is taken at random from the (large) lot. The number of defectives is then counted and compared to the first sample's acceptance number $a1$ and rejection number $r1$. Denote the number of defectives in sample 1 by $d1$ and in sample 2 by $d2$, then:
  - If $d1 <= a1$, the lot is accepted.
    If $d1 >= r1$, the lot is rejected.
    If $a1 < d1 < r1$, a second sample is taken.

- If a second sample of size $n2$ is taken, the number of defectives, $d2$, is counted. The total number of defectives is $D2 = d1 + d2$. Now this is compared to the acceptance number $a2$ and the rejection number $r2$ of sample 2. In double sampling, $r2 = a2 + 1$ to ensure a decision on the sample.
  - If $D2 <= a2$, the lot is accepted.
    If $D2 >= r2$, the lot is rejected.

Vary n and c

Vary n and c
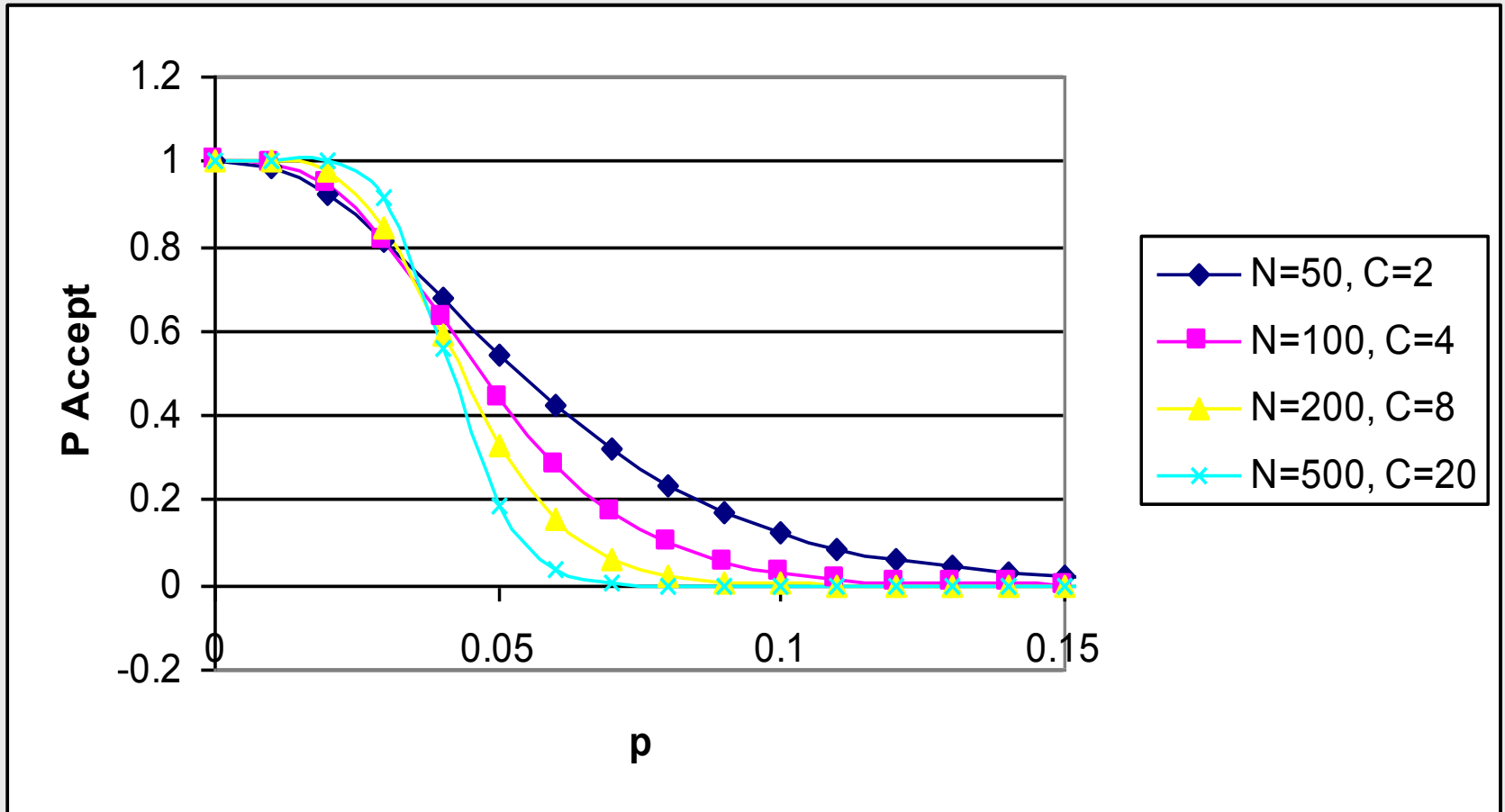
Class Problem

- Acceptance Sampling Plan – n=30,c=1

- Draw the OC Curve

- What is Producer's risk if AQL is 0.02.
- What is Consumer's risk if LQL is 0.1.

OC Curve Worksheet

p	np (=u)	Paccept (x<=1)
0		1
0.02		
0.04		
0.06		
0.08		
0.1		
0.12		

n=30, c=1

Plot Paccept vs. p