Ch 12- Control Charts for Attributes

- p chart – fraction defective
- np chart – number defective

- c, u charts – number of defects

Defect vs. Defective

- ‘Defect’ – a single nonconforming quality characteristic.

- ‘Defective’ – items having one or more defects.

Legal Concerns with Term ‘Defect’

- Often called ‘nonconformity’.
- Possible Legal Dialog
  - *Does your company make a lot of ‘defects’?*
  - *Enough to track them on a chart ?*
  - *If they are not ‘bad’, why do you call them ‘defects’, sounds bad to me.*
  - *So you knowingly track and ship products with ‘defects’?*

Summary of Control Chart Types and Limits
Table 12.3

These are again ‘3 sigma’ control limits

p, np - Chart

- P is fraction nonconforming.
- np is total nonconforming.

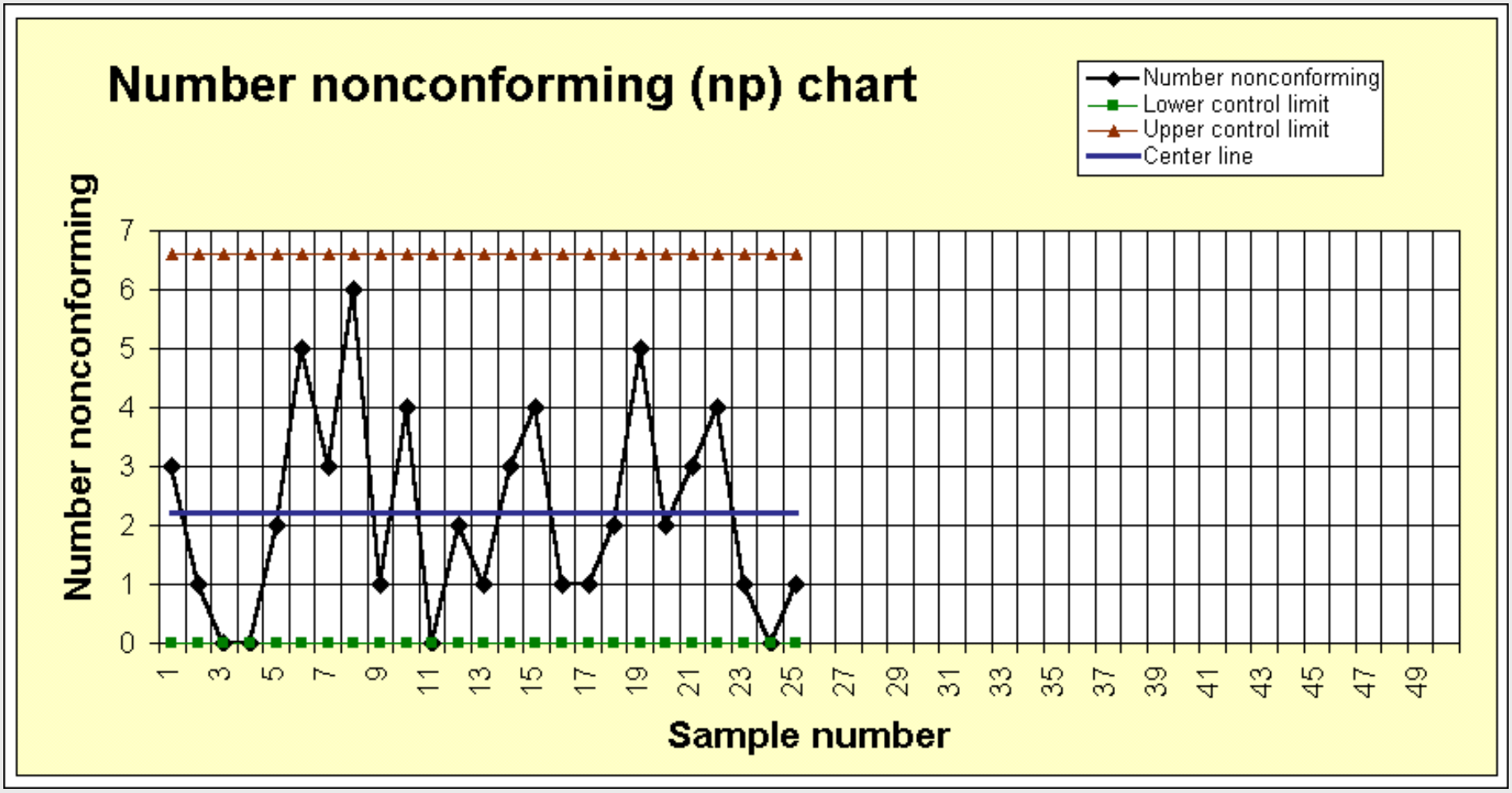
- Charts based on Binomial distribution.
- Sample size must be large enough *(example p=2%)*
- Definition of a nonconformity.
- Probability the same from item to item.

c, u - Charts

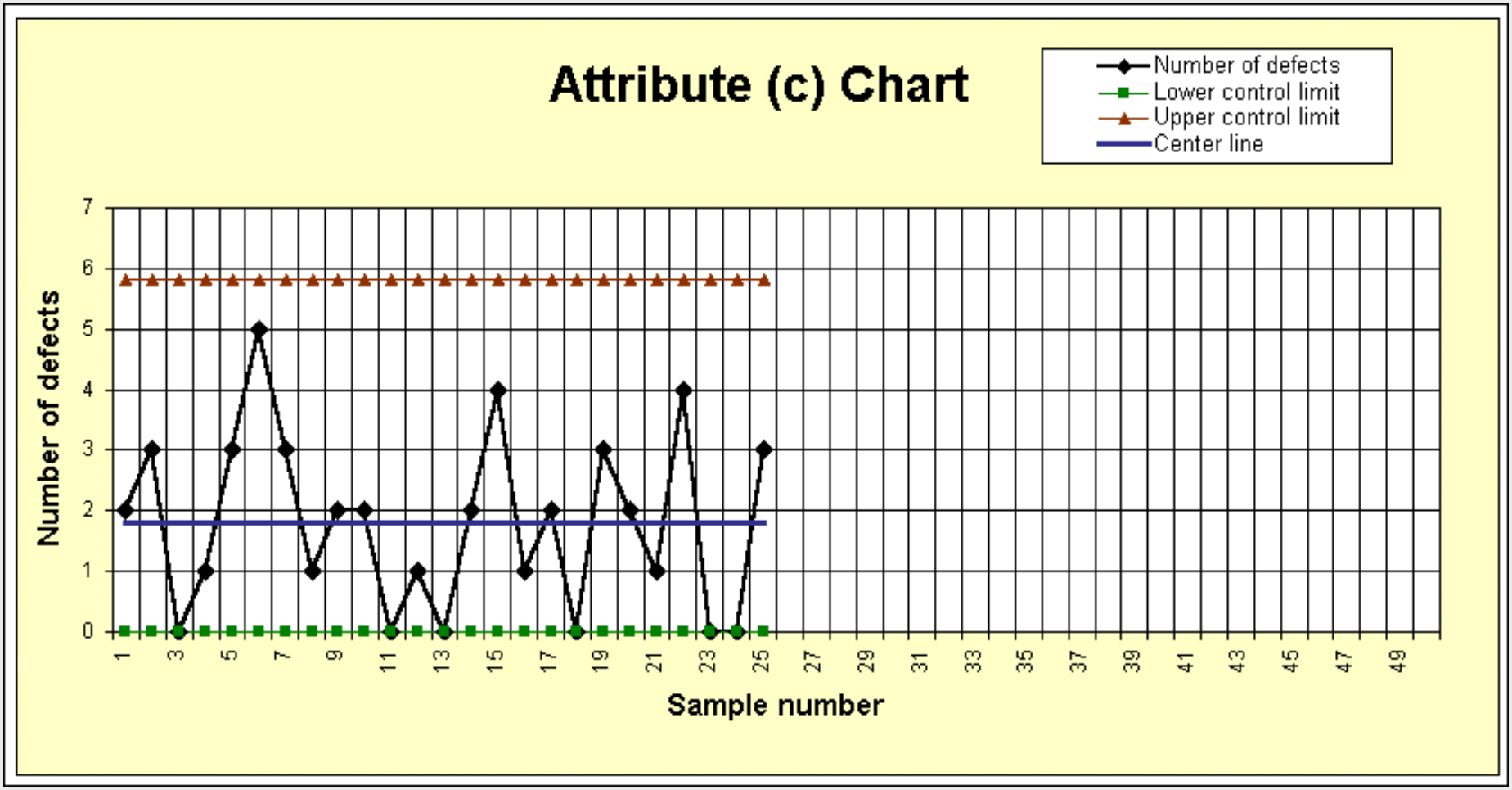
- c and u charts deal with nonconformities.
  - c Chart – total number of nonconformities.
  - u Chart – nonconformities per unit.

- Charts based on Poisson distribution.
- Sample size, constant probabilities.

	A	B	C	D	E	F	G	H	I
1	**Number Nonconforming (np) Chart**								
2	This spreadsheet is designed for up to 50 samples. Enter data ONLY in yellow-shaded cells.								
3	Each sample must have a constant sample size; enter this in cell C6.								
4	Click on the sheet tab to display the control chart (some rescaling may be needed).								
5									
6	**Sample size**		100						
7									
8	**Average (np-bar)**		**2.2**						
9	**Standard deviation**		**1.466833324**						
10									
11		**Number**							
12	**Sample**	**Nonconforming**	**LCLnp**	**CL**	**UCLnp**				
13	1	3	0	2.2	6.6005				
14	2	1	0	2.2	6.6005				
15	3	0	0	2.2	6.6005				
16	4	0	0	2.2	6.6005				
17	5	2	0	2.2	6.6005				
18	6	5	0	2.2	6.6005				
19	7	3	0	2.2	6.6005				
20	8	6	0	2.2	6.6005				
21	9	1	0	2.2	6.6005				
22	10	4	0	2.2	6.6005				
23	11	0	0	2.2	6.6005				
24	12	2	0	2.2	6.6005				
25	13	1	0	2.2	6.6005				
26	14	3	0	2.2	6.6005				
27	15	4	0	2.2	6.6005				
28	16	1	0	2.2	6.6005				
29	17	1	0	2.2	6.6005				
30	18	2	0	2.2	6.6005				
31	19	5	0	2.2	6.6005				
32	20	2	0	2.2	6.6005				
33	21	3	0	2.2	6.6005				
34	22	4	0	2.2	6.6005				
35	23	1	0	2.2	6.6005				
36	24	0	0	2.2	6.6005				
37	25	1	0	2.2	6.6005				


Number nonconforming (np) chart
Number nonconforming
Lower control limit
Upper control limit
Center line
Number nonconforming
0
1
2
3
4
5
6
7
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
49
Sample number

	A	B	C	D	E	F	G	H
1	**Average Number of Defects (c) Chart**							
2	This spreadsheet is designed for up to 50 samples. Enter data ONLY in yellow-shaded cells.							
3	Click on the sheet tab to display the control chart (some rescaling may be needed).							
4								
5	**Average (c-bar)**		**1.8**					
6	**Standard deviation**		**1.341640786**					
7								
8		**Number**						
9	**Sample**	**of Defects**	**LCLc**	**CL**	**UCLc**			
10	1	2	0	1.8	5.824922			
11	2	3	0	1.8	5.824922			
12	3	0	0	1.8	5.824922			
13	4	1	0	1.8	5.824922			
14	5	3	0	1.8	5.824922			
15	6	5	0	1.8	5.824922			
16	7	3	0	1.8	5.824922			
17	8	1	0	1.8	5.824922			
18	9	2	0	1.8	5.824922			
19	10	2	0	1.8	5.824922			
20	11	0	0	1.8	5.824922			
21	12	1	0	1.8	5.824922			
22	13	0	0	1.8	5.824922			
23	14	2	0	1.8	5.824922			
24	15	4	0	1.8	5.824922			
25	16	1	0	1.8	5.824922			
26	17	2	0	1.8	5.824922			
27	18	0	0	1.8	5.824922			
28	19	3	0	1.8	5.824922			
29	20	2	0	1.8	5.824922			
30	21	1	0	1.8	5.824922			
31	22	4	0	1.8	5.824922			
32	23	0	0	1.8	5.824922			
33	24	0	0	1.8	5.824922			
34	25	3	0	1.8	5.824922			


Attribute (c) Chart
Number of defects
Lower control limit
Upper control limit
Center line
Number of defects
Sample number
0
1
2
3
4
5
6
7
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
49

How to Interpret Attribute Charts

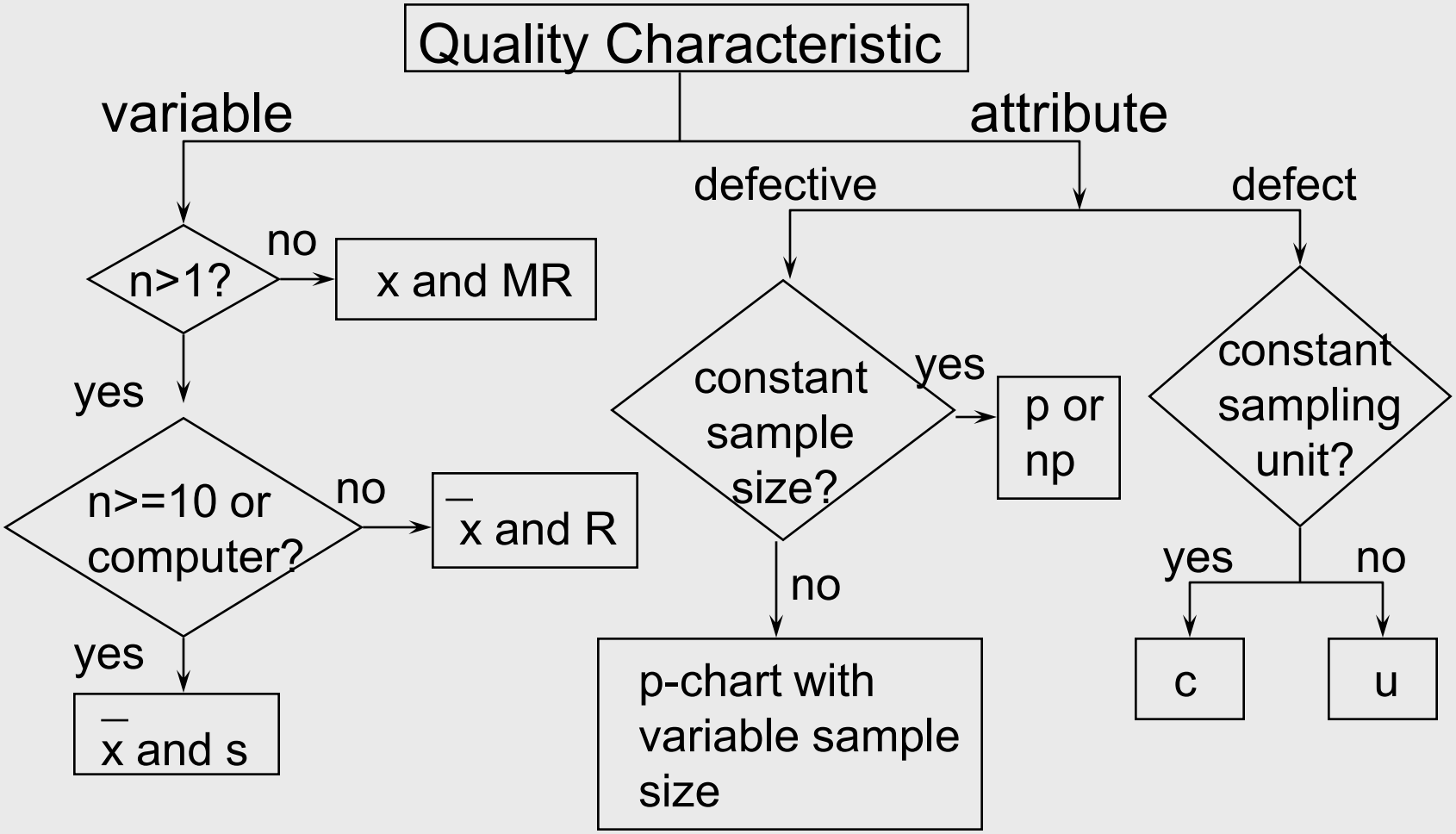
- Points beyond limits- primary test.
  - *Below lower limits means process has improved.*
- Zone rules do not apply.
- Rules for trends, shifts do apply.

Only get One Chart !!

Examples of When to Use

- p,np charts–
  - Number of nonconforming cables is found for 20 samples of size 100.
  - Number of nonconforming floppy disks is found for samples of 200 for 25 trials.
- c,u charts-
  - Number of paint blemishes on auto body observed for 30 samples.
  - Number of imperfections in bond paper – by area inspected and number of imperfections.

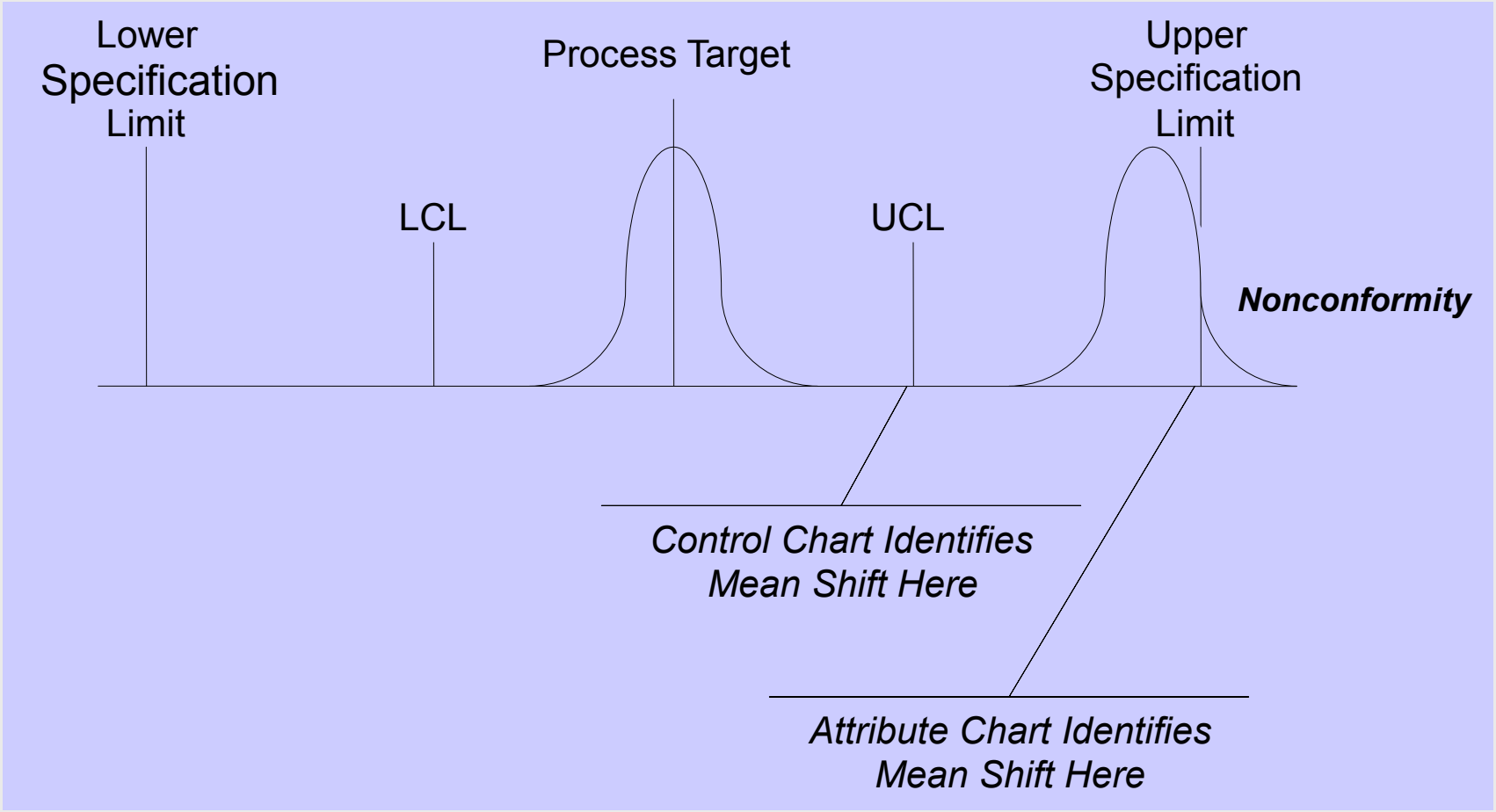
Control Chart Selection



Comparison of Variables v. Attributes

- **Variables**
  - Fit certain cases.
  - Both mean and variation information.
  - More expensive?
  - Identify mean shifts sooner before large number nonconforming.
- **Attributes**
  - Fit certain cases – taste, color, etc.
  - Larger sample sizes.
  - Provides summary level performance.
  - Must define nonconformity.

When are Shifts Detected ?



Variables v. Attributes

- Both have advantages.
- At High levels - Attribute charts, identify problem areas.

- At Lower levels – Variables charts, quantitative problem solving tools.

Intro to Acceptance Sampling

- **Acceptance Sampling** –

  a historically significant topic but less used today.

- Part of Ch. 11 on Inspection Methods.

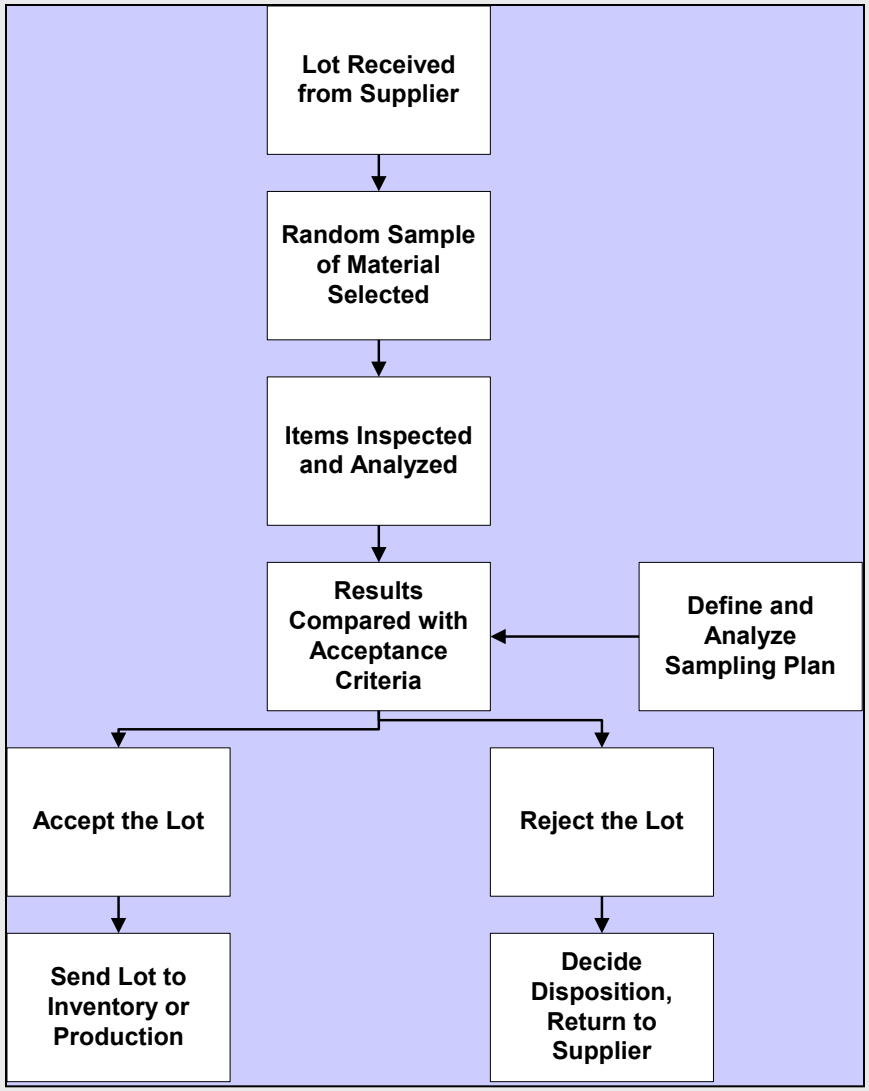
- Still used in some applications today.

History and Status

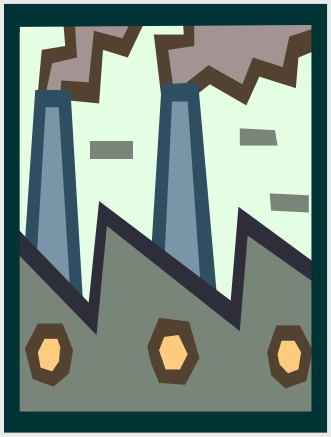
- Used extensively in WW II.
- Many Mil-Spec plans developed (105-E, ANSI/ASQC Z1.4-1993).

- Still popular as a defense procurement tool.
  - Very large lots, screening tool.
  - Low bid suppliers – no history.

Acceptance Sampling Flow Chart



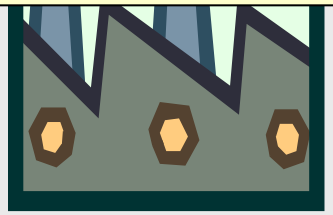
Role of Producer and Consumer



Producer

Risk is a 'good' lot will be rejected and sent back.

Take a Sample Size 'n', Accept if 'c' or less.



Consumer

Risk is a 'bad' lot will be accepted.

Terminology

- **Producer's risk** – risk associated with rejecting a lot of 'good' quality.
- **Acceptable Quality Level (AQL)** – Numerical definition of a 'good' lot, associated with the producer's risk.

- **Consumer's risk** – risk of accepting a 'poor' lot.
- **Limiting Quality Level -** Numerical definition of a 'poor' lot, associated with the consumer's risk.

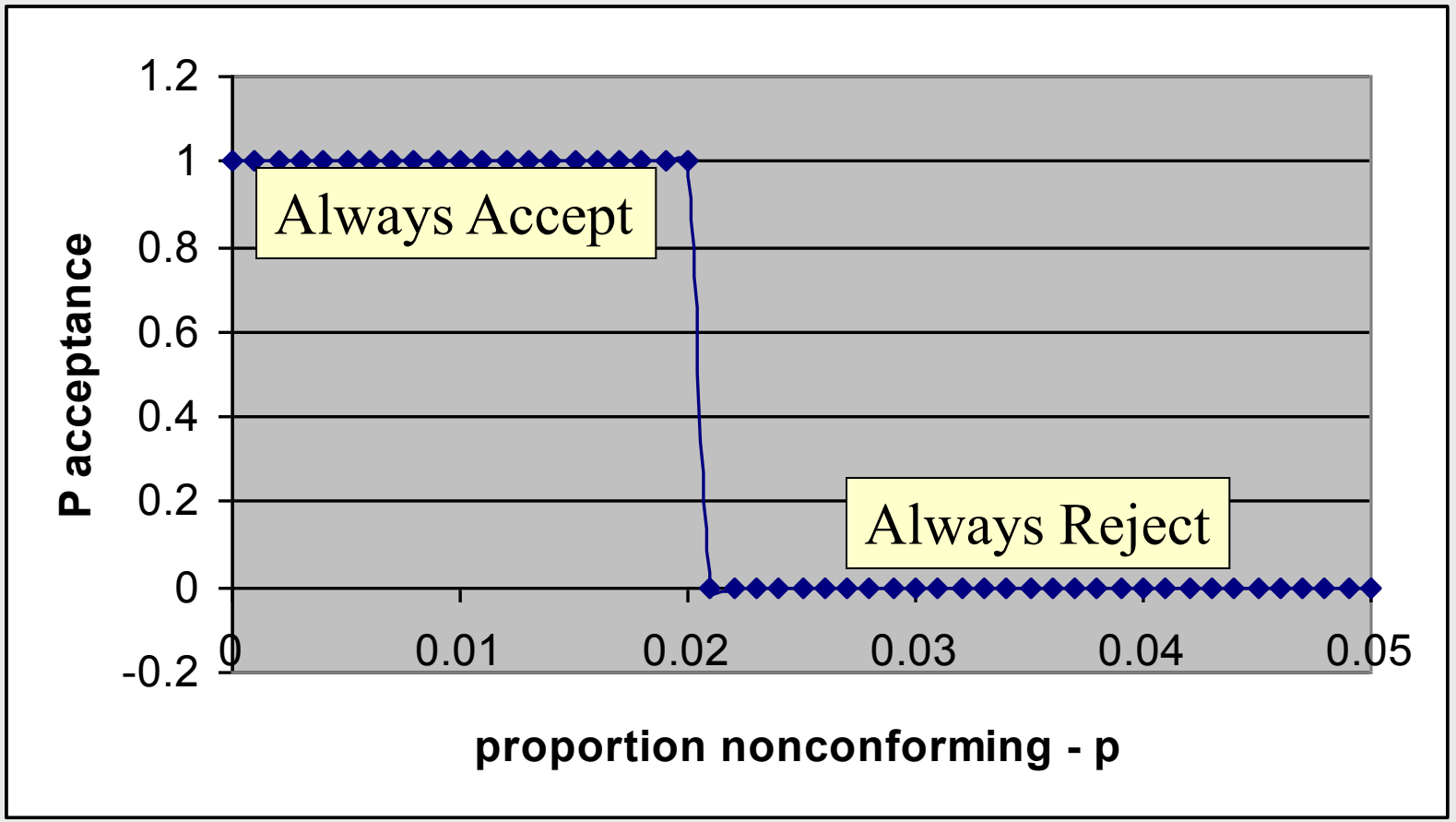
Examples

- Producer's risk is 5% for an AQL of 0.02.
  - Means batches that are 2% nonconforming are good and prefer to reject these no more than 5% of the time.
- Consumer's risk is 10% for an LQL of 0.08.
  - Means batches that are 8% nonconforming are bad and prefer to accept these only 10% of the time.

Operating Characteristic (OC) Curve

- Defines the performance of a sampling plan.
- Plots
  - probability of acceptance versus
  - proportion nonconforming (p).

Ideal OC Curve



Actual OC Curves

- Are determined by sample size [n] and acceptance number [c].
  - *Accept the lot if 'c' or fewer nonconforming are obtained, reject if more*.
- OK to assume Binomial distribution (if lot size is 10x sample size).
- Calculate Paccept for range of incoming p levels.

Sample problem

- Given a lot size of N=2000, a sample size n=50, and an acceptance number c=2.

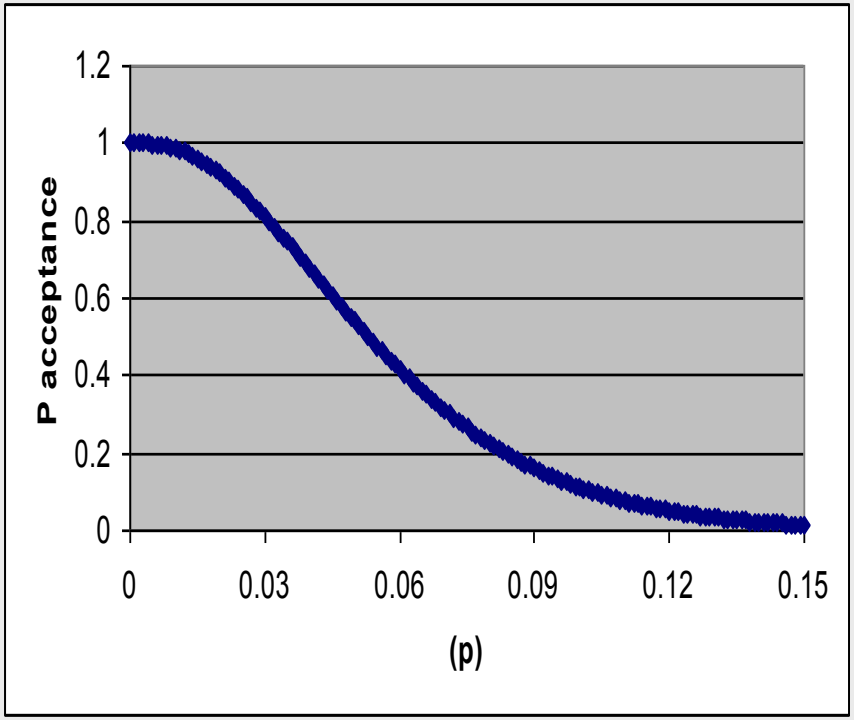
- Calculate the OC curve for this plan.

Create OC Curve

$$b(x) = \binom{n}{x} p^x (1-p)^{(n-x)} \qquad x = 0,1,..n$$

Probability of accepting is obtaining c=2 or less non-conforming items in samples of size n=50.

Vary p from 0 to 0.15 (what if p = ….)



Acceptance Sampling

- **Pros**

1. Vary level of risk in decisions.
2. Inexpensive, less work than 100% inspection.
3. Flexibility – vary plan based on history.
4. Lots rejected – pressure on supplier.

- **Cons**

1. Plan to accept bad quality.
2. Detects bad quality, not prevention or improvement.
3. Deming views on inspection.
4. Risk of rejecting ‘good’ lots.

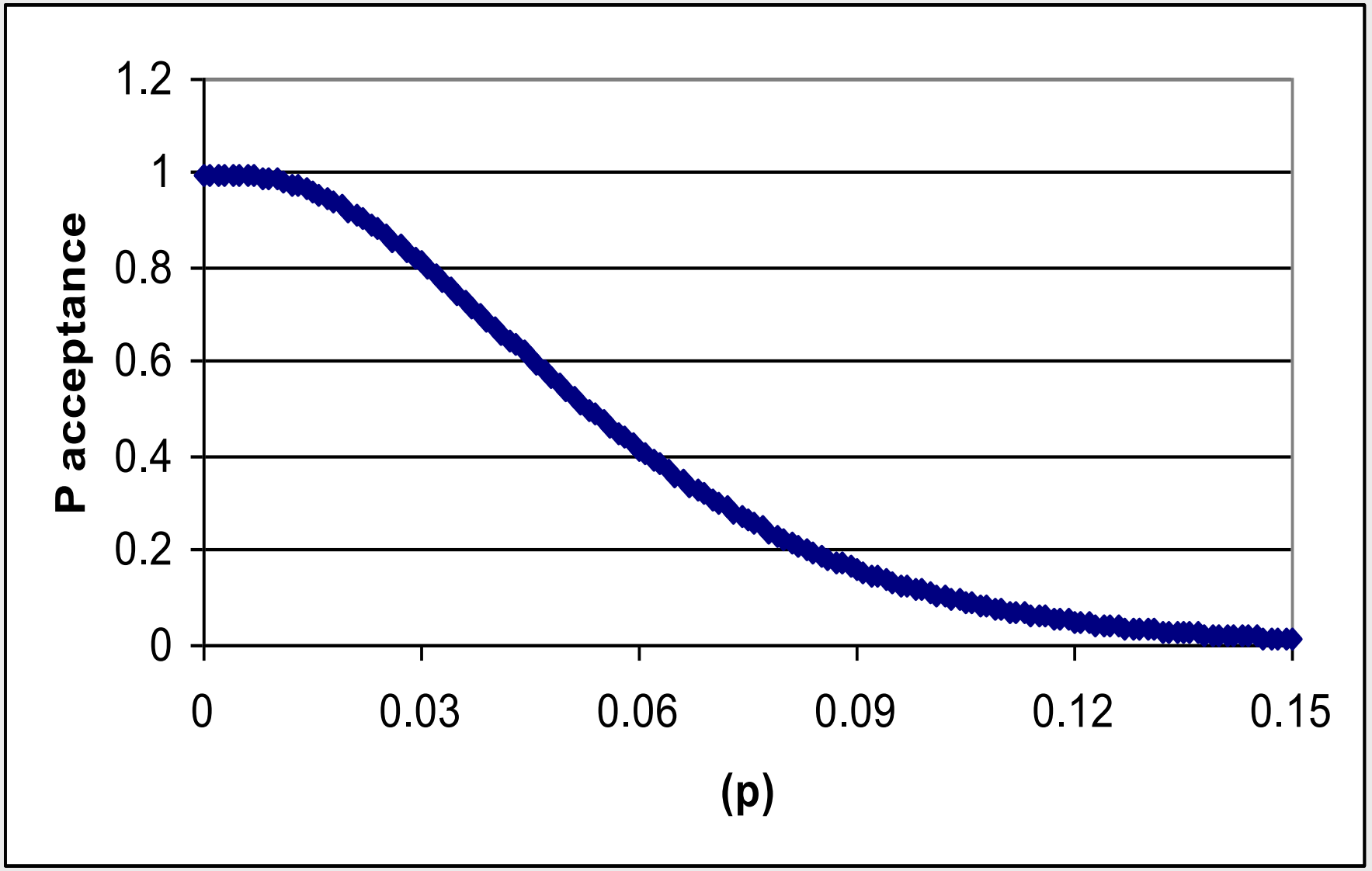
Sample Calculations

- Binomial table only goes up to n=20.
- Approximate Binomial by Poisson, u=np.
- Calculate p(2 or less). This is Paccept.

- Example n=50, p=0.03, u=1.5. $P(x \leq 2)=0.809$.

- Vary p from 0 to 0.15.

Single sampling plan n = 50, c= 2



Producer and Consumer Risk

- Assume AQL(acceptable quality level) is 0.01. Then Paccept = .986.
- Producer's Risk is 1-0.986 = 0.014.

- Assume LQL(limiting quality level) is 0.11. Then Paccept = 0.076.
- Consumer's Risk is 0.076.

Designing Plan Performance

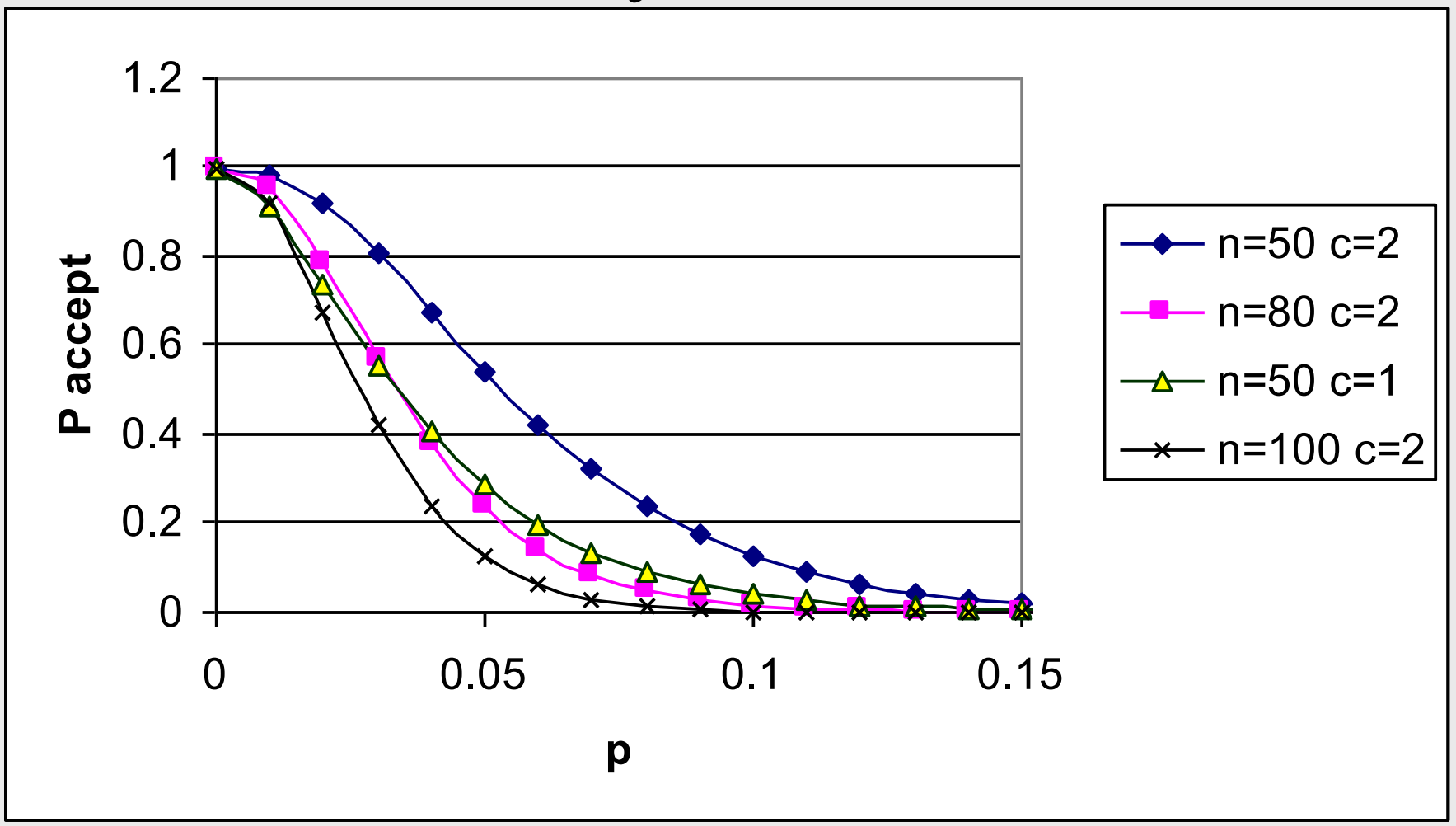
- Vary n and c to obtain different OC curves.
- Single and multiple sampling.
- Refer to standard published sampling plans.

Double Sampling Plan

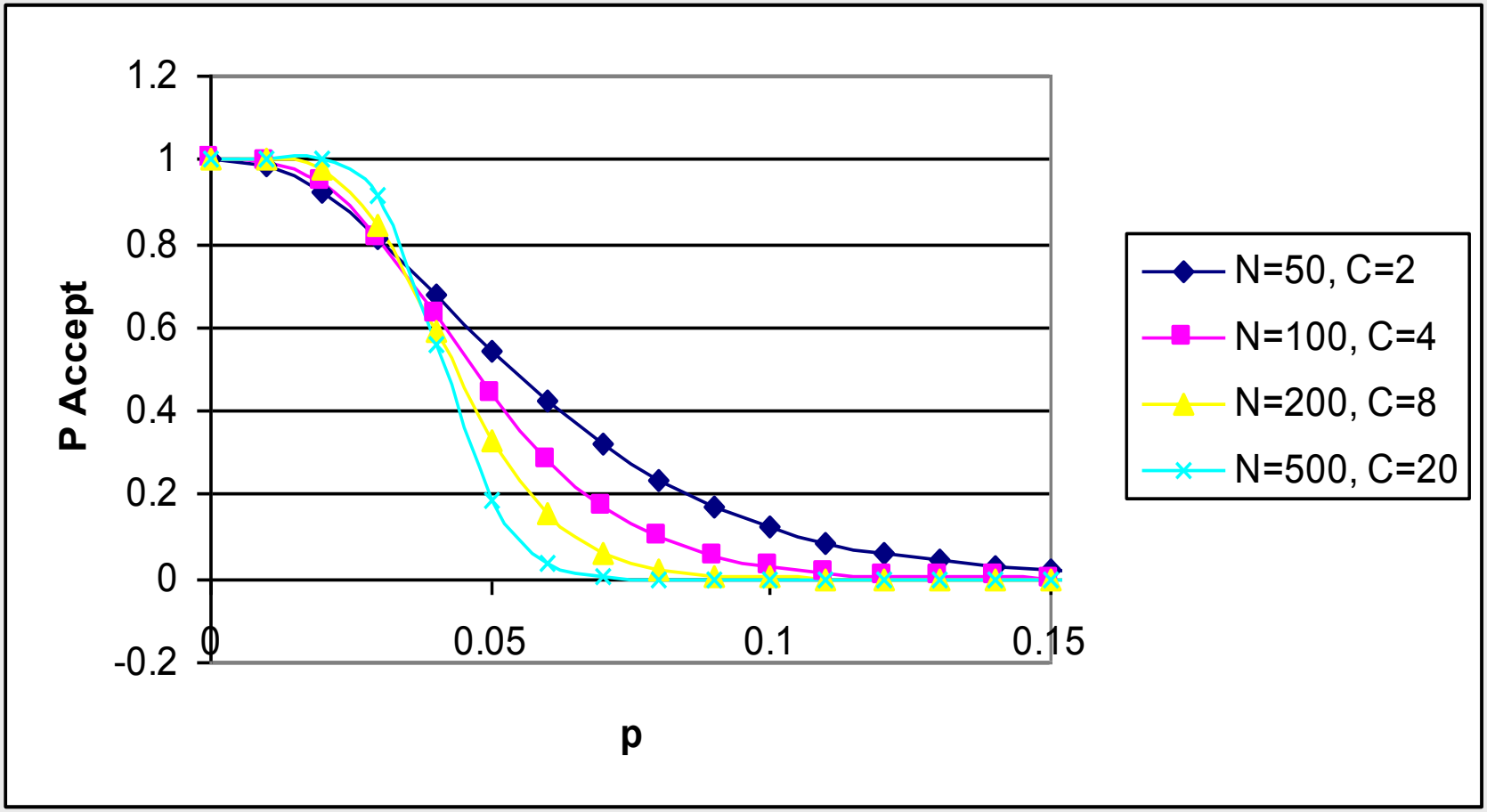
- Application of double sampling requires that a first sample of size *n1* is taken at random from the (large) lot. The number of defectives is then counted and compared to the first sample's acceptance number *a1* and rejection number *r1*. Denote the number of defectives in sample 1 by *d1* and in sample 2 by *d2*, then:
  - If $d1 <= a1$, the lot is accepted.
    If $d1 >= r1$, the lot is rejected.
    If $a1 < d1 < r1$, a second sample is taken.

- If a second sample of size *n2* is taken, the number of defectives, *d2*, is counted. The total number of defectives is $D2 = d1 + d2$. Now this is compared to the acceptance number *a2* and the rejection number *r2* of sample 2. In double sampling, $r2 = a2 + 1$ to ensure a decision on the sample.
  - If $D2 <= a2$, the lot is accepted.
    If $D2 >= r2$, the lot is rejected.

Vary n and c


1.2
1
0.8
0.6
0.4
0.2
0
P accept
0
0.05
0.1
0.15
p
n=50 c=2
n=80 c=2
n=50 c=1
n=100 c=2

Vary n and c


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
P Accept
0
0.05
0.1
0.15
p
N=50, C=2
N=100, C=4
N=200, C=8
N=500, C=20

Class Problem

- Acceptance Sampling Plan – n=30,c=1
- Draw the OC Curve
- What is Producer's risk if AQL is 0.02.
- What is Consumer's risk if LQL is 0.1.

OC Curve Worksheet

n=30, c=1

p	np (=u)	Paccept (x<=1)
0		1
0.02		
0.04		
0.06		
0.08		
0.1		
0.12		

Plot Paccept vs. p