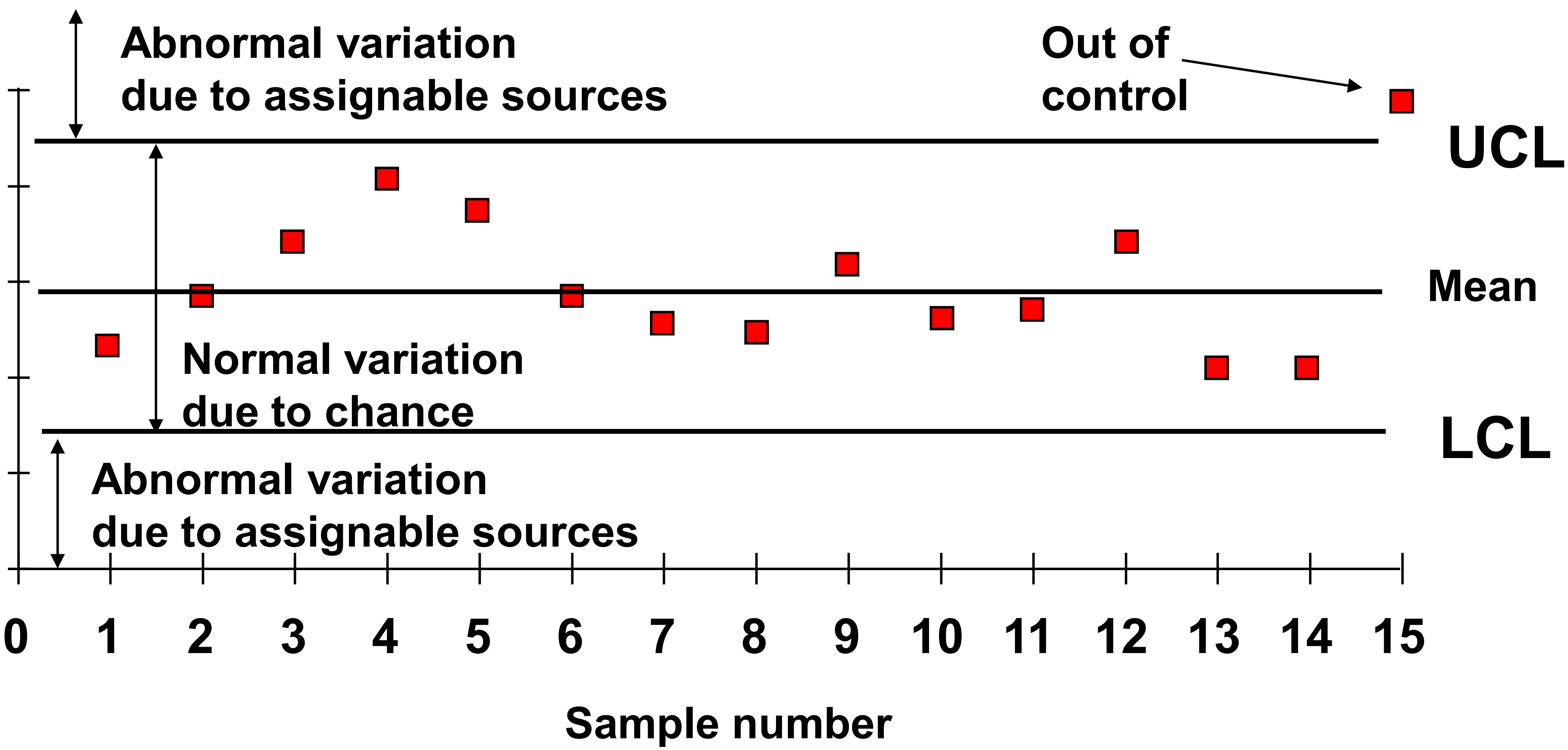
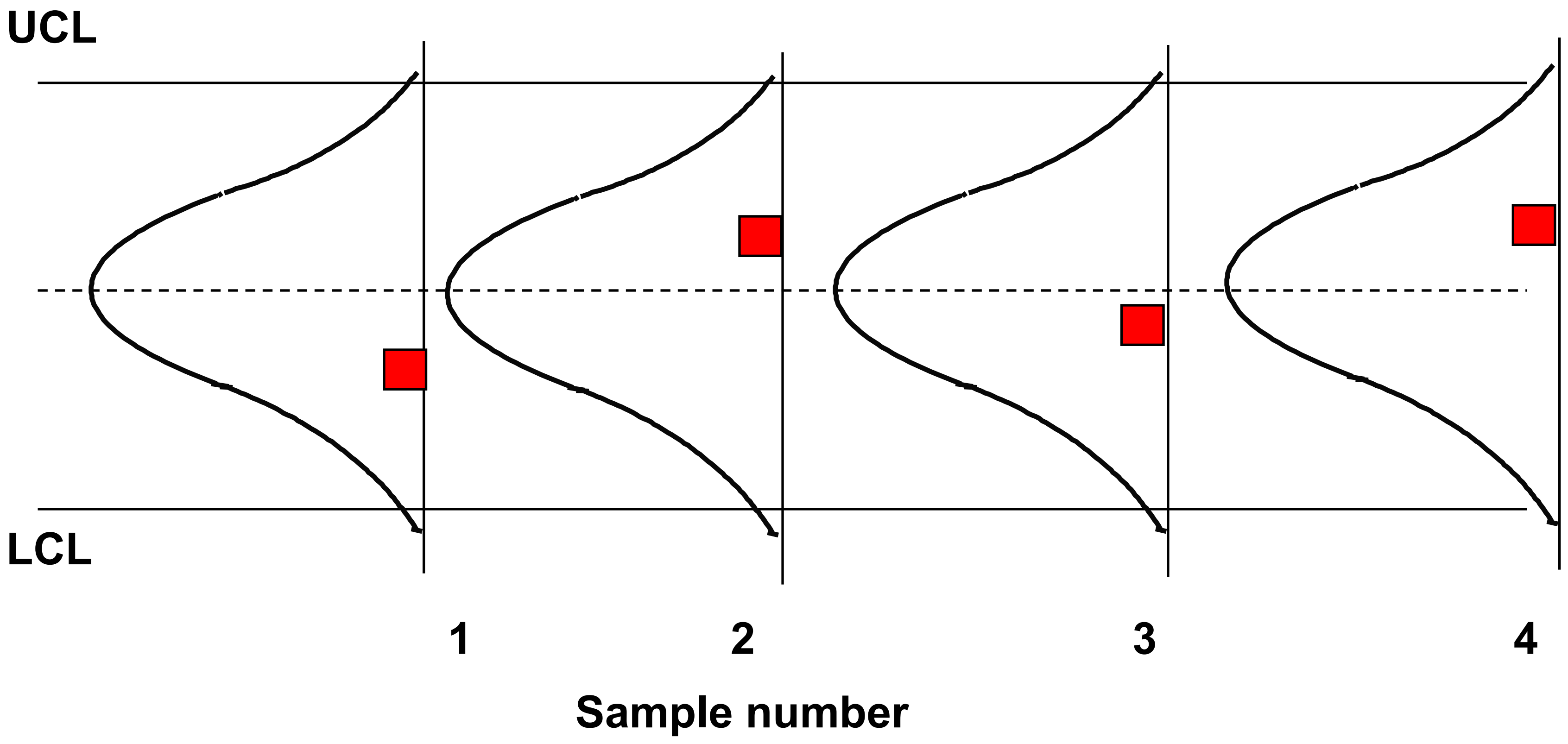
Control Chart



Observations from Sample Distribution



Control Charts

- Control charts for variables (measurable quantities), e.g. length, temperature
  - Mean control charts
    - To check mean
  - Range control charts
    - To check variability
- Control charts for attributes, e.g. fit, defective
  - p-charts
    - To check proportion of defectives (occurrences)
  - c-charts
    - To check the number of defectives (occurrences)

Mean control chart

$$Grand\ mean\ \bar{\bar{x}} = average\ of\ \bar{x}$$

$$UCL = \bar{\bar{x}} + z\sigma_{\bar{x}} = \text{grand mean plus a multiple of standard deviation}$$

$$LCL = \bar{\bar{x}} - z\sigma_{\bar{x}} = \text{grand mean minus a multiple of standard deviation}$$

$$\mathrm{z} = \frac{UCL - \bar{\bar{x}}}{\sigma_{\bar{\mathrm{x}}}} = \frac{\mathrm{norminv}(1 - \alpha/2, \bar{\bar{\mathrm{x}}}, \sigma_{\bar{\mathrm{x}}}) - \bar{\bar{x}}}{\sigma_{\bar{\mathrm{x}}}}$$

Most often z is set to 2 or 3.

If the standard deviation of the sample means is not known, use the average of sample ranges to get the limits:

$$\bar{R} = \text{average of sample ranges R}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = \text{grand mean plus a multiple of the average of sample ranges}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = \text{grand mean minus a multiple of the average of sample ranges}$$

Multiplier A_2 depends on n and is available in Table 10-2.

Range Control Chart

$$UCL = D_4\overline{R} = \text{A multiple of the average of sample ranges}$$
$$LCL = D_3\overline{R} = \text{A multiple of the average of sample ranges}$$

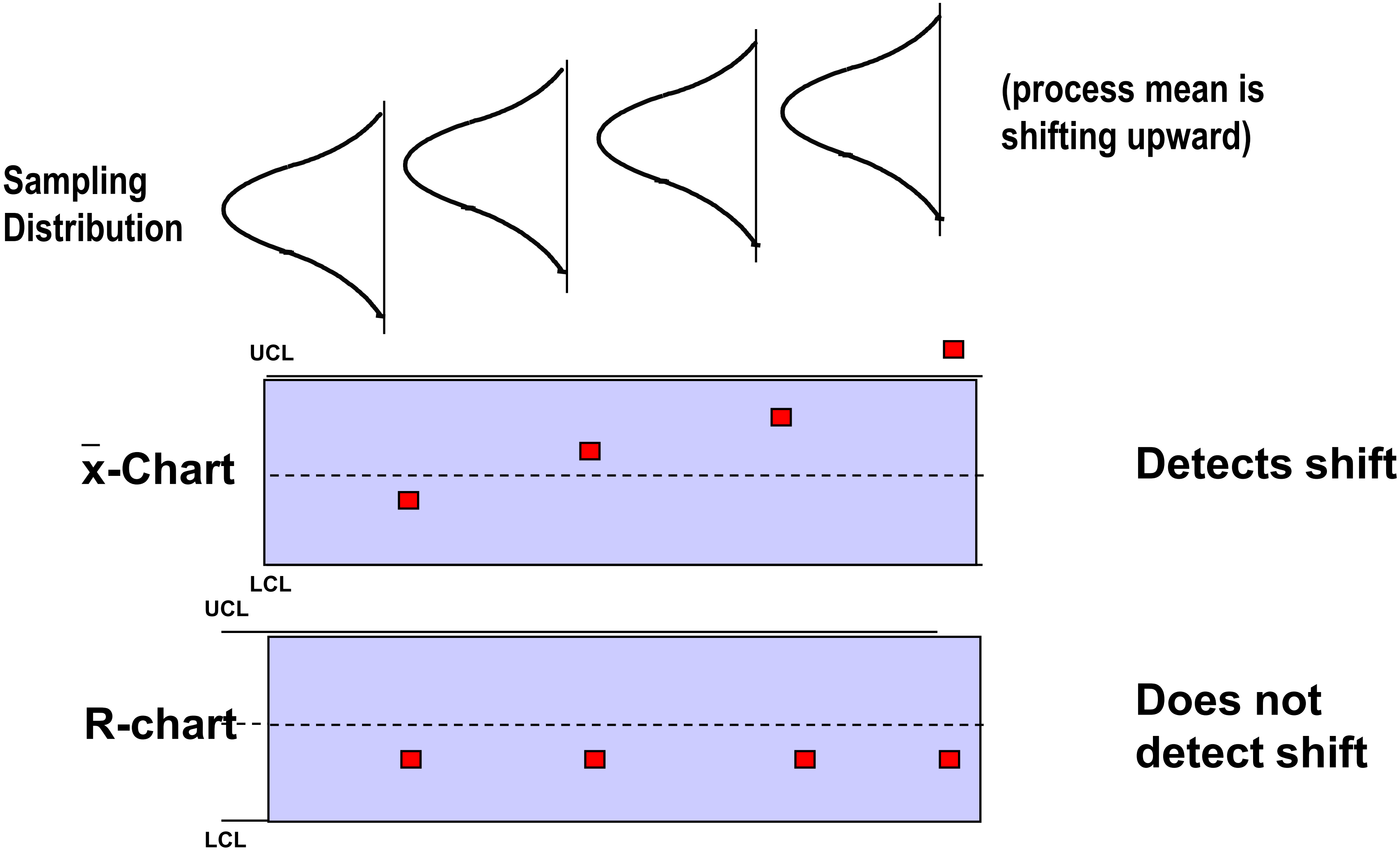
Multipliers D_4 and D_3 depend on n and are available in Table 10-2.

EX: In the last five years, the range of GMAT scores of incoming PhD class is 88, 64, 102, 70, 74. If each class has 6 students, what are UCL and LCL for GMAT ranges?

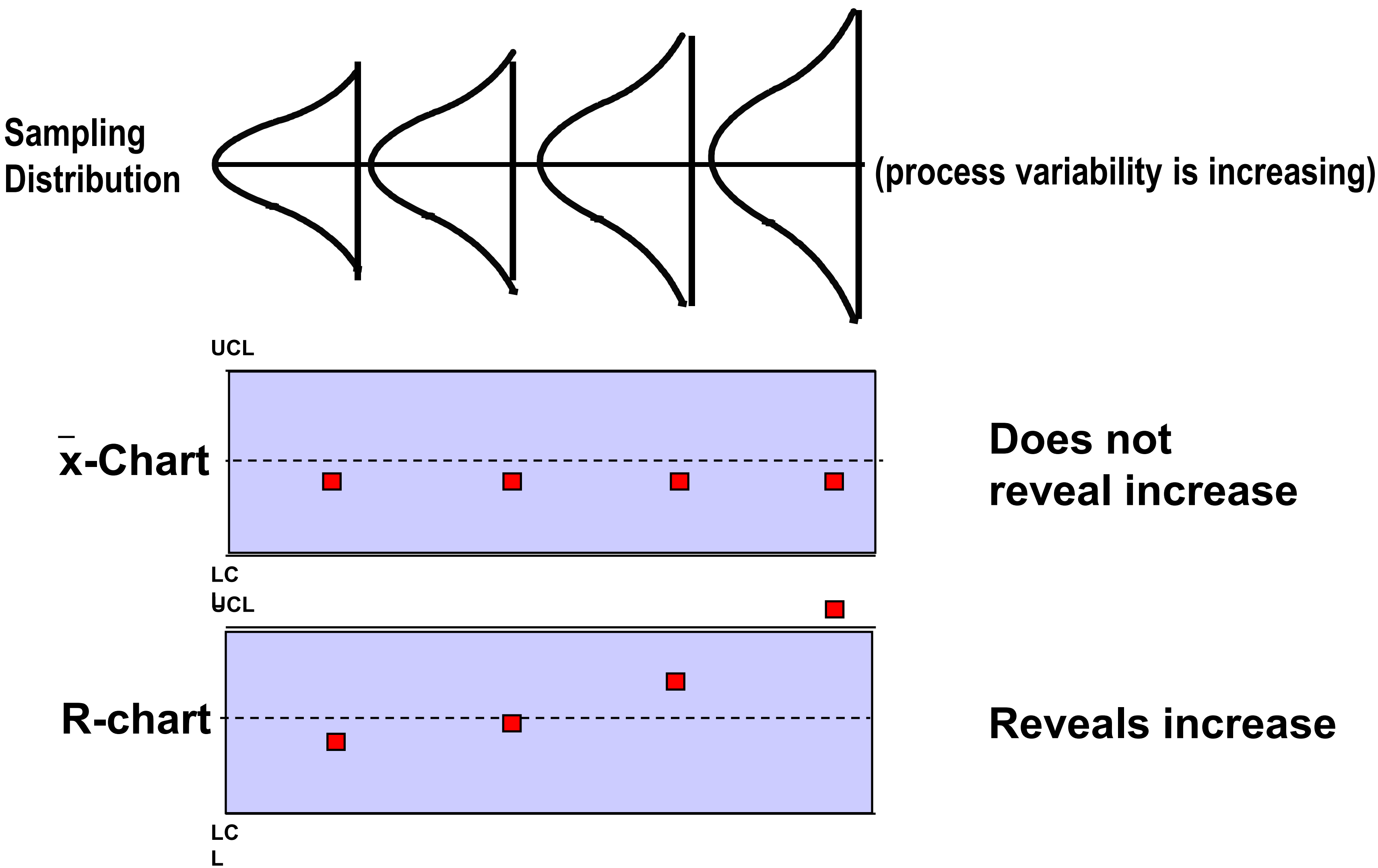
$$\overline{R} = (88 + 64 + 102 + 70 + 74)/5 = 79.6.\ \text{For n} = 6, \text{D}_4 = 2,\ \text{D}_3 = 0.$$
$$UCL = D_4\overline{R} = 2 * 79.6 = 159.2 \quad LCL = D_3\overline{R} = 0 * 79.6 = 0$$

Are the GMAT ranges in control?

Mean and Range Charts: Which?



Mean and Range Charts: Which?



Use of p-Charts

- p=proportion defective, assumed to be known
- When observations can be placed into two categories.
  - Good or bad
  - Pass or fail
  - Operate or don't operate
  - Go or no-go gauge

$$UCL = p + z\sigma_p \qquad \text{LCL} = p - z\sigma_p$$

$$where \quad \sigma_p = \sqrt{\frac{p(1-p)}{n}} \text{ , } z \text{ as before}$$

Use of c-Charts

- c=number of occurrences per unit
- Use only when the number of occurrences per unit can be counted.
    - Scratches, chips, dents, or errors per item
    - Cracks or faults per unit of distance
    - Breaks or Tears per unit of area
    - Bacteria or pollutants per unit of volume
    - Calls, complaints, failures per unit of time

$$UCL = c + z\sqrt{c} \qquad \mathrm{LCL} = c - z\sqrt{c}$$

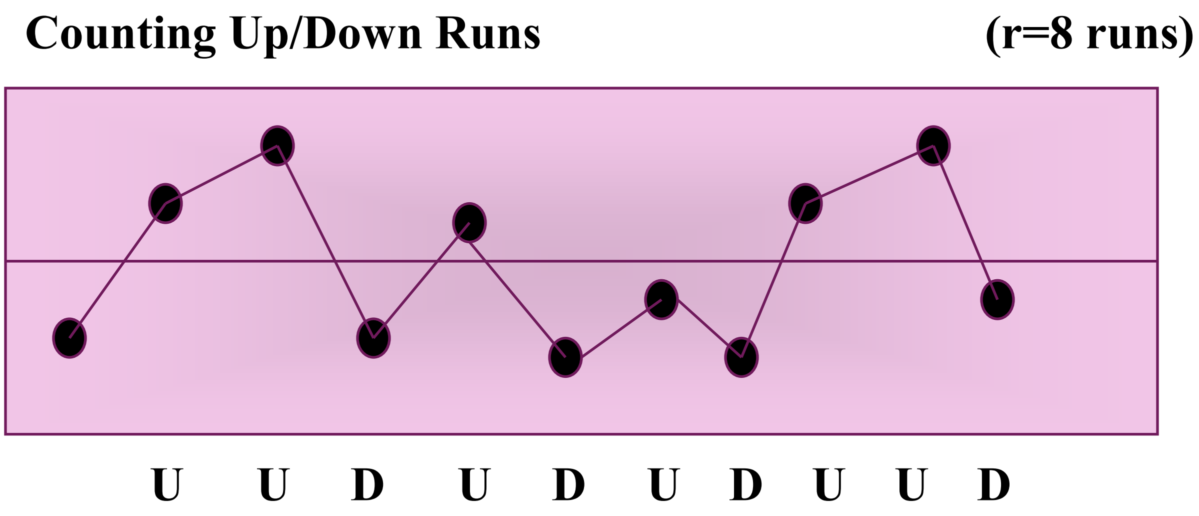
$$\text{if c is not known, use the average } \bar{\mathrm{c}}$$

C-chart Example

- While the nuclear submarine Kursk was being raised in the Barents sea (between Svalbard, No and Novaya Zemlya, Ru), which took 15 hours, engineers took a reading of number of Geiger counts per hour to detect any increase in radiation levels. Should they have stopped before 5th or 10th hour given 3-sigma control and the readings data: 42, 48, 50, 45, 52, 66, 64, 84, 92, 76.
  - At the 5th hour, average number of counts=47.4, stdev of counts=6.88, UCL=47.4+3*6.88=68.05, LCL=47.4-3*6.88=26.75. Do not stop.
  - At the 10th hour, average number of counts=61.9, stdev of counts=7.87, UCL=61.9+3*7.87=85.51, LCL=61.9-3*7.87=38.29. Stop, 9th reading is out of control.

Up and Down Run Charts

- If all readings are in control, is the process really in control?
- There could be trends in readings even when they are in control.



Up and Down Run Charts

$$UCL = E(r) + z\sigma_r = \text{Expected runs plus a multiple of stdev of runs}$$

$$LCL = E(r) - z\sigma_r = \text{Expected runs minus a multiple of stdev of runs}$$

$$\text{E(r)} = \frac{\text{2K - 1}}{3} \quad \text{and} \quad \sigma_r = \sqrt{\frac{16K - 29}{90}}$$

$$\text{K} = \text{Number of samples}$$

EX: What are 3-sigma UCL and LCL for the number of runs in 50 samples?

$$\text{K} = 50,\ \text{E(r)} = \frac{\text{2K - 1}}{3} = 33 \quad \text{and} \quad \sigma_r = \sqrt{\frac{16K - 29}{90}} = 2.92$$

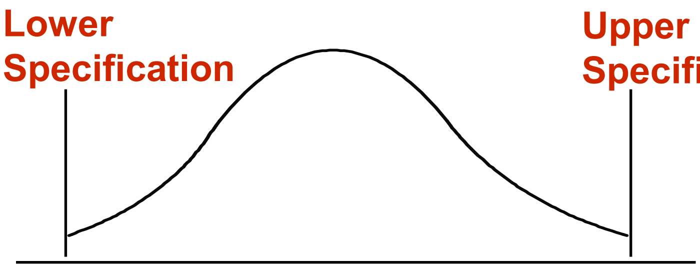
$$UCL = E(r) + z\sigma_r = 33 + 3 * 2.92$$

$$LCL = E(r) - z\sigma_r = 33 - 3 * 2.92$$

Process Capability

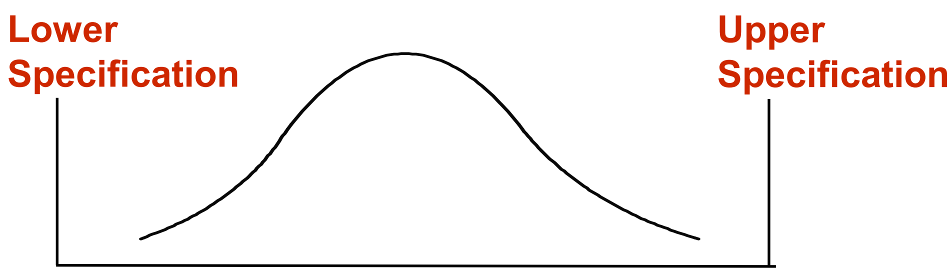
- Tolerances/Specifications
  - Requirements of the design or customers
- Process variability
  - Natural variability in a process
  - Variance of the measurements coming from the process
- Process capability
  - Process variability relative to specification
  - Capability=Process specifications / Process variability

Process Capability:
Specification limits are not control chart limits

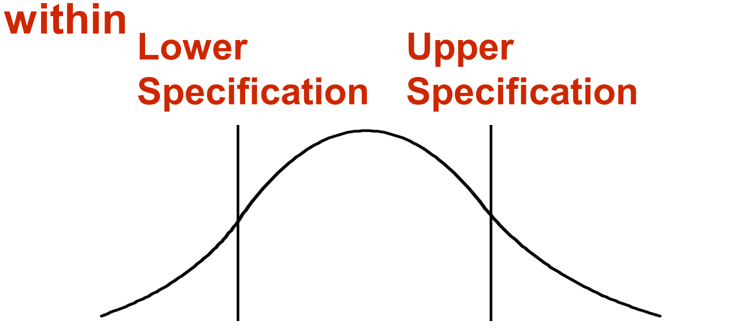


Process variability matches specifications

Sampling Distribution is used



Process variability well within specifications



Process variability exceeds specifications

Process Capability Ratio

When the process is **centered**, process capability ratio

$$Cp = \frac{\text{Upper specification} - \text{lower specification}}{6\sigma}$$

A capable process has large Cp.

Example: The standard deviation, of sample averages of the midterm 1scores obtained by students whose last names start with R, has been 7. The SOM management requires the scores not to differ by more than 50% in an exam. That is the highest score can be at most 50 points above the lowest score. Suppose that the scores are centered, what is the process capability ratio?

Answer: 50/42

Process Capability Ratio

When the process is **not centered**, process capability ratio

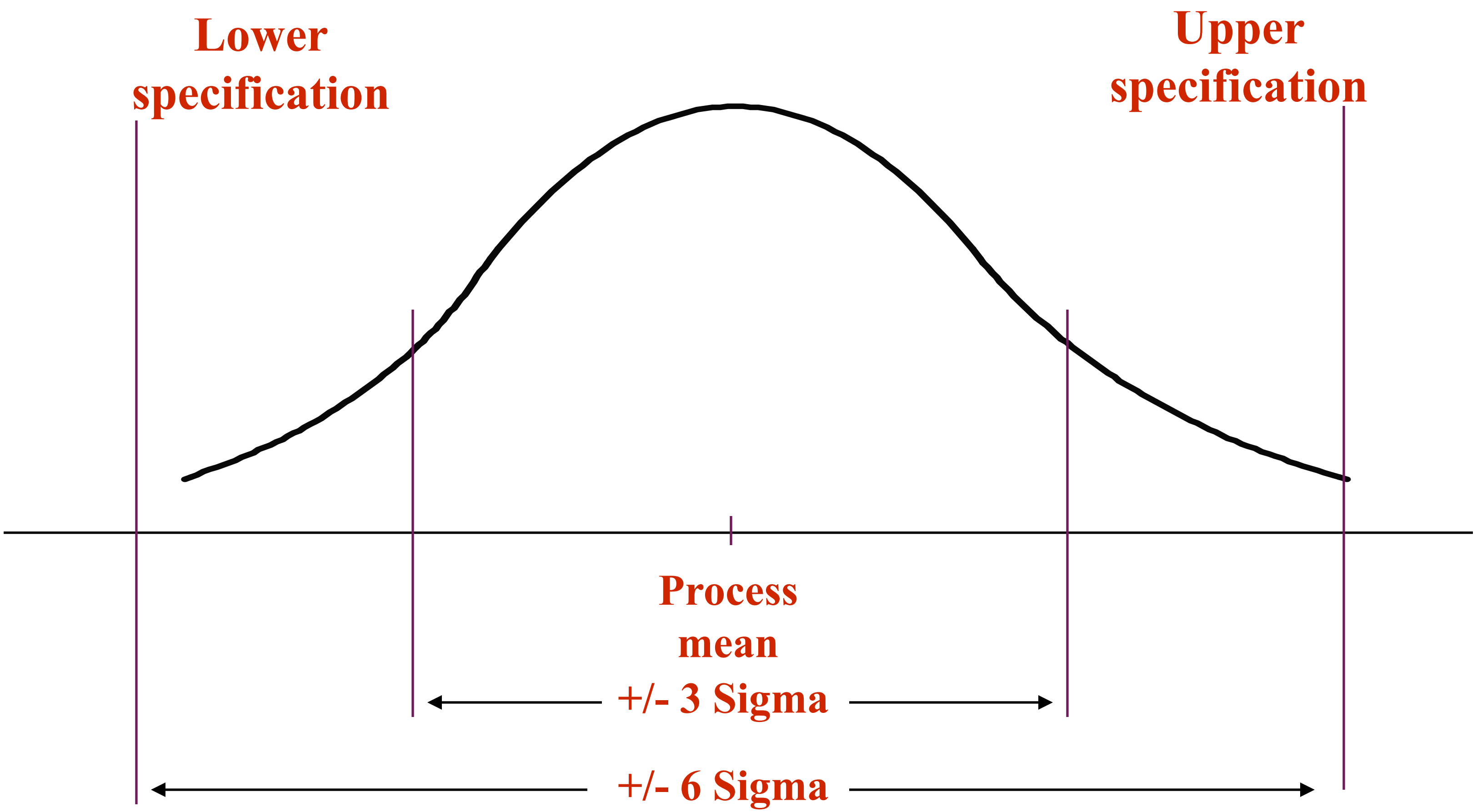
$$Cpk = \frac{\text{Min}\{\text{Process mean - lower spec , Upper spec - Process mean}\}}{3\sigma}$$

When the process is not centered, the closest spec to mean determines the capability of the process because that spec is likely to be more of a limiting factor than the other.

Example: Suppose that the process is not centered in the previous example and the SOM wants all the scores to fall within 50% and 100%. What is the Capability ratio if the average score was 70?

Answer: From the lower limit, we have (70-50)/21
From the upper limit, we have (100-70)/21
Then the ratio is 20/21

3 Sigma and 6 Sigma Quality



Chapter 10 Supplement

Acceptance Sampling

Acceptance Sampling

- Acceptance sampling: Is a lot of N products good if a random sample of n ($n<N$) products contain only c defects?
  - For example take a sample of 10(=n) milk bottles out of every 100(=N). If 1(=c) or more bottles do not fit specifications, reject the entire lot of 100 bottles.
- c is determined to balance type I and type II errors.
- This is a smart compromise between 100% inspection and no inspection.
- Generally used for input/output inspection.

Why not to emphasize Acceptance Sampling (AS)

- AS plans have no clearly stated economic objective. They target some levels of type I and II errors.
- AS incorporate an attitude of punishment by rejecting entire lots after examining small samples. This feeds the mistrust between supplier and the customer.
- AS does not attempt to find the root cause of defectives. It merely detects defectives. Real problem is actually finding the root cause. Some people say that:
  - "AS provides elegant solutions to balance type I and II errors by making a type III error: solving the wrong problem".