## Lecture <br> On

## Signal Flow Graph

## Flow of PPT

- What is Signal Flow Graph (SFG)?
- Definitions of terms used in SFG
- Rules for drawing of SFG
- Mason's Gain formula
- SFG from simultaneous eqns
- SFG from differential eqns
- Examples
- Solution of a problem by Block diagram reduction technique and SFG
- SFG from a given Transfer function
- Examples


## What is Signal Flow Graph?

- SFG is a diagram which represents a set of simultaneous equations.
- This method was developed by S.J.Mason. This method does n't require any reduction technique.
- It consists of nodes and these nodes are connected by a directed line called branches.
- Every branch has an arrow which represents the flow of signal.
- For complicated systems, when Block Diagram (BD) reduction method becomes tedious and time consuming then SFG is a good choice.


## Comparison of BD and SFG

## block diagram:



In this case at each step block diagram is to be redrawn. That's why it is tedious method. So wastage of time and space.

## signal flow graph:



Only one time SFG is to be drawn and then Mason's gain formula is to be evaluated.
So time and space is saved.

## SFG

Alternative to block diagram;
Consists only branches (systems), and nodes (signals)


## Definition of terms required in SFG

Node: It is a point representing a variable.

$$
\mathrm{x}_{2}=\mathrm{t}_{12} \mathrm{x}_{1}+\mathrm{t}_{32} \mathrm{x}_{3}
$$



In this SFG there are 3 nodes.

Branch : A line joining two nodes.


Input Node: Node which has only outgoing branches.
$X_{1}$ is input node.

## Output node/ sink node: Only incoming branches.

Mixed nodes: Has both incoming and outgoing branches.

Transmittance : It is the gain between two nodes. It is generally written on the branch near the arrow.


- Path : It is the traversal of connected branches in the direction of branch arrows, such that no node is traversed more than once.
- Forward path: A path which originates from the input node and terminates at the output node and along which no node is traversed more than once.
- Forward Path gain : It is the product of branch transmittances of a forward path.


$$
P_{1}=G_{1} G_{2} G_{3} G_{4}, \quad P_{2}=G_{5} G_{6} G_{7} G_{8}
$$

Loop : Path that originates and terminates at the same node and along which no other node is traversed more than once.
Self loop: Path that originates and terminates at the same node.

Loop gain: it is the product of branch transmittances of a loop.
Non-touching loops: Loops that don't have any common node or branch.


$$
\begin{aligned}
& L_{1}=G_{2} H_{2} \quad L_{2}=H_{3} \\
& L_{3}=G_{7} H_{7}
\end{aligned}
$$

Non-touching loops are L1 \& L2, L1 \& L3, L2 \&L3

## SFG terms representation



## Rules for drawing of SFG from Block diagram

- All variables, summing points and take off points are represented by nodes.
- If a summing point is placed before a take off point in the direction of signal flow, in such a case the summing point and take off point shall be represented by a single node.
- If a summing point is placed after a take off point in the direction of signal flow, in such a case the summing point and take off point shall be represented by separate nodes connected by a branch having transmittance unity.


## Mason's Gain Formula

- A technique to reduce a signal-flow graph to a single transfer function requires the application of one formula.
- The transfer function, $C(s) / R(s)$, of a system represented by a signal-flow graph is

$$
G(s)=\frac{C(s)}{R(s)}=\frac{\sum_{k} \mathcal{R}_{k} \Delta_{k}}{\Delta}
$$

$k=$ number of forward path
$P_{k}=$ the kth forward path gain
$\Delta=1-(\Sigma$ loop gains $)+(\Sigma$ non-touching loop gains taken two at a time) - ( $\Sigma$ non-touching loop gains taken three at a time)+ so on.
$\Delta_{k}=1$ - (loop-gain which does not touch the forward path)

## Ex: SFG from BD



EX: To find T/F of the given block diagram

(a)

(b)

## Identification of Forward Paths



$$
\begin{aligned}
& P_{2}=1.1 \cdot G_{2} \cdot G_{3} \cdot 1 \\
& =G_{2} G_{3}
\end{aligned}
$$

## Individual Loops


$L_{1}=G_{1} G_{2} H_{1}$
$L_{2}=-G_{2} G_{3} H_{2}$
$L_{3}=-G_{4} H_{2}$


$\mathrm{L}_{4}=-\mathrm{G}_{1} \mathrm{G}_{4}$

(b)
$\mathrm{L}_{5}=-\mathrm{G}_{1} \mathrm{G}_{2} \mathrm{G}_{3}$

## Construction of SFG from simultaneous equations

$$
\begin{aligned}
& y_{2}=t_{21} y_{1}+t_{29} y_{9} \\
& y_{9}=t_{92} y_{2}+t_{39} y_{3}+t_{31} y_{1} \\
& y_{4}=t_{43} y_{9}+t_{42} y_{2} \\
& y_{5}=t_{54} y_{4} \\
& y_{6}=t_{65} y_{51}+t_{64} y_{4}
\end{aligned}
$$

$$
\underbrace{y_{2}=t_{21} y_{1}+t_{2 \exists} y_{G}}_{r_{1}}
$$

## $y_{5}=t_{54} y_{4}$



