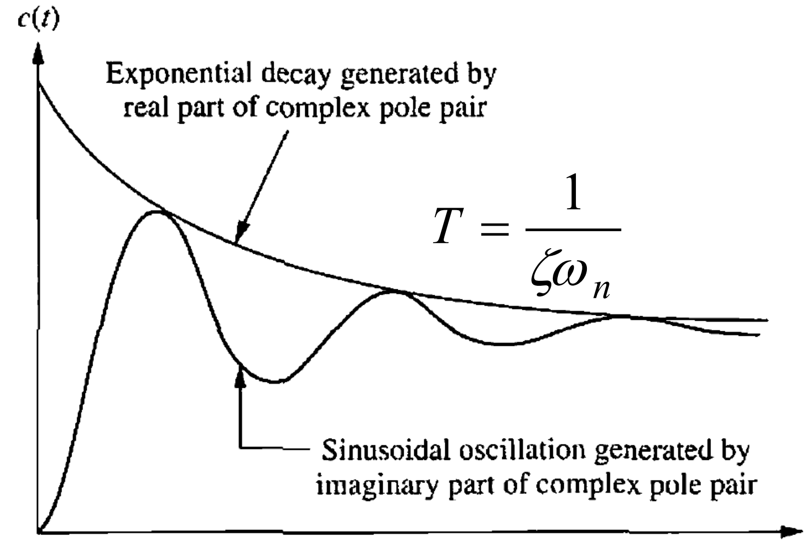
Time Domain Specifications (Settling Time)

- Settling time (2%) criterion
  - Time consumed in exponential decay up to 98% of the input.

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$



- Settling time (5%) criterion
  - Time consumed in exponential decay up to 95% of the input.

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

Summary of Time Domain Specifications

Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}}$$

Peak Time

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

Settling Time (2%)

$$t_s = 4T = \frac{4}{\zeta \omega_n}$$

$$t_s = 3T = \frac{3}{\zeta \omega_n}$$

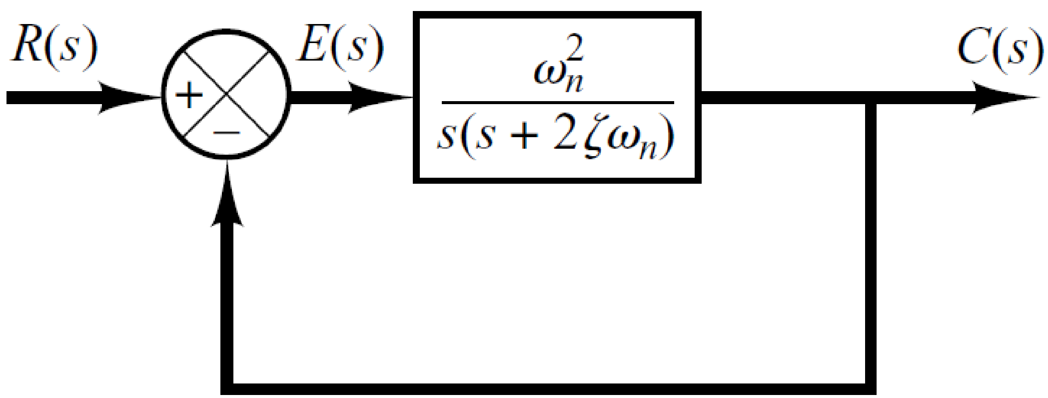
Settling Time (4%)

Maximum Overshoot

$$M_p = e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \times 100$$

Example#5

- Consider the system shown in following figure, where damping ratio is 0.6 and natural undamped frequency is 5 rad/sec. Obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time 2% and 5% criterion t_s when the system is subjected to a unit-step input.



Example#5

Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d}$$

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

Settling Time (2%)

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

Settling Time (4%)

Maximum Overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

Example#5

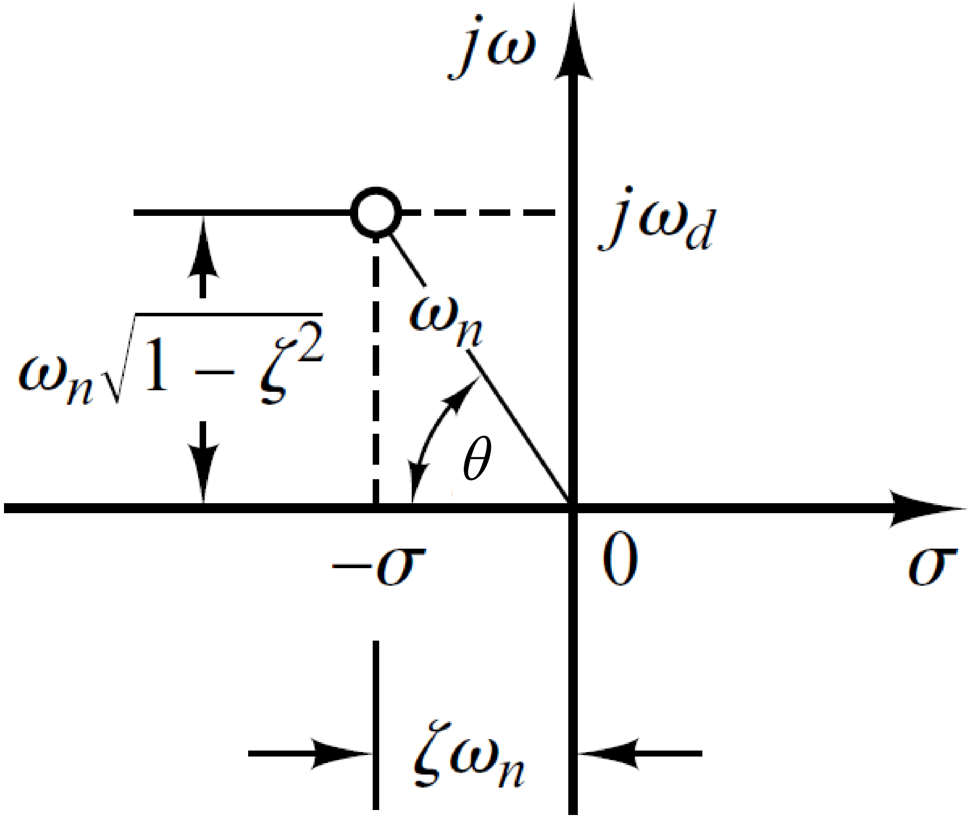
Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$t_r = \frac{3.141 - \theta}{\omega_n \sqrt{1-\zeta^2}}$$

$$\theta = \tan^{-1}\left(\frac{\omega_n \sqrt{1-\zeta^2}}{\zeta \omega_n}\right) = 0.93 \text{ rad}$$

$$t_r = \frac{3.141 - 0.93}{5\sqrt{1-0.6^2}} = 0.55s$$



Example#5

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{3.141}{4} = 0.785s$$

Settling Time (2%)

$$t_s = \frac{4}{\zeta\omega_n}$$

$$t_s = \frac{4}{0.6 \times 5} = 1.33s$$

Settling Time (4%)

$$t_s = \frac{3}{\zeta\omega_n}$$

$$t_s = \frac{3}{0.6 \times 5} = 1s$$

Example#5

Maximum Overshoot

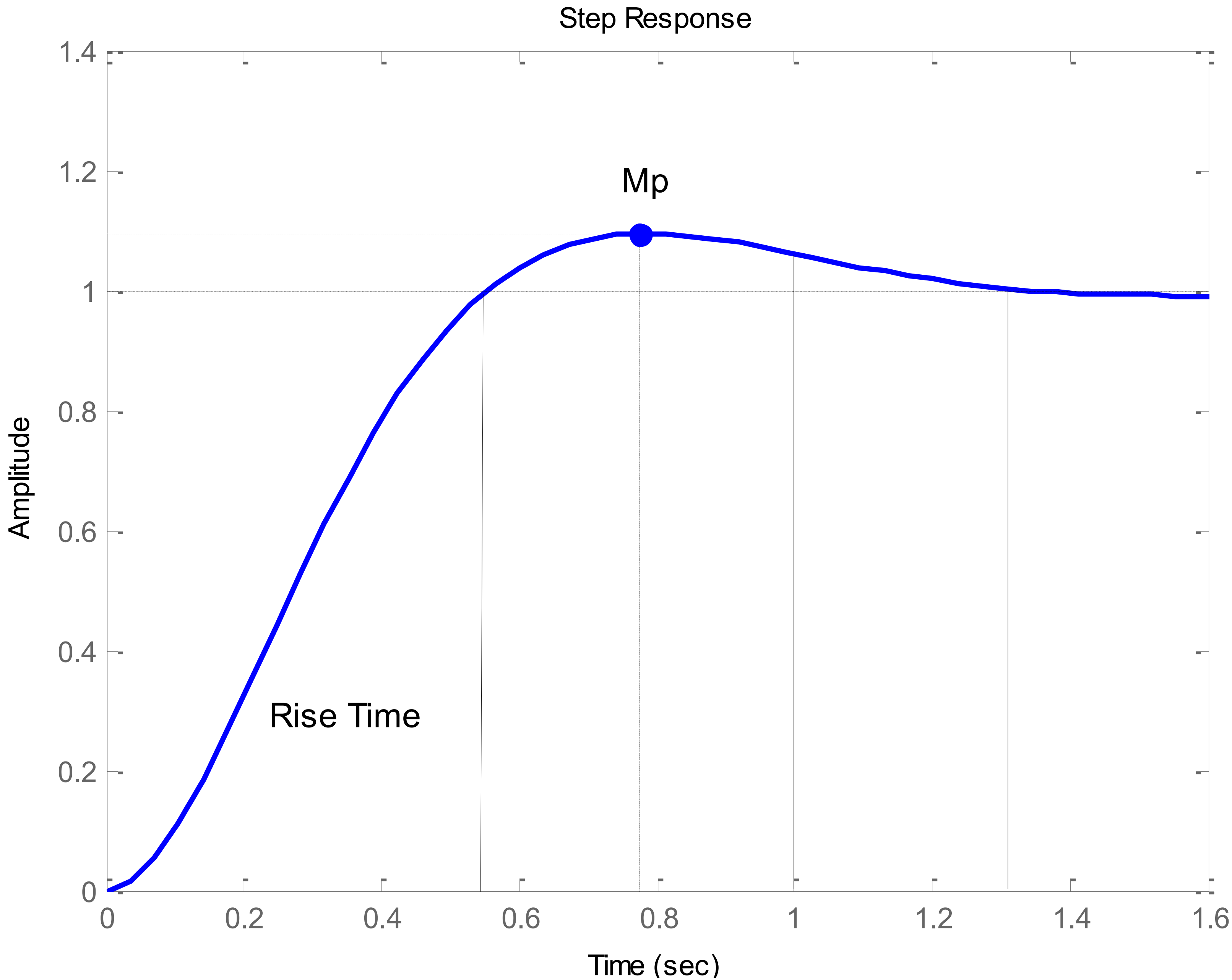
$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$M_p = e^{-\frac{3.141\times 0.6}{\sqrt{1-0.6^2}}} \times 100$$

$$M_p = 0.095 \times 100$$

$$M_p = 9.5\%$$

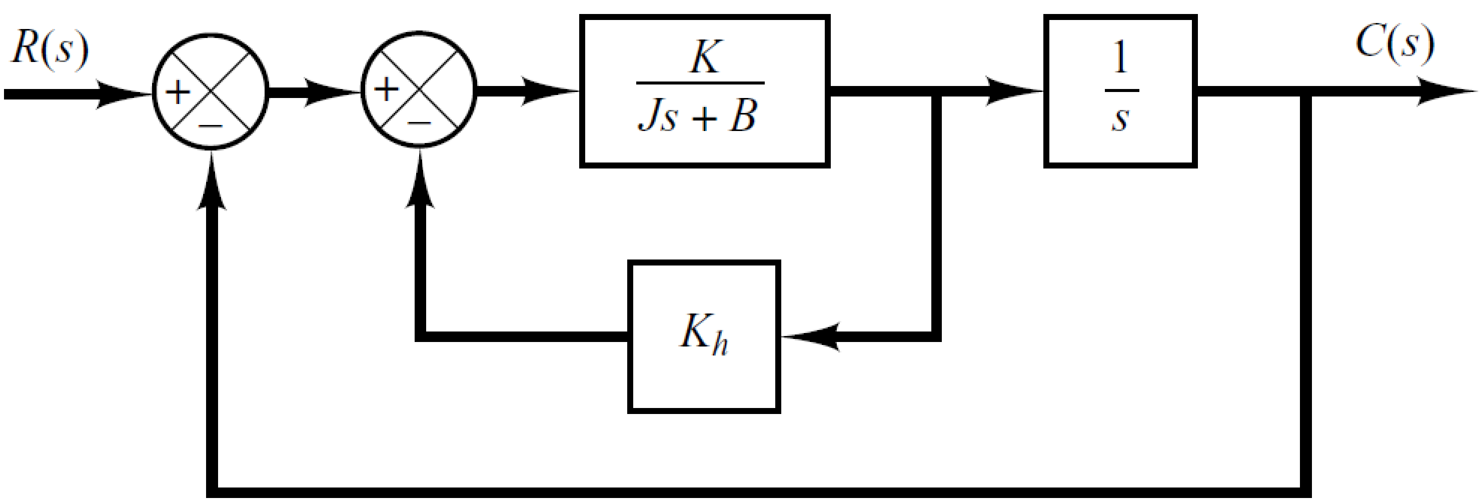
Example#5



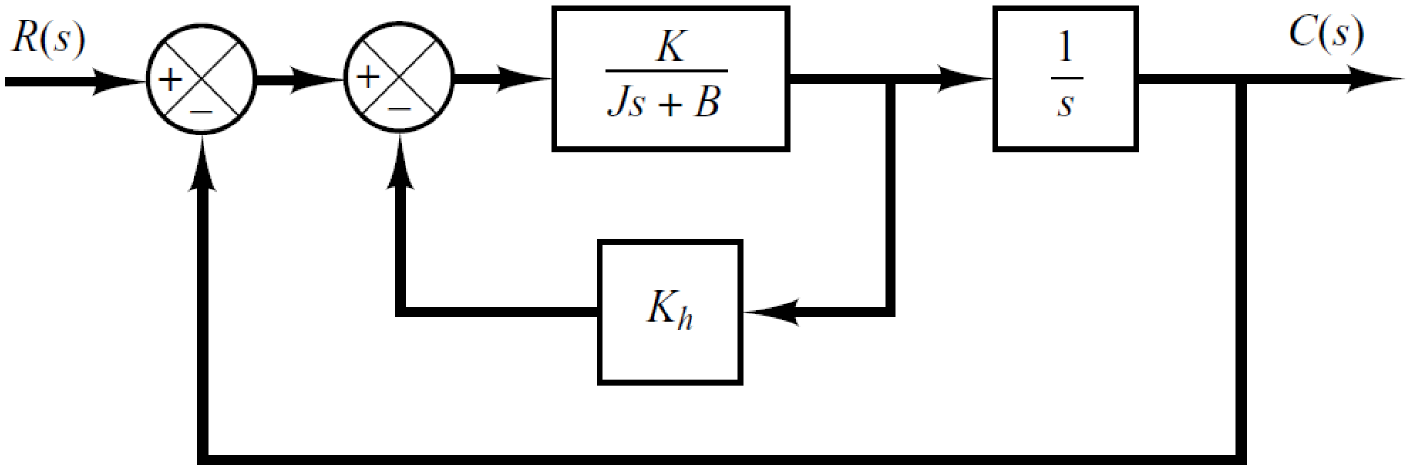
Step Response
Mp
Rise Time
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

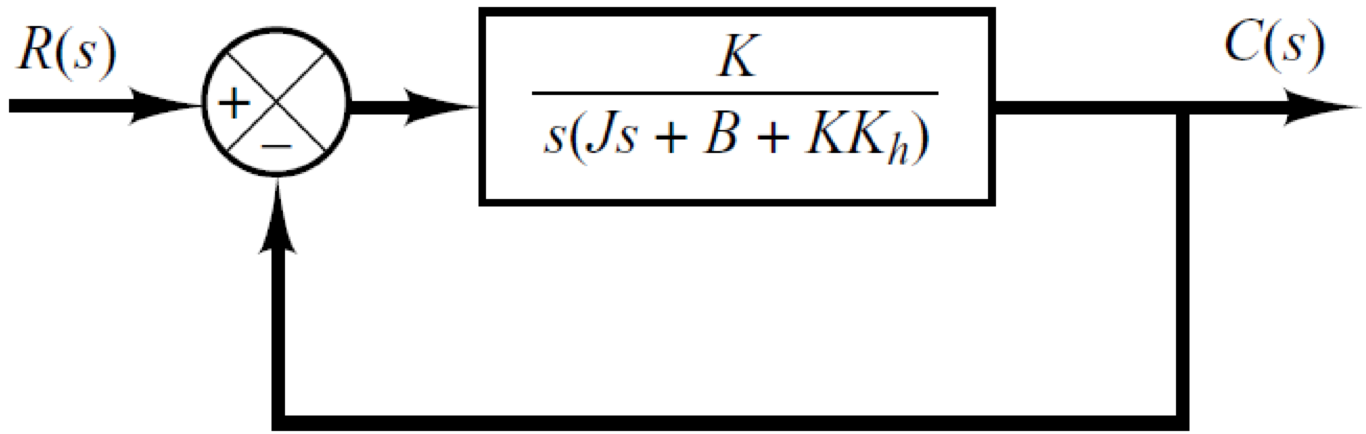
Example#6

- For the system shown in Figure-(a), determine the values of gain K and velocity-feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that J=1 kg-m^2 and B=1 N-m/rad/sec.



Example#6





$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Example#6

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Since $J = 1\ kgm^2$ and $B = 1$ Nm/rad/se

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + (1 + KK_h)s + K}$$

- Comparing above T.F with general 2nd order T.F

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K} \qquad \zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

Example#6

$$\omega_n = \sqrt{K}$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

- Maximum overshoot is 0.2.

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

$$\ln(e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}) = \ln(0.2)$$

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

$$\zeta = 0.456$$

- The peak time is 1 sec

$$t_p = \frac{\pi}{\omega_d}$$

$$1 = \frac{3.141}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = \frac{3.141}{\sqrt{1-0.456^2}}$$

$$\omega_n = 3.53$$

Example#6

$$\zeta = 0.456$$

$$\omega_n = 3.96$$

$$\omega_n = \sqrt{K}$$

$$3.53 = \sqrt{K}$$

$$3.53^2 = K$$

$$K = 12.5$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

$$0.456 \times 2\sqrt{12.5} = (1 + 12.5K_h)$$

$$K_h = 0.178$$

Example#6

$\zeta = 0.456$

$\omega_n = 3.96$

$$t_r = \frac{\pi - \theta}{\omega_n \sqrt{1-\zeta^2}}$$

$$t_s = \frac{4}{\zeta\omega_n}$$

$t_r = 0.65s$

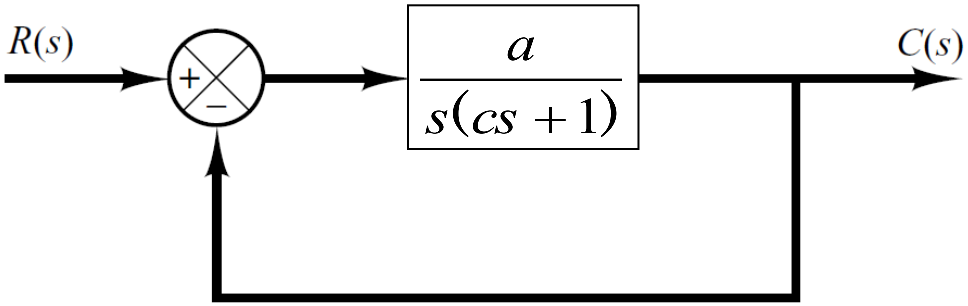
$t_s = 2.48s$

$$t_s = \frac{3}{\zeta\omega_n}$$

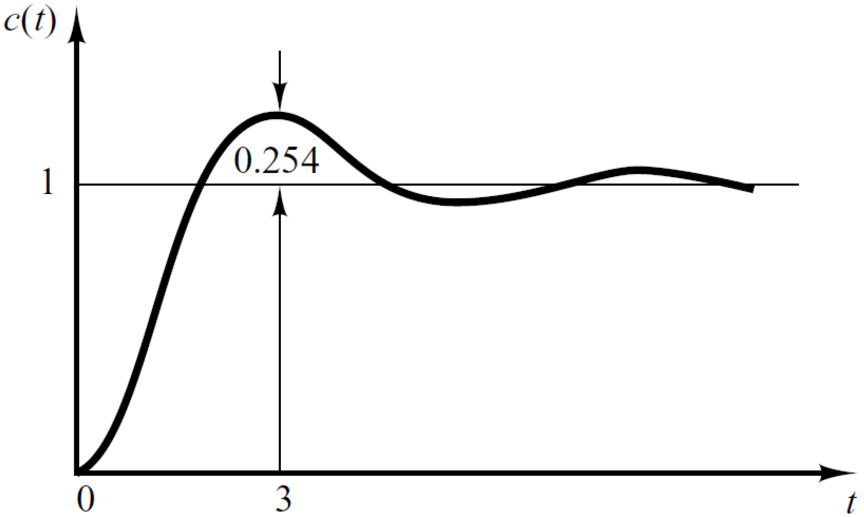
$t_s = 1.86s$

Example#7

When the system shown in Figure(a) is subjected to a unit-step input, the system output responds as shown in Figure(b). Determine the values of *a* and *c* from the response curve.



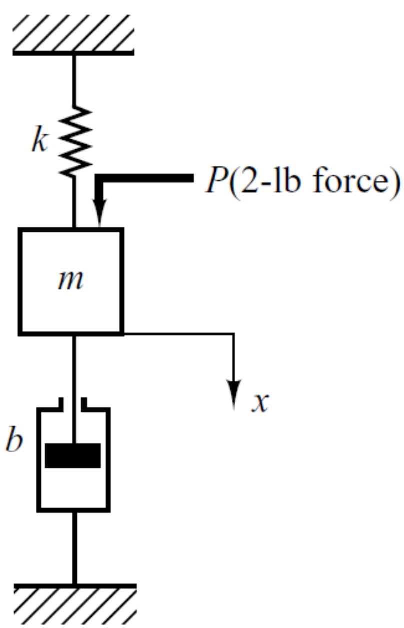
(a)



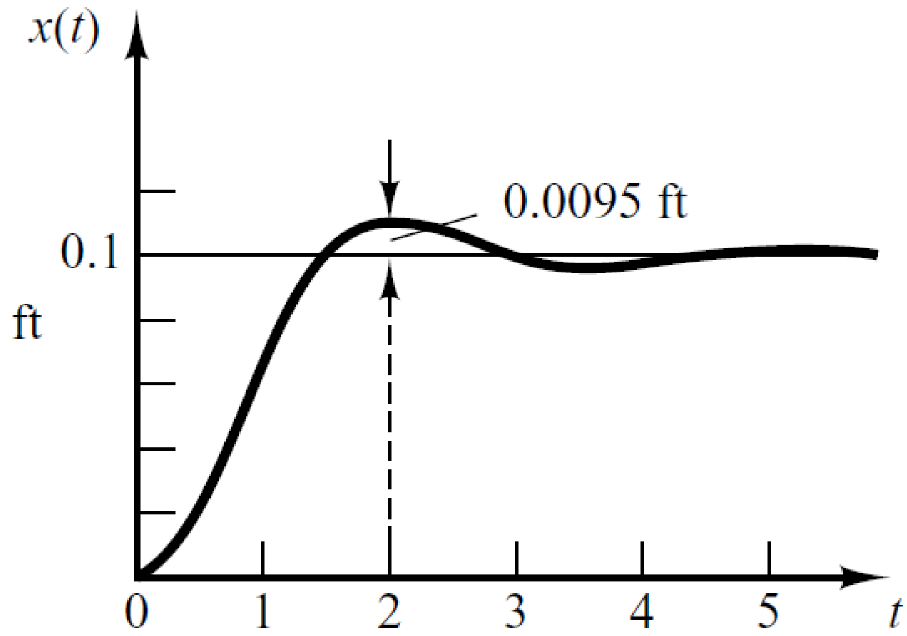
(b)

Example#8

Figure (a) shows a mechanical vibratory system. When 2 lb of force (step input) is applied to the system, the mass oscillates, as shown in Figure (b). Determine m, b, and k of the system from this response curve.



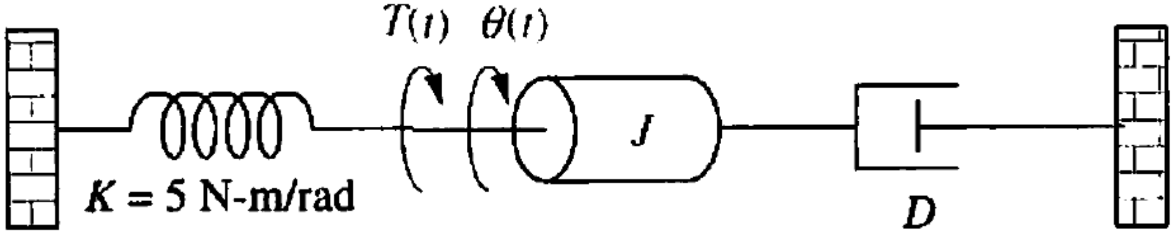
(a)



(b)

Example#9

Given the system shown in following figure, find *J and D to yield 20% overshoot* and a settling time of 2 seconds for a step input of torque *T(t).*



$$G(s) = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$T_s = 2 = \frac{4}{\zeta\omega_n}$$

$$2\zeta\omega_n = 4$$

$$\zeta = \frac{4}{2\omega_n} = 2\sqrt{\frac{J}{K}}$$

Example#9

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$\zeta = 2\sqrt{\frac{J}{K}}$$

20% overshoot implies $\zeta = 0.456$. Therefore,

$$\zeta = 2\sqrt{\frac{J}{K}} = 0.456$$

Hence,

$$\frac{J}{K} = 0.052$$

From the problem statement, $K = 5$ N-m/rad.

$$J = 0.26 \text{ kg-m}^2$$

Example#9

$$G(s) = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

$$2\zeta\omega_n = \frac{D}{J}$$

$$D = 1.04 \text{ N-m-s/rad}$$

Step Response of critically damped System ($\zeta = 1$)

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2} \xrightarrow{\text{Step Response}} C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

- The partial fraction expansion of above equation is given as

$$\frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$c(t) = 1 - e^{-\omega_n t} - \omega_n e^{-\omega_n t} t$$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$