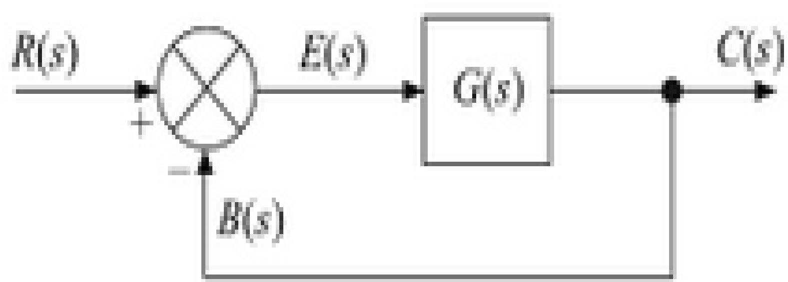
Steady State Errors And Error Constants Design Specifications Of Second Order Systems

STEADY-STATE ERRORS AND ERROR CONSTANTS

Steady-state errors constitute an extremely important aspect of system performance; for it would be meaningless to design for dynamic accuracy if the steady output differed substantially from the desired value for one reason or other. The steady-state error is a measure of system accuracy. These errors arise from the nature of the inputs, type of system and from nonlinearities of system components such as static friction backlash, etc. These are generally aggravated by amplifier drifts, ageing or deterioration.

Control systems may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs and so on as type-0, type-1, type-2 and so on systems. The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system. Consider a unity feedback system shown in Figure 4.14.



F Unity feedback system.

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

The error signal is

$$E(s) = C(s)/G(s)$$

$$E(s) = \frac{R(s)}{1+G(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

The steady-state error e_{ss} may be found by using the final-value theorem.

$$e_{ss} = \underset{t \to \infty}{\text{Lt}}\ e(t) = \underset{s \to 0}{\text{Lt}}\ sE(s) = \underset{s \to 0}{\text{Lt}}\ \frac{sR(s)}{1 + G(s)} \tag{4.20}$$

Equation (4.20) for e_{ss} shows that the steady-state error depends upon the input $R(s)$ and the forward path transfer function.

4.5.1 Static Position Error Constant K_p

The steady-state error of the system for a unit-step input [$r(t) = 1$, $R(s) = 1/s$] is

$$e_{ss} = \underset{s \to 0}{\text{Lt}}\ sE(s) = \underset{s \to 0}{\text{Lt}}\ s \cdot \frac{R(s)}{1+G(s)} = \underset{s \to 0}{\text{Lt}}\ \frac{s \cdot 1/s}{1+G(s)}$$

$$= \frac{1}{1 + \underset{s \to 0}{\text{Lt}}\ G(s)} = \frac{1}{1+G(0)} = \frac{1}{1+K_p} \tag{4.21}$$

where $K_p = \underset{s \to 0}{\text{Lt}}\ G(s) = G(0)$ is defined as the *position error constant*.

4.5.2 Static Velocity Error Constant K_v

The steady-state error of the system for a unit-ramp input [$r(t) = t$, $R(s) = 1/s^2$] is

$$e_{ss} = \underset{s\to 0}{\mathrm{Lt}}\ sE(s) = \underset{s\to 0}{\mathrm{Lt}}\ s \cdot \frac{R(s)}{1+G(s)} = \underset{s\to 0}{\mathrm{Lt}} \frac{s \cdot \frac{1}{s^2}}{1+G(s)}$$

$$= \frac{1}{\underset{s\to 0}{\mathrm{Lt}}\ sG(s)} = \frac{1}{K_v}$$

where $K_v = \underset{s\to 0}{\mathrm{Lt}}\ sG(s)$ is defined as the *velocity error constant.*

Static Acceleration Error Constant K_a

The steady-state error of the system for a unit-parabolic input $\left[r(t) = \frac{t^2}{2}, R(s) = \frac{1}{s^3} \right]$ is

$$e_{ss} = \underset{s \to 0}{\text{Lt}}\, sE(s) = \underset{s \to 0}{\text{Lt}}\, s \cdot \frac{R(s)}{1 + G(s)} = \underset{s \to 0}{\text{Lt}} \frac{s \cdot \frac{1}{s^3}}{1 + G(s)}$$

$$= \frac{1}{\underset{s \to 0}{\text{Lt}}\, s^2 G(s)} = \frac{1}{K_a} \qquad (4.23)$$

where $K_a = \underset{s \to 0}{\text{Lt}}\, s^2 G(s)$ is defined as the *acceleration error constant*.

TYPES OF CONTROL SYSTEMS

The open-loop transfer function of a unity feedback system can be written in two standard forms: the time-constant form and the pole-zero form. In these two forms, $G(s)$ is given as follows:

$$G(s) = \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{s^n(1+T_{p1}s)(1+T_{p2}s)\cdots} \qquad \text{(time-constant form)} \tag{4.24}$$

$$G(s) = \frac{K'(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots} \qquad \text{(pole-zero form)} \tag{4.25}$$

The gains in the two forms are related by

$$K = K' \frac{\Pi_i z_i}{\Pi_j p_j}, \; i = 1, 2, \ldots \quad j = 1, 2, \ldots \tag{4.26}$$

The term s^n in the denominator of Eqs. (4.24) and (4.25) corresponds to the number of integrations in the system. As s tends to zero, this term dominates in determining the steady-state error. Control systems are therefore classified in accordance with the number of integrations in the open-loop transfer function $G(s)$, as

Type-0 system ($n = 0$, no integration, i.e. no pole of $G(s)$ at the origin of s-plane)

Type-1 system ($n = 1$, one integration, i.e. one pole of $G(s)$ at the origin of s-plane)

Type-2 system ($n = 2$, two integrations, i.e. two poles of $G(s)$ at the origin of s-plane) and so on

4.6.1 Steady-State Error: Type-0 System

For a type-0 system,

$$G(s) = \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

$$K_p = \underset{s\to 0}{\text{Lt}}\ \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{(1+T_{p1}s)(1+T_{p2}s)\cdots} = K$$

$$\therefore \qquad e_{ss}(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+K} = \text{ finite value}$$

$$e_{ss}(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+K} = \text{ finite value}$$

$$K_v = \underset{s\to 0}{\text{Lt}}\ sG(s) = \underset{s\to 0}{\text{Lt}}\ s\cdot\frac{K(1+T_{z1})(1+T_{z2}s)\cdots}{(1+T_{p1}s)(1+T_{p2}s)\cdots} = 0$$

$$e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{0} = \infty$$

$$K_a = \underset{s\to 0}{\text{Lt}}\ s^2G(s) = \underset{s\to 0}{\text{Lt}}\ s^2\frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{(1+T_{p1}s)(1+T_{p2}s)\cdots} = 0$$

$\therefore$

$$e_{ss}\,(\text{acceleration}) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Thus, a system with $n = 0$, or no integration in $G(s)$ (type-0 system) has a constant position error, infinite velocity and acceleration errors. The position error constant is given by the open-loop gain of the transfer function in the time-constant form.

Steady-State Error: Type-1 System

For a type-1 system,

$$G(s) = \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{p1}s)(1+T_{p2}s)\cdots}$$

$$K_p = \underset{s\to 0}{\mathrm{Lt}}\, G(s) = \underset{s\to 0}{\mathrm{Lt}}\, \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{p1}s)(1+T_{p2}s)\cdots} = \infty$$

$$\therefore \quad e_{ss}(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$K_v = \underset{s\to 0}{\mathrm{Lt}}\, sG(s) = \underset{s\to 0}{\mathrm{Lt}}\, \frac{sK(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{p1}s)(1+T_{p2}s)\cdots} = K$$

$$\therefore \quad e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{K} = \text{finite value}$$

$$K_a = \underset{s\to 0}{\mathrm{Lt}}\, s^2G(s) = \underset{s\to 0}{\mathrm{Lt}}\, s^2 \frac{K(1+T_{z1}s)(1+T_{z2}s)\cdots}{s(1+T_{p1}s)(1+T_{p2}s)\cdots} = 0$$

$$\therefore \quad e_{ss}(\text{acceleration}) = \frac{1}{K_a} = \frac{1}{0} = \infty$$