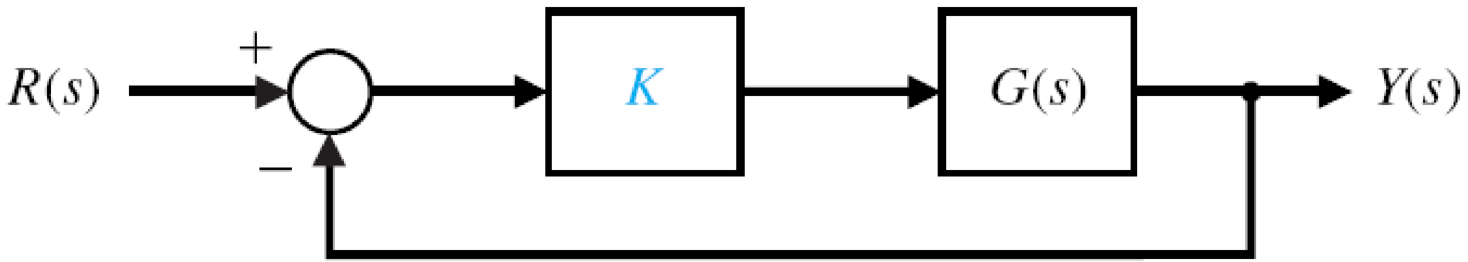
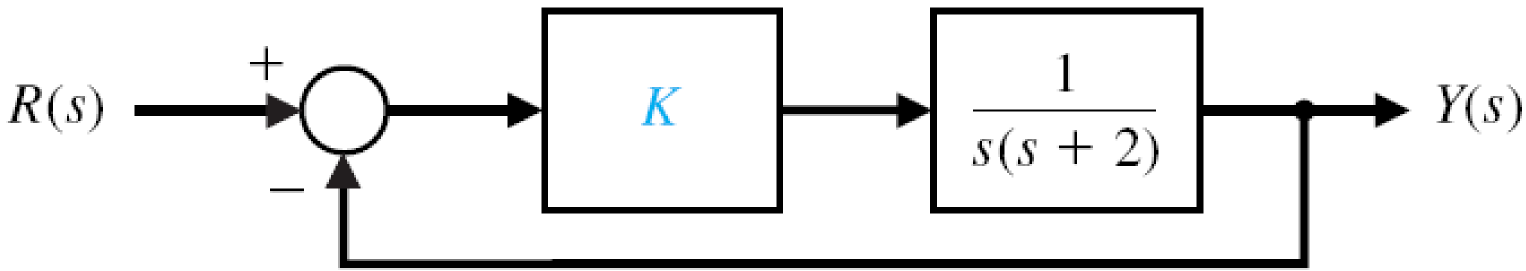
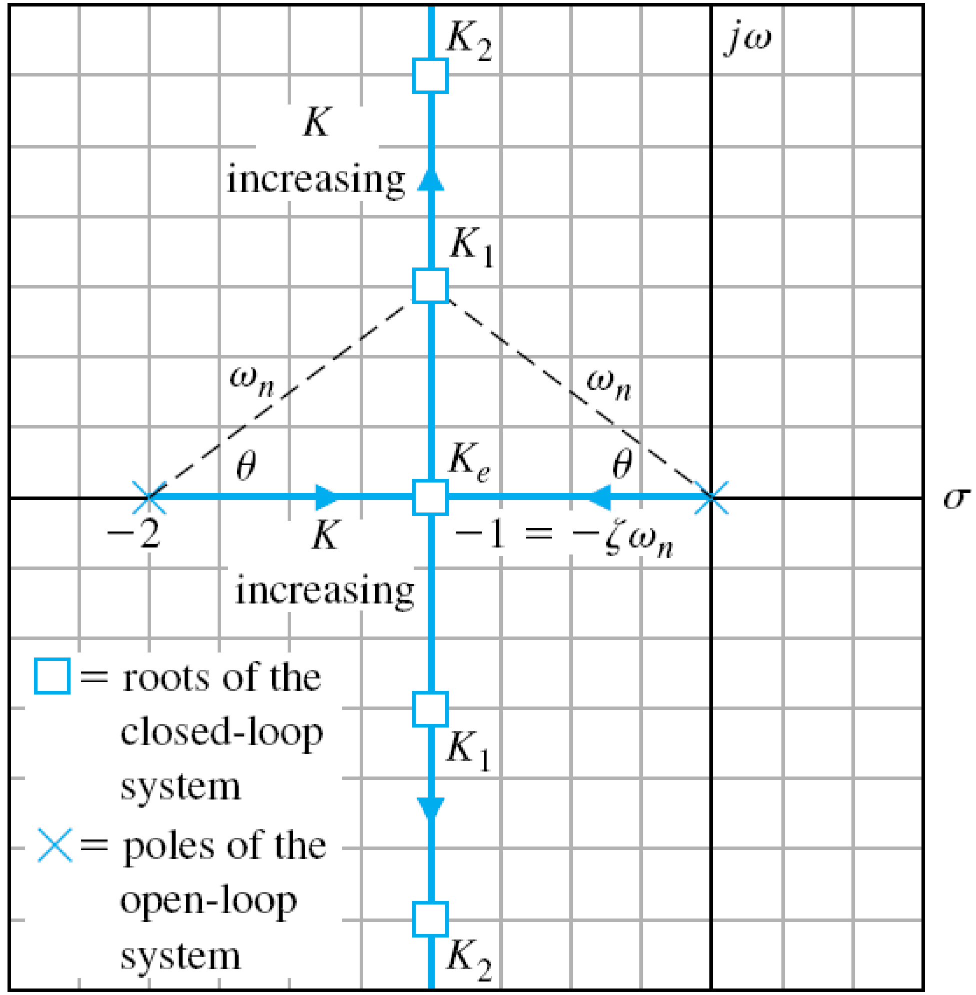
Root Locus



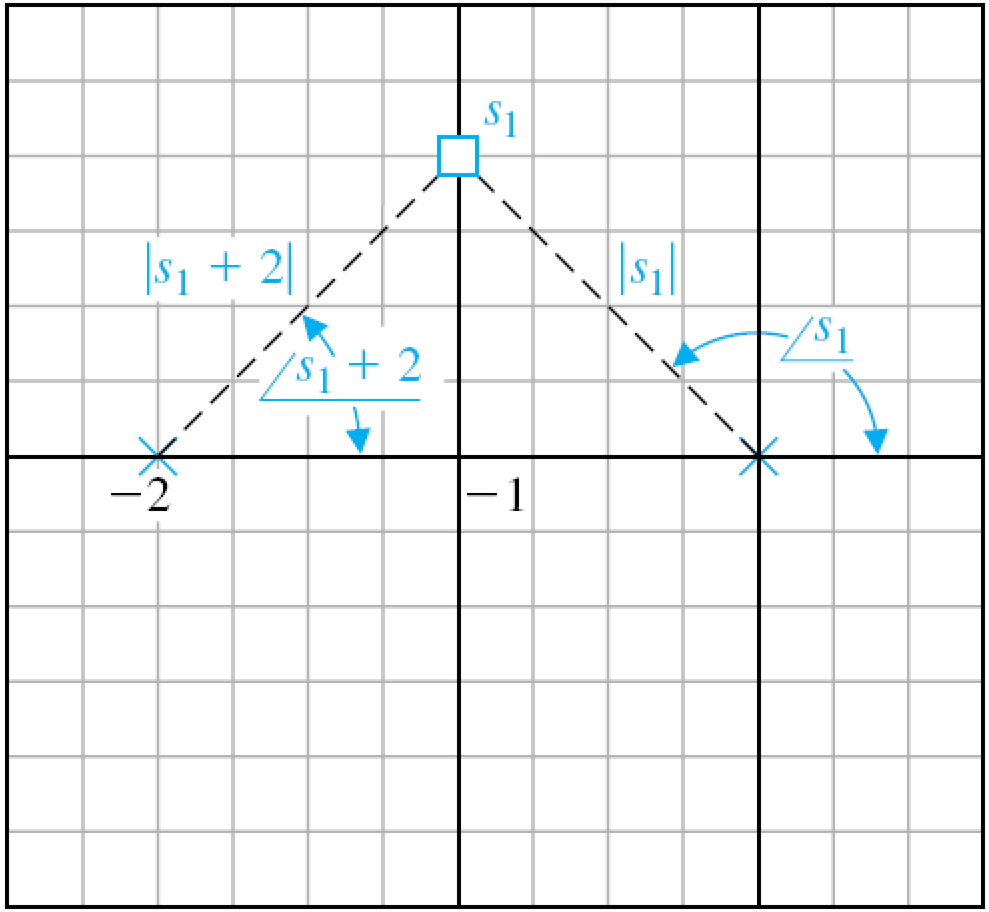
Closed-loop control system with a variable parameter K.



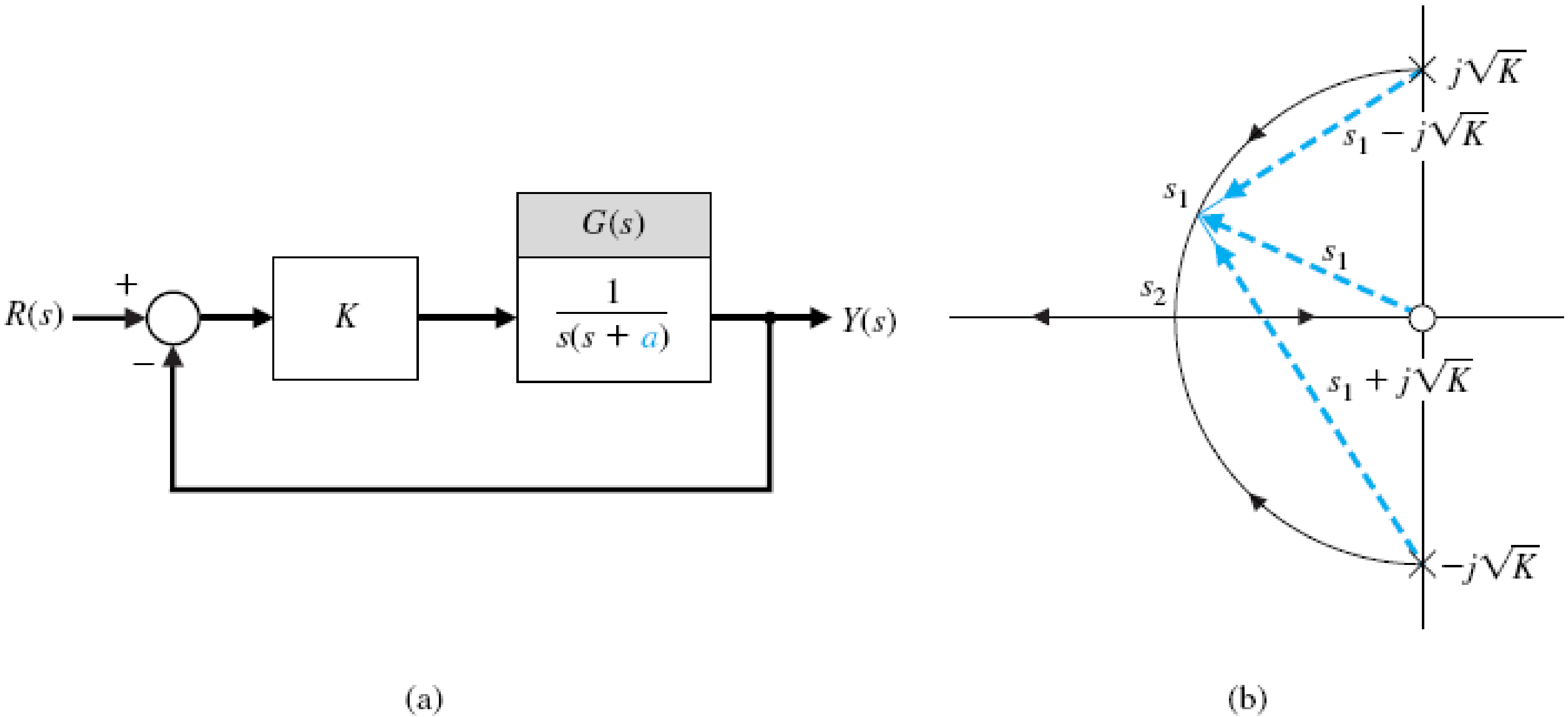
Unity feedback control system. The gain K is a variable parameter.



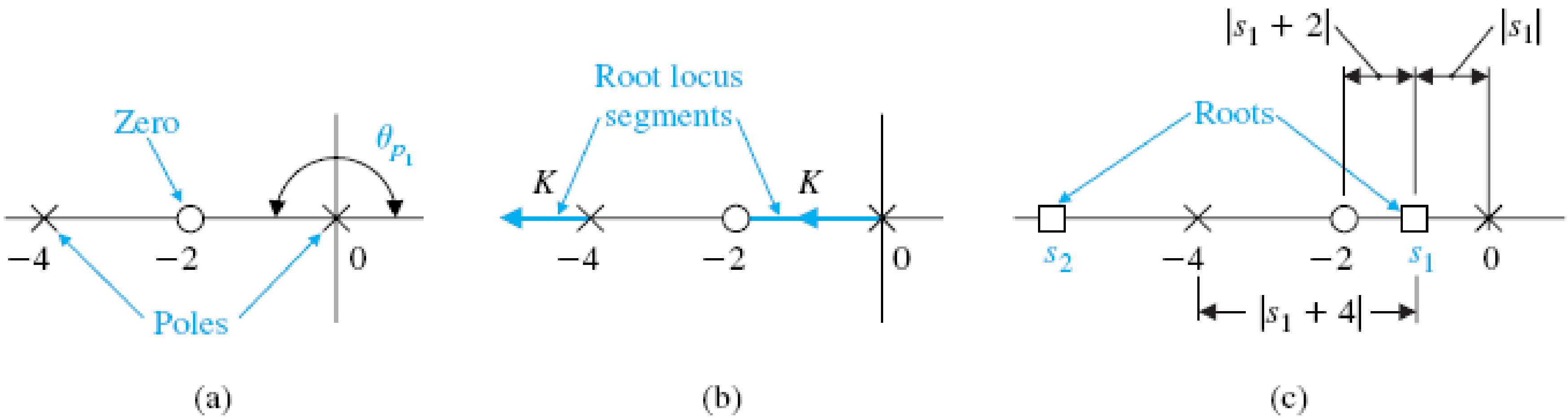
Root locus for a second-order system when $K_e<K_1<K_2$. The locus is shown in color.



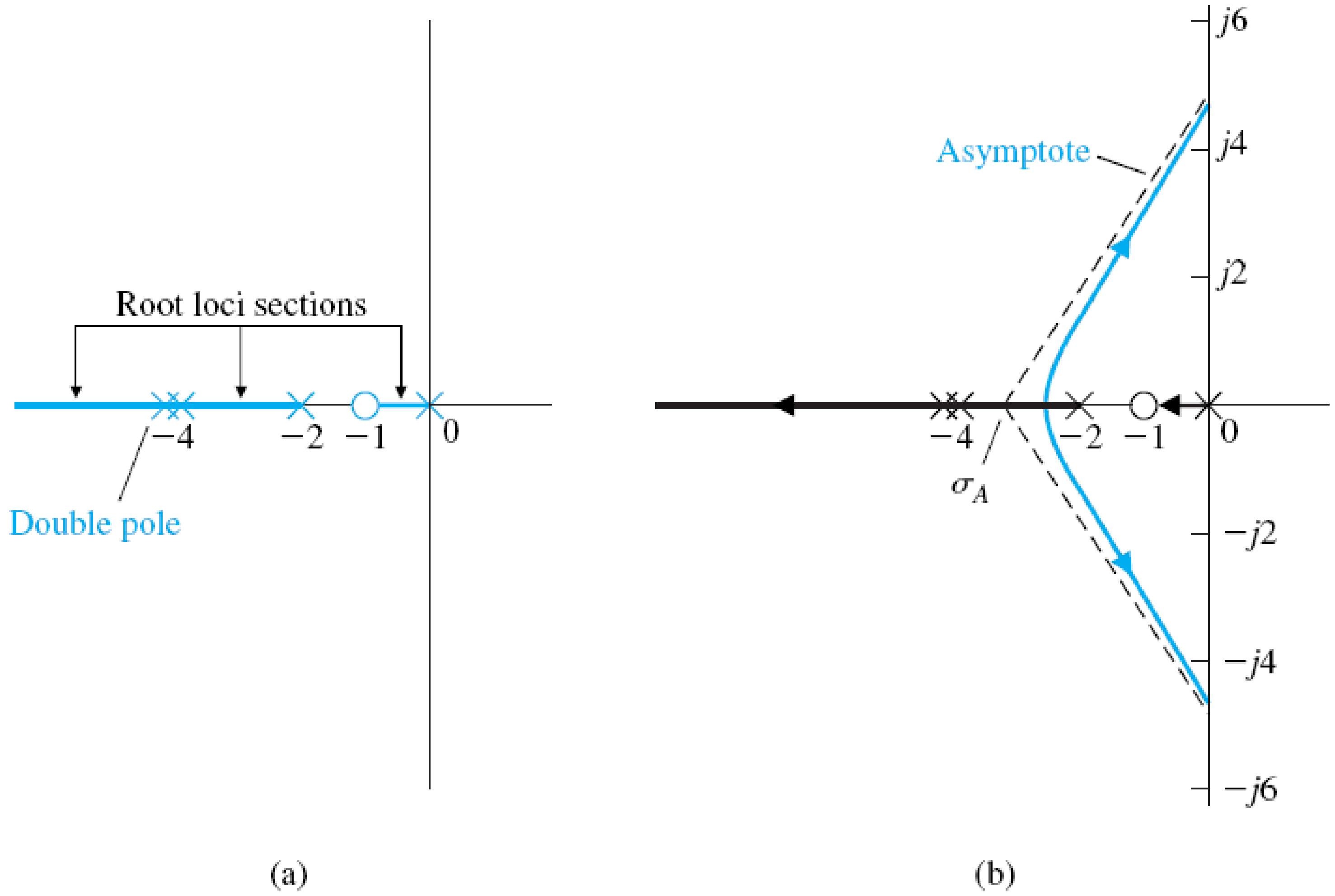
Evaluation of the angle and gain at s_1,for gain $K=K_1$.



(a) Single-loop system. (b) Root locus as a function of the parameter a.



(a) The zero and poles of a second-order system,(b) the root locus segments,and (c) the magnitude of each vector at s1.



A fourth-order system with (a) a zero and (b) root locus.

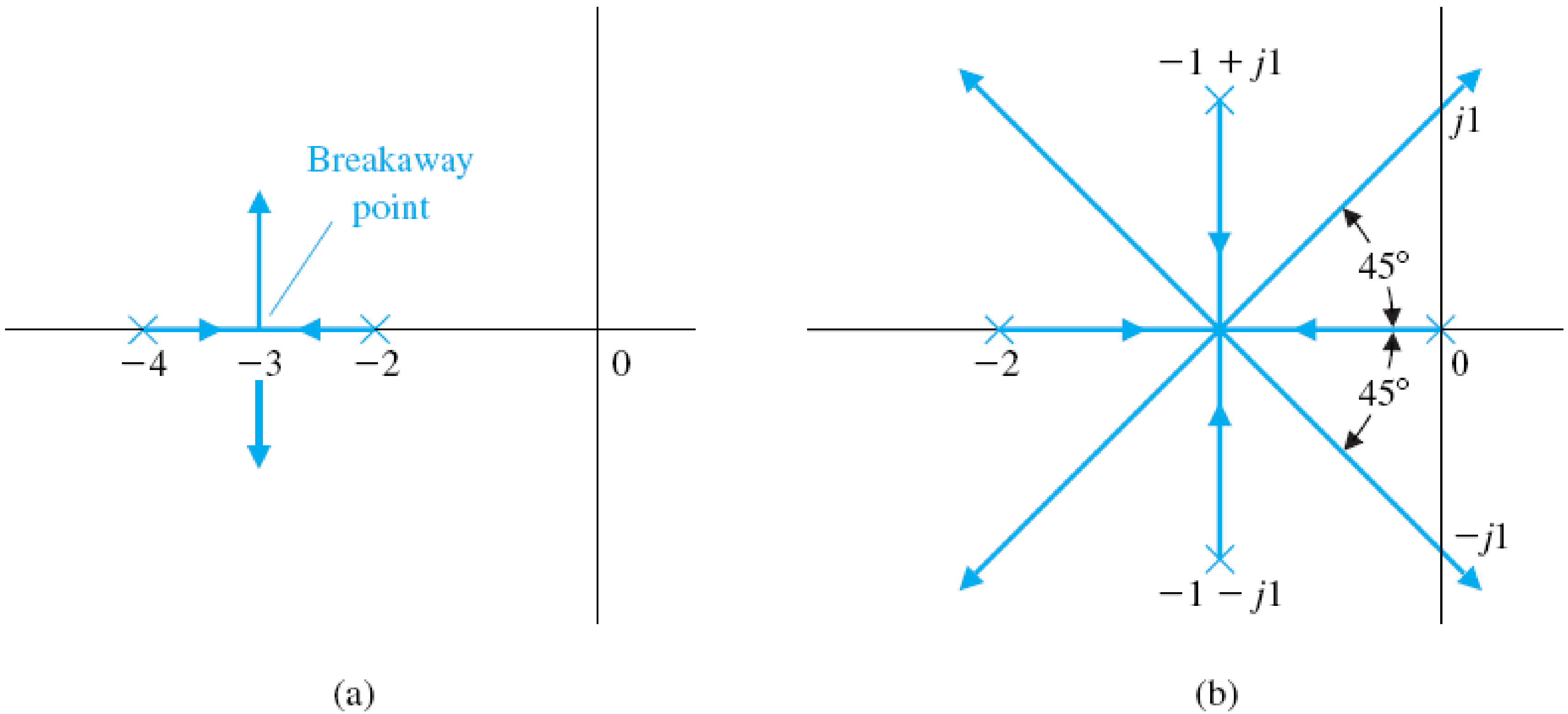
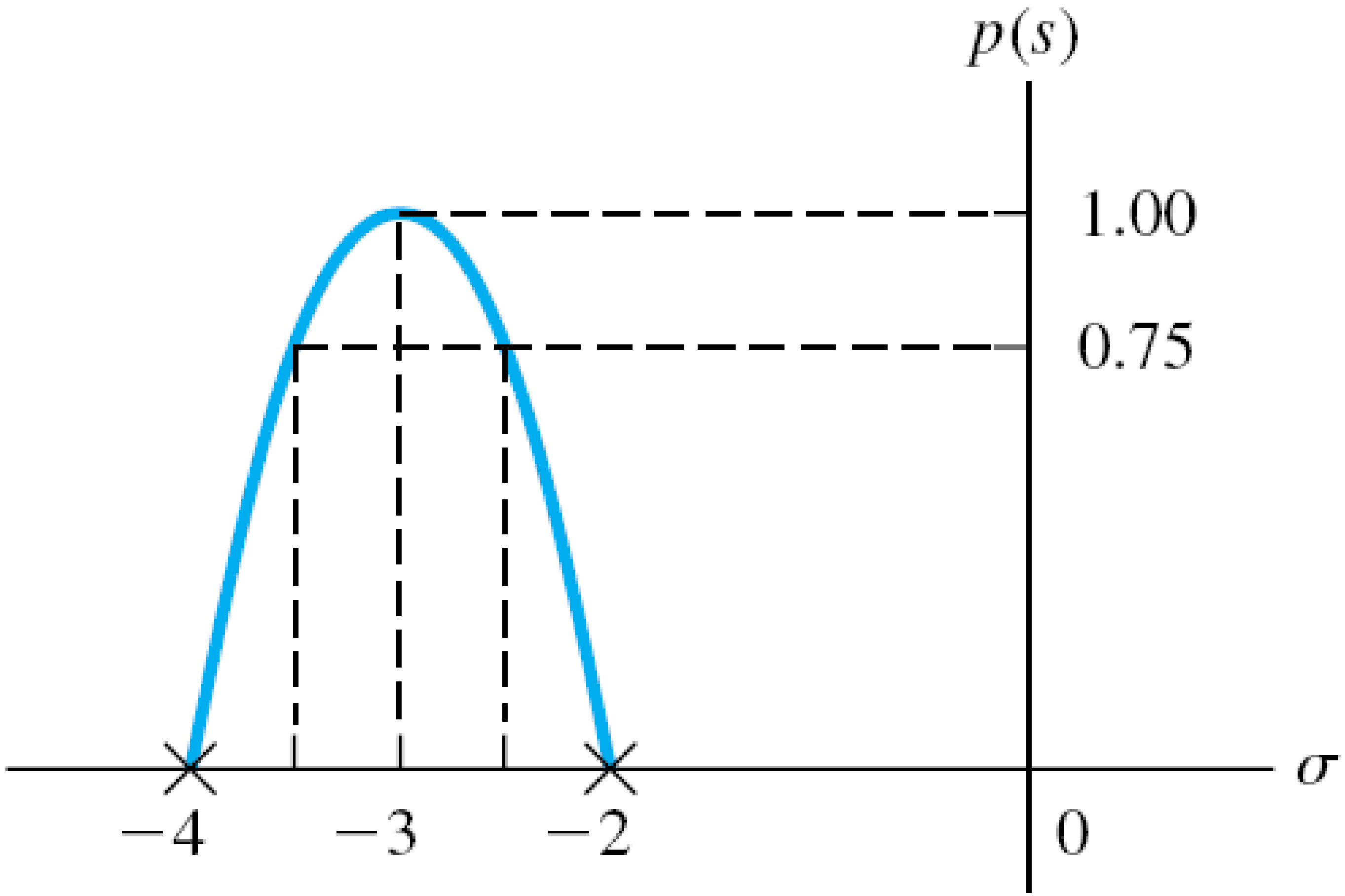
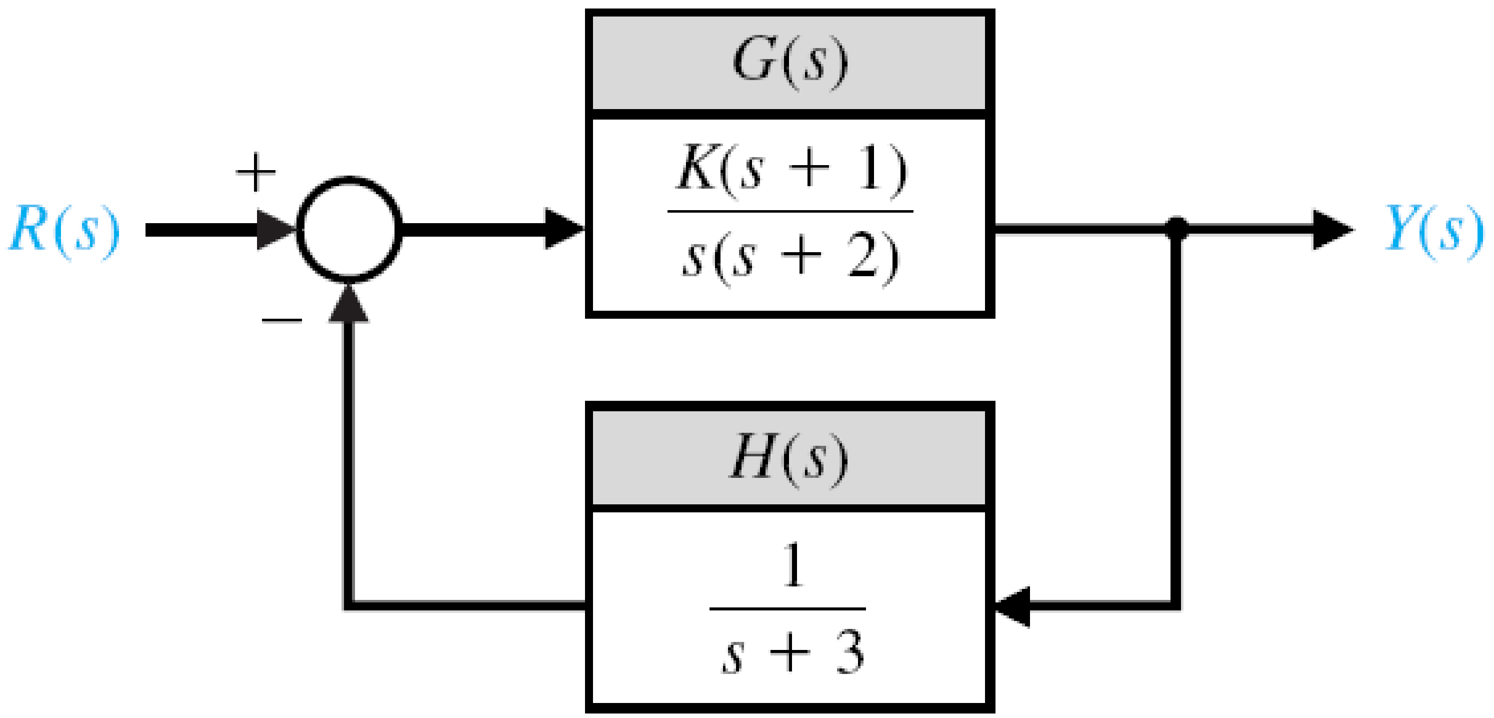


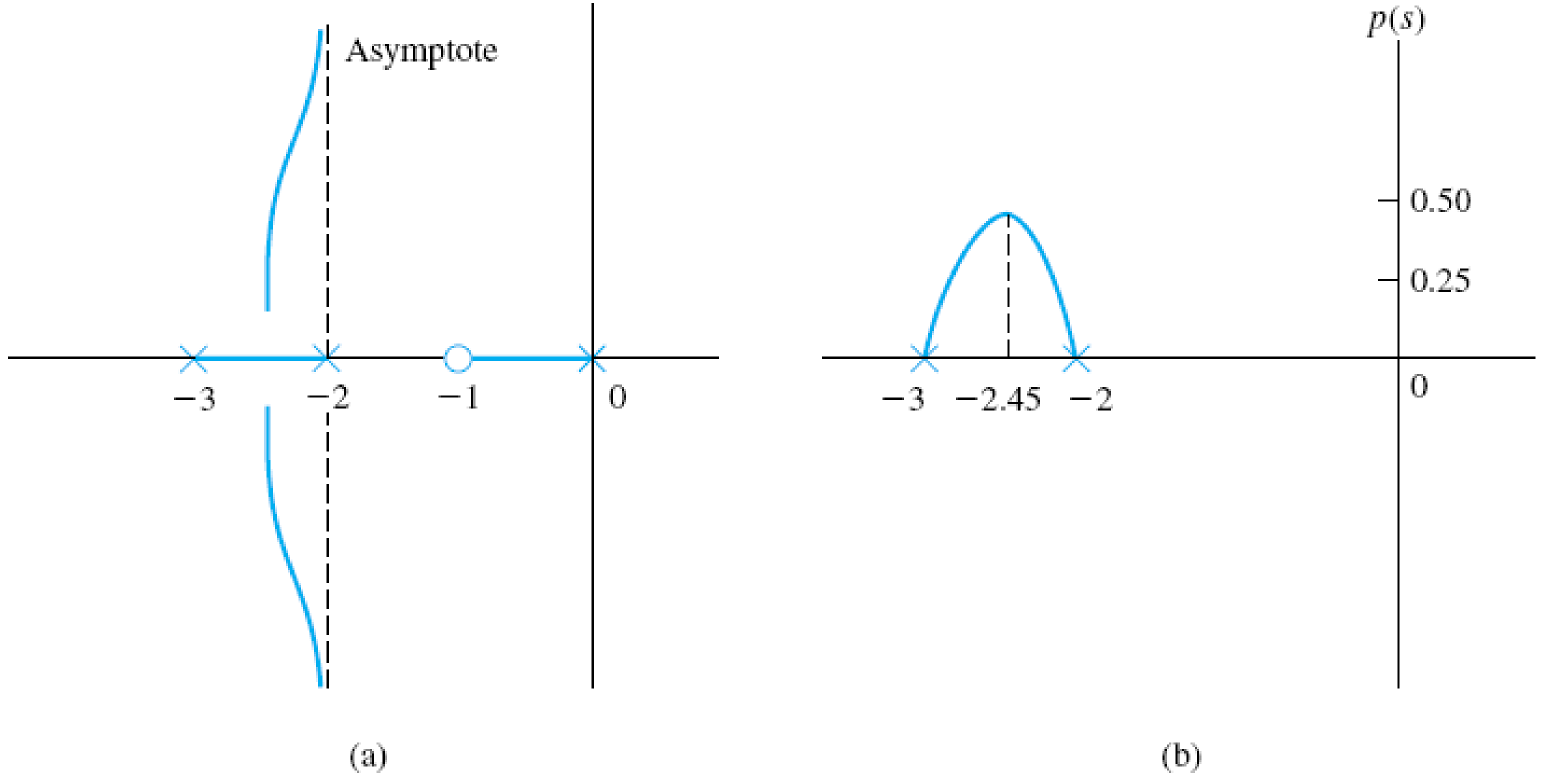
Illustration of the breakaway point (a) for a simple second-order system and (b) for a fourth-order system.



A graphical evaluation of the breakaway point.



Closed-loop system.



Evaluation of the (a) asymptotes and (b) breakaway point.

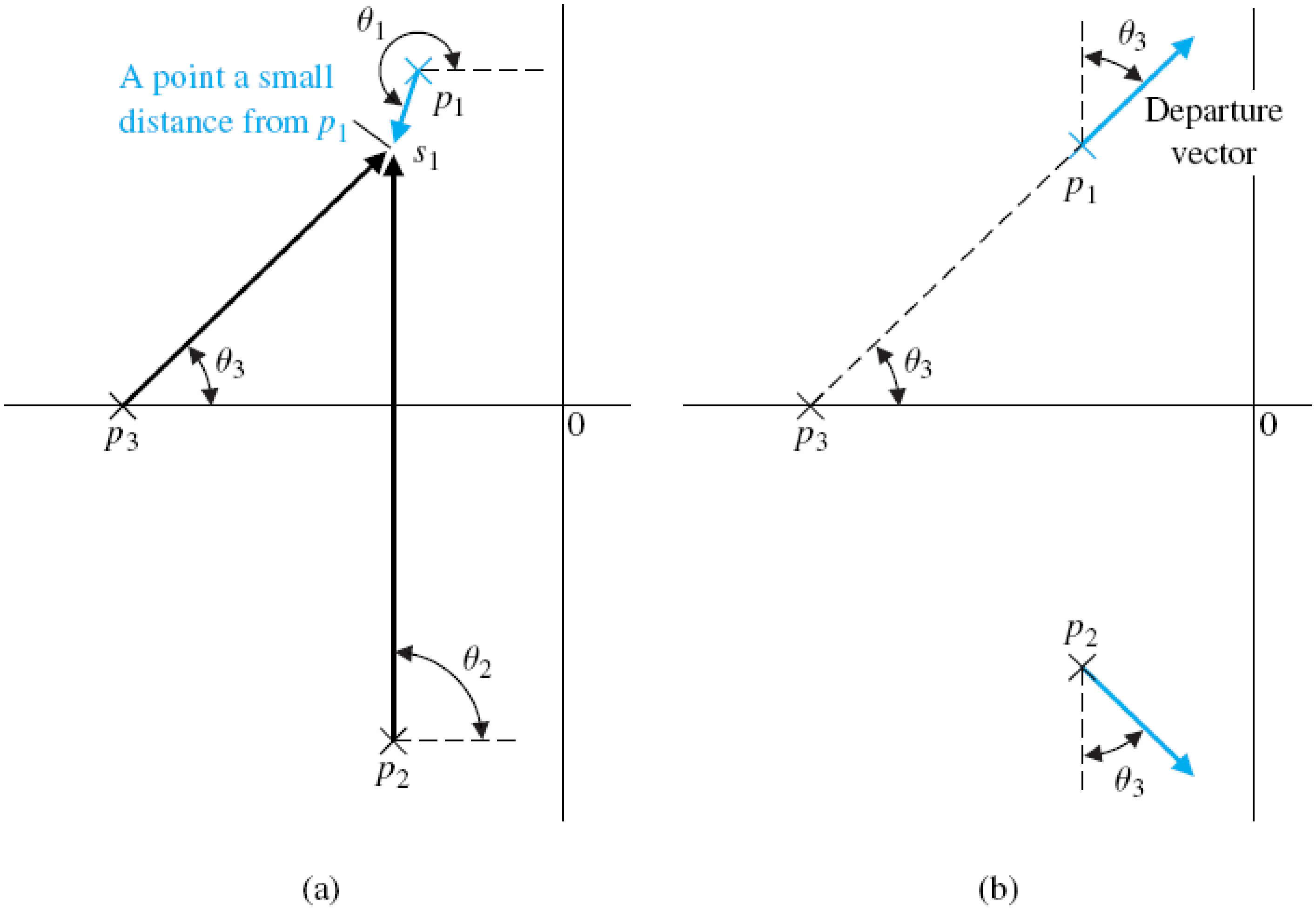
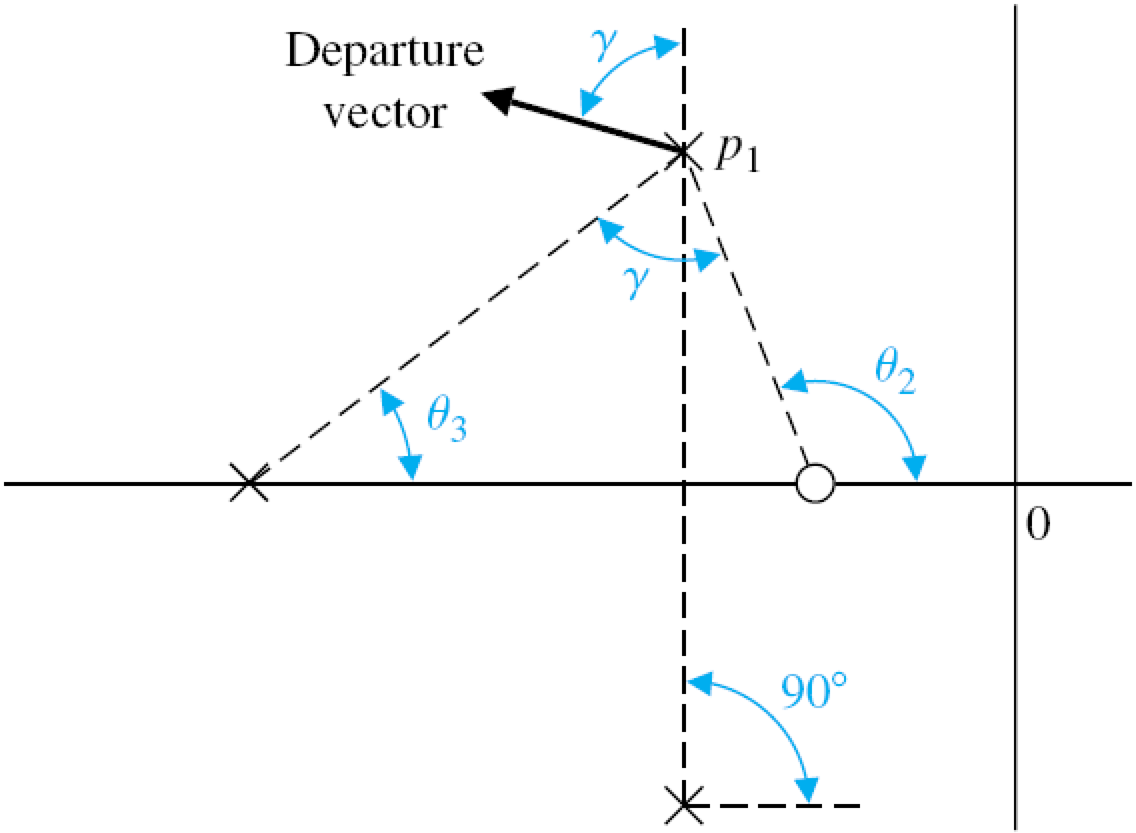


Illustration of the angle of departure:(a) Test point infinitesimal distance from p_1;(b) actual departure vector at p_1



Evaluation of the angle of departure.

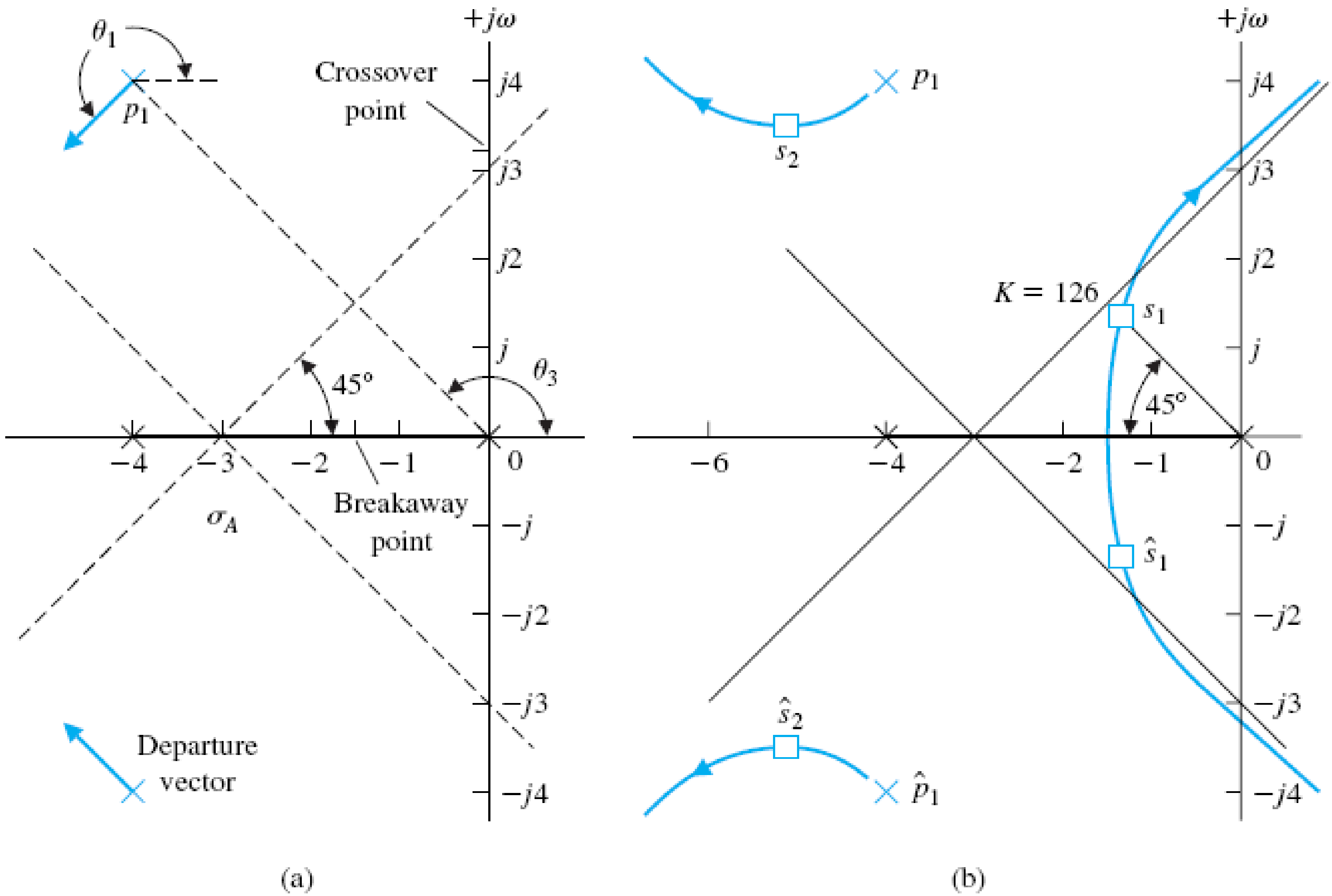
$$1 + \frac{K}{s(s+4)(s+4+j4)(s+4-j4)} = 0$$

$$s(s+4)(s^2+8s+32) + K = s^4 + 12s^3 + 64s^2 + 128s + K = 0.$$

s^4	1	64	K
s^3	12	128	
s^2	b_1	K	
s	c_1		
s^0	K		

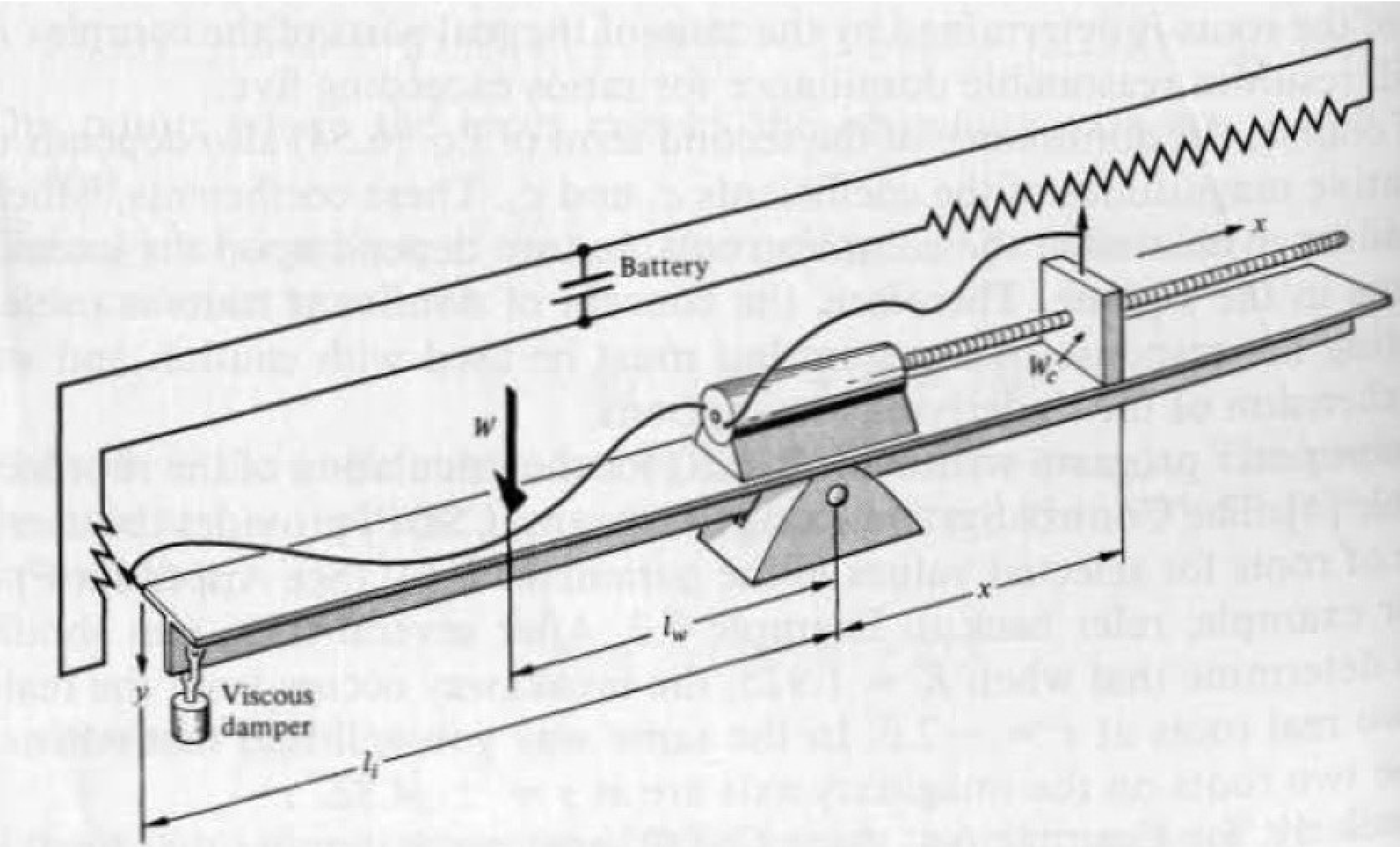
$$b_1 = \frac{12(64) - 128}{12} = 53.33$$

$$c_1 = \frac{53.33(128) - 12K}{53.33}.$$



The root locus for Example 7.4:locating (a) the poles and (b) the asymptotes.

1. Write the characteristic equation in pole-zero form so that the parameter of interest k appears as 1+kF(s)=O.
2. Locate the open-loop poles and zeros of F(s) in the s-plane.
3. Locate the segments of the real axis that are root loci.
4. Determinethenumberofseparateloci.
5. Locate the angles of the asymptotes and the center of the asymptotes.
6. Determine the break away point on the real axis(if any).
7. By utilizing the Routh-Hurwitz criterion, determine the point at which the locus crosses the imaginary axis(if it does so).
8. Estimate the angle of locus departure from complex poles and the angle locus arrival at complex zeros.

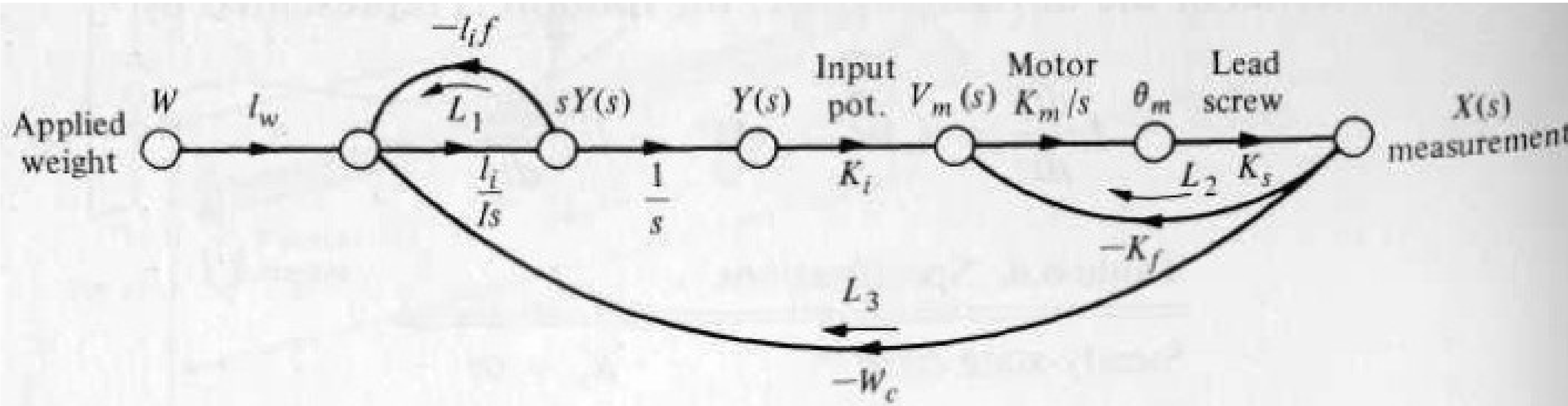


Battery
x
Wc
W
y
Viscous
damper
lw
x
li

1. Select the parameters and the specifications of the feedback system.
2. Obtain a model and signal-flow diagram representing the system.
3. Select the gain K based on a root locus diagram.
4. Determine the dominant mode of response.

Table 6.4. Specifications

Steady-state error	$K_p = \infty$
Underdamped response	$\zeta = 0.5$
Settling time	Less than 2 sec



$$\frac{X(s)}{W(s)} = \frac{l_w l_i K_i K_m K_s}{s(Is + l_i^2 f)(s + K_m K_s K_f) + W_c K_m K_s K_i l_i}.$$

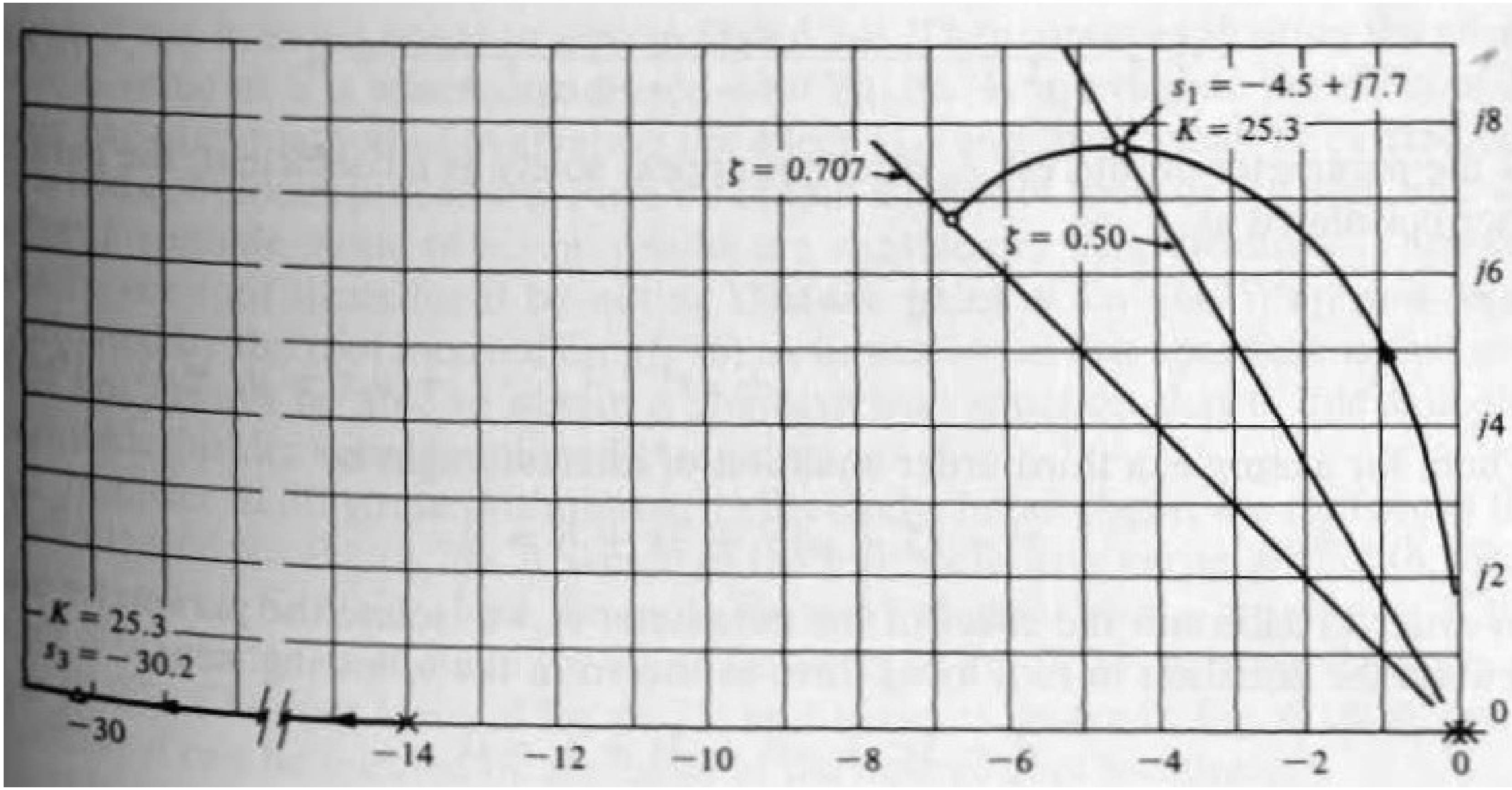
Table 6.3.

$W_r = 2$ Ng	Lead screw gain $K_s = \dfrac{1}{4000\pi}$ m/rad
$J = 0.05$ kg-m^2	
$l_w = 5$ cm	Input potentiometer gain $K_i = 4800$ v/m
$l_t = 20$ cm	
$f = 10\sqrt{3}$ kg/m/sec	Feedback potentiometer gain $K_f = 400$ v/m

$$s(s + 8\sqrt{3})\left(s + \frac{K_m}{10\pi}\right) + \frac{96K_m}{10\pi} = 0$$

$$1 + KP(s) = 1 + \frac{(K_m/10\pi)[s(s + 8\sqrt{3}) + 96]}{s^2(s + 8\sqrt{3})} = 0$$

$$= 1 + \frac{(K_m/10\pi)(s + 6.93 + j6.93)(s + 6.93 - j6.93)}{s^2(s + 8\sqrt{3})}$$



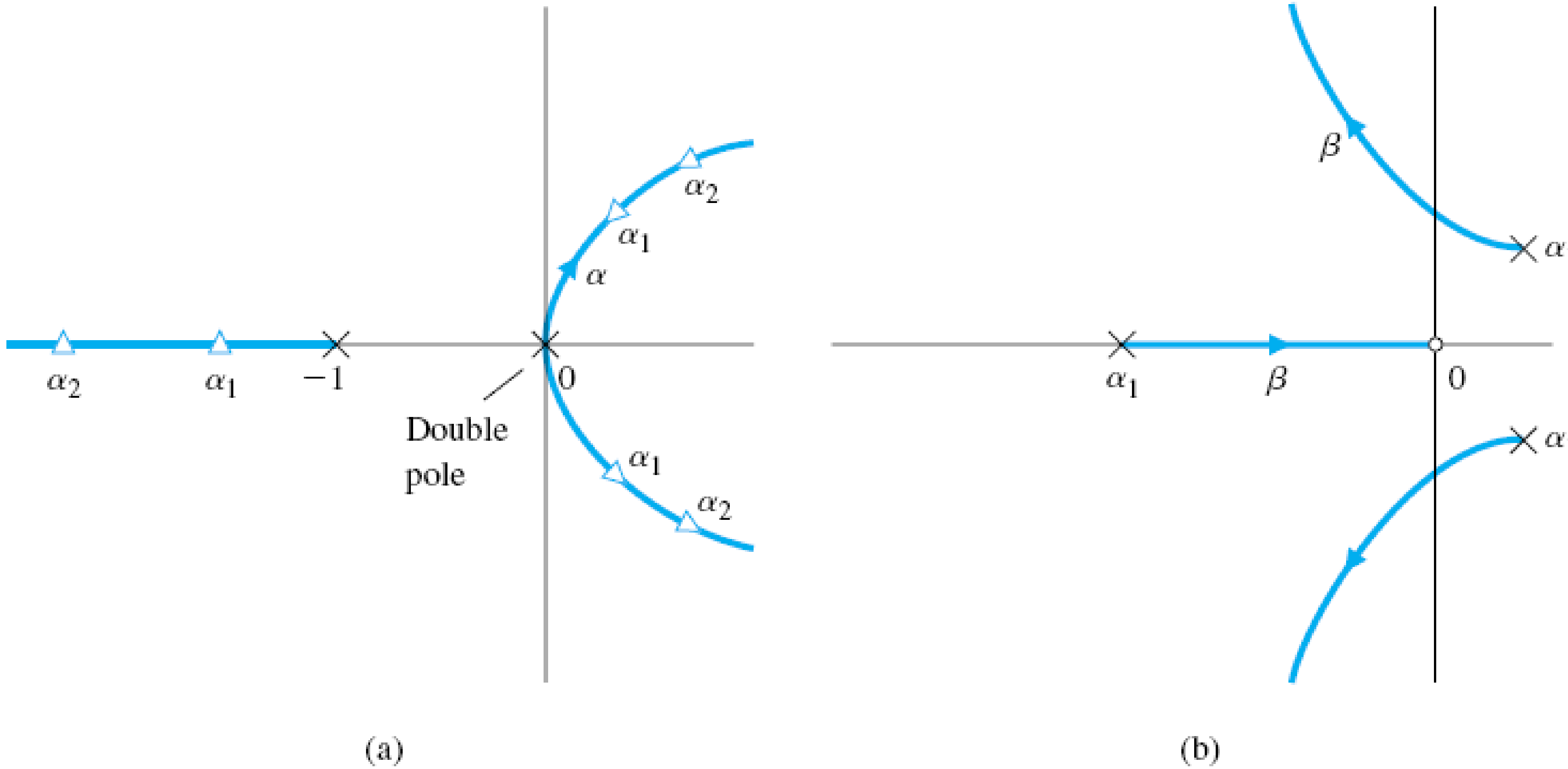
$s_1 = -4.5 + j7.7$
$K = 25.3$
$\zeta = 0.707$
$\zeta = 0.50$
$K = 25.3$
$s_3 = -30.2$
j8
j6
j4
j2
0
−30
−14
−12
−10
−8
−6
−4
−2
0

$$s^3 + s^2 + \beta s + \alpha = 0$$

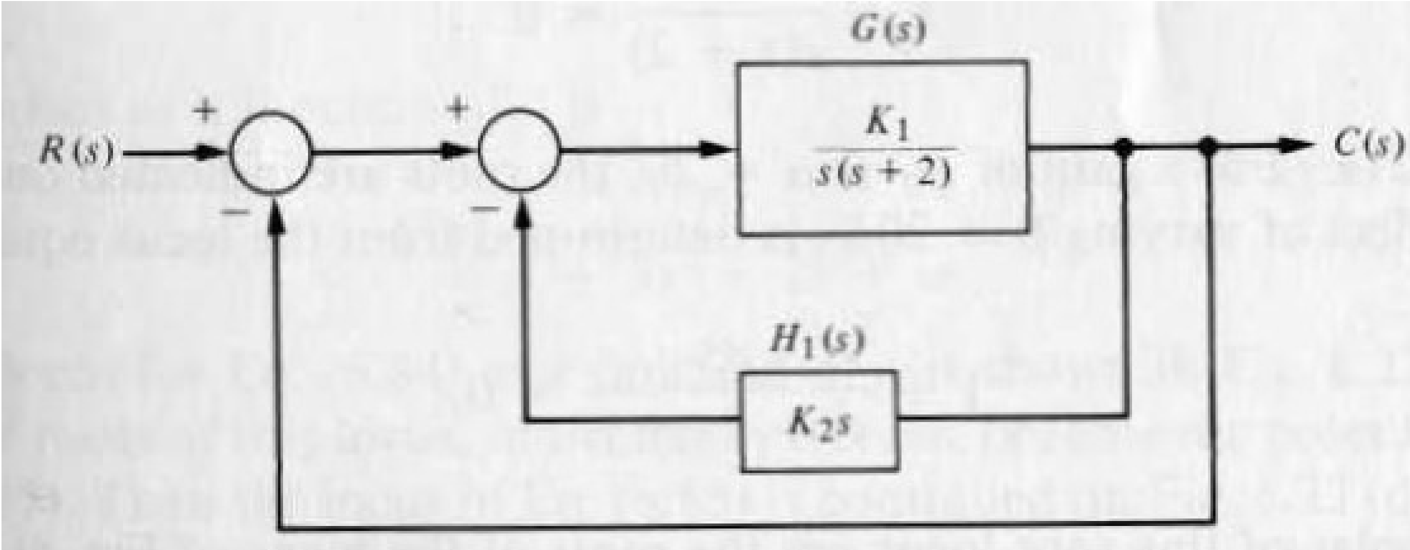
$$1 + \frac{\beta s}{s^3 + s^2 + \alpha} = 0.$$

$$s^3 + s^2 + \alpha = 0.$$

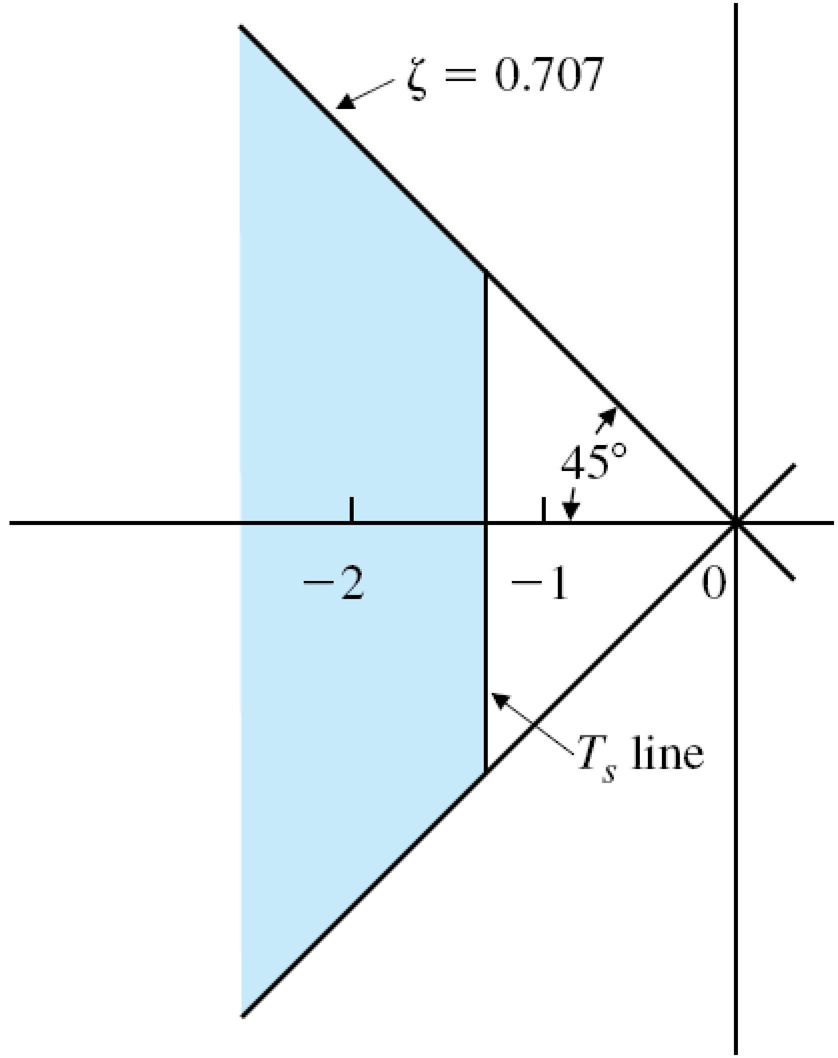
$$1 + \frac{\alpha}{s^2(s + 1)} = 0,$$



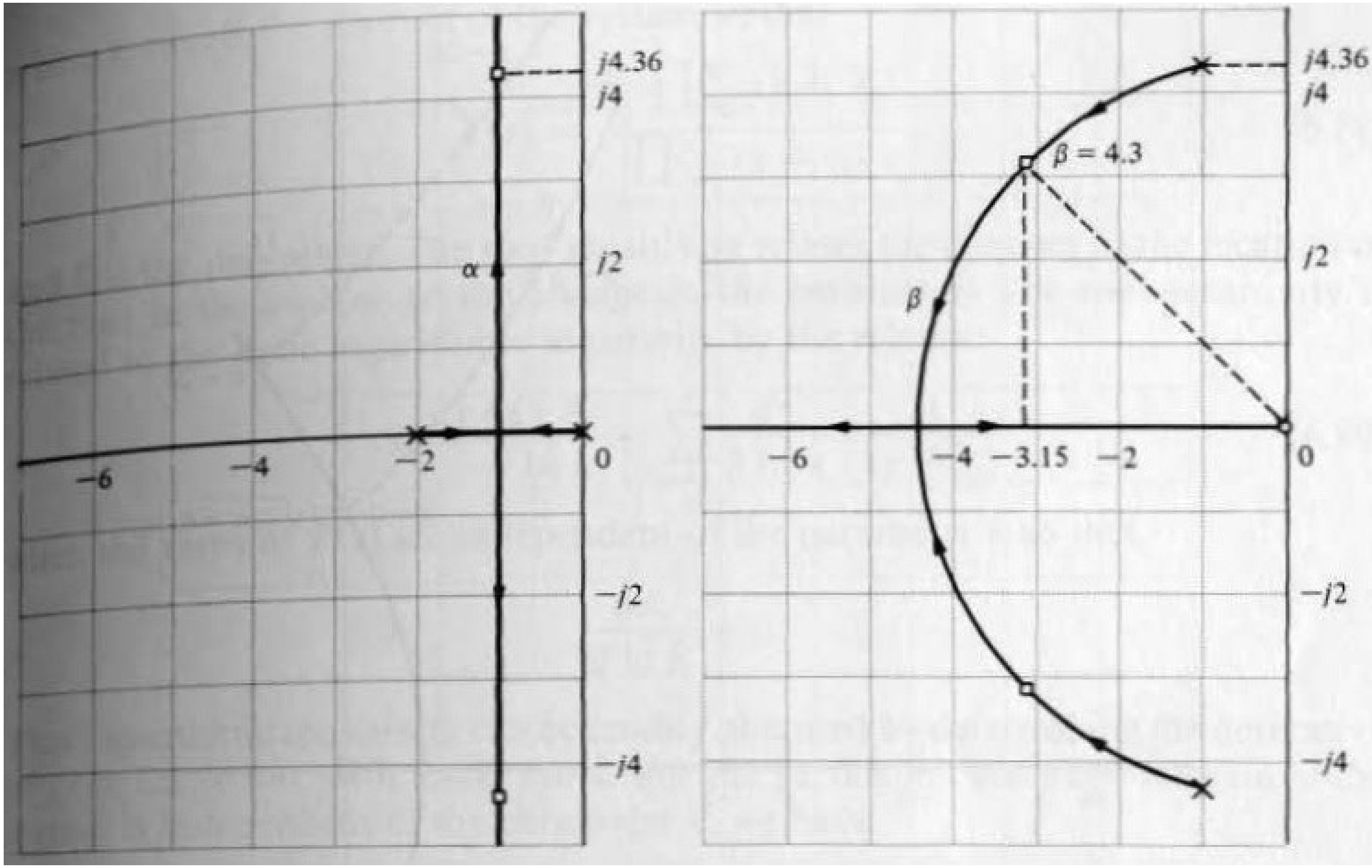
Root loci as a function of α and β:(a) loci as α varies; (b) loci as β varies for one value of $\alpha = \alpha_1$



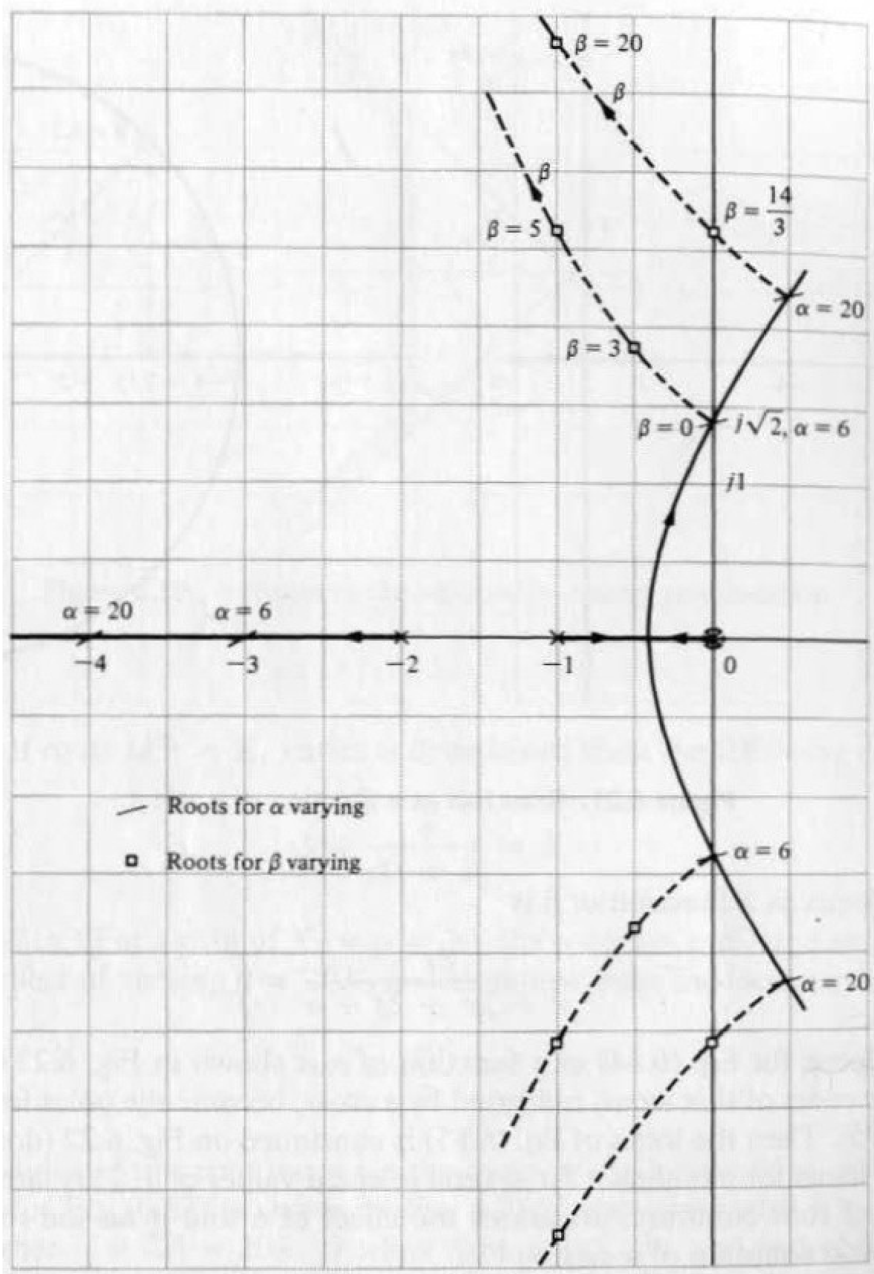
1. Steady-state error for a ramp input ≤ 35% of input slope
2. Damping ratio of dominant roots ≥ 0.707
3. Settling time of the system ≤ 3 sec



A region in the s-plane for desired root location.



j4.36
j4
j2
α
−j2
−j4
−6
−4
−2
0
j4.36
j4
β = 4.3
j2
β
−6
−4
−3.15
−2
0
−j2
−j4



β = 20
β
β
β = 14/3
β = 5
α = 20
β = 3
β = 0
j√2, α = 6
j1
α = 20
α = 6
−4
−3
−2
−1
0
Roots for α varying
Roots for β varying
α = 6
α = 20