Bode Plot

Poles & Zeros and Transfer Functions

Transfer Function: A transfer function is defined as the ratio of the Laplace transform of the output to the input with all initial conditions equal to zero. Transfer functions are defined only for linear time invariant systems.

Considerations: Transfer functions can usually be expressed as the ratio of two polynomials in the complex variable, s.

Factorization: A transfer function can be factored into the following form.

$$G(s) = \frac{K(s + z_1)(s + z_2)\ldots(s + z_m)}{(s + p_1)(s + p_2)\ \ldots\ (s + p_n)}$$

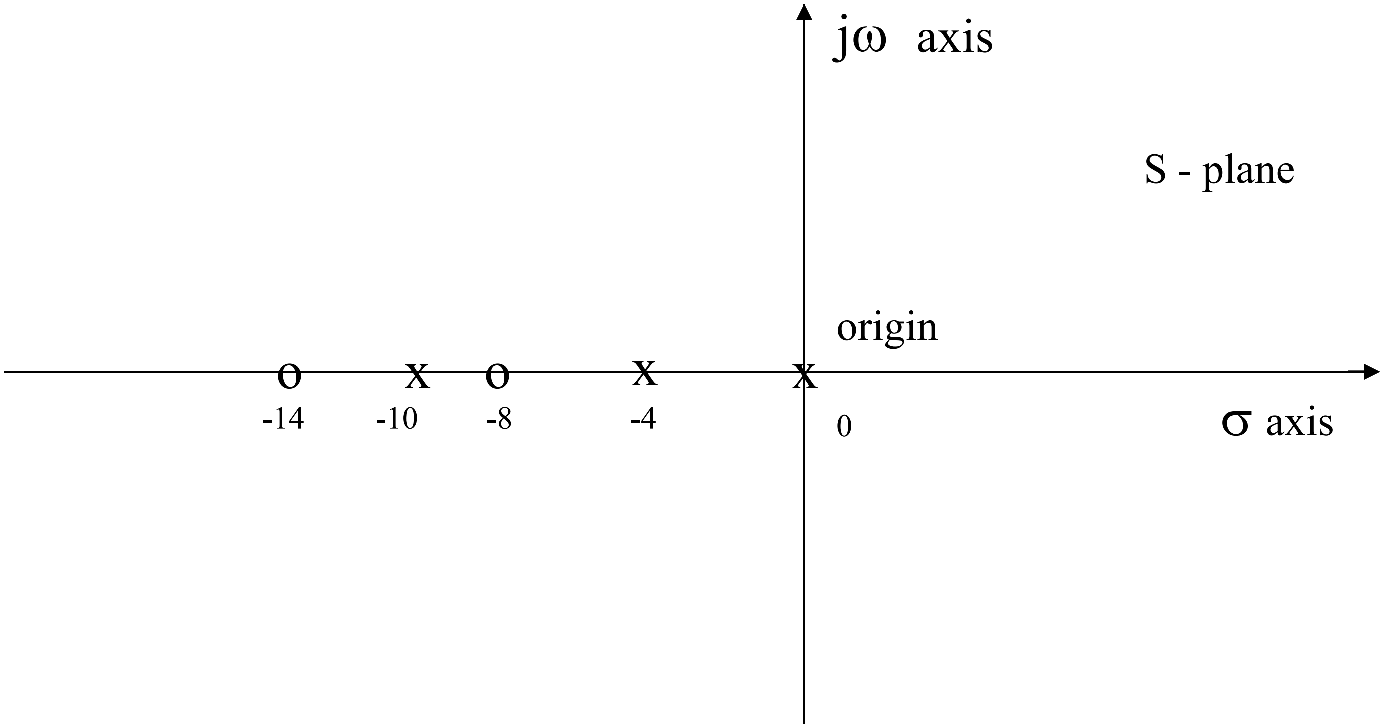
The roots of the numerator polynomial are called zeros.

The roots of the denominator polynomial are called poles.

Poles, Zeros and S-Plane

An Example: You are given the following transfer function. Show the poles and zeros in the s-plane.

$$G(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$



Bode Plot

- It is graphical representation of transfer function to find out the stability of control system.
- It consists of two plots
  - Magnitude (in dB) Vs frequency plot
  - Phase angle Vs frequency plot

Bode Plot…

- **Consider following T.F**

$$G(s)=\frac{K1(s+z_1)(s+z_2)\ldots(s+z_m)}{s^N(s+p_1)(s+p_2)\ \ldots\ (s+p_n)\left\{1+2\xi\frac{s}{w_n}+\left(\frac{s}{w_n}\right)^2\right\}}$$

- **Put s=jw**

$$G(jw)=\frac{K1(jw+z_1)(jw+z_2)\ldots(jw+z_m)}{(jw)^N(jw+p_1)(jw+p_2)\ \ldots\ (jw+p_n)\left\{1+2\xi\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right\}}$$

- **Arrange in following form**

$$G(jw)=\frac{K1(1+jwT_1)(1+jwT_2)\ldots(1+jwT_m)}{(jw)^N(1+jwT_a)(1+jwT_b)\ \ldots\ (1+jwT_n)\left\{1+2\xi\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right\}}$$

$$G(jw)=|G(jw)|\angle G(jw)$$

Bode Plot…

- **So**

$$G(jw) = \underset{\substack{\uparrow \\ \text{Magnitude}}}{|G(jw)|}\,\underset{\substack{\uparrow \\ \text{Phase Angle}}}{\angle G(jw)}$$

$$Magnitude\ in\ dB = 20\log_{10}|G(jw)|$$

- **Hence Bode Plot consists of two plots**
  - Magnitude ($20\log_{10}|G(jw)|$ dB) Vs frequency plot (w)
  - Phase angle ($\angle G(jw)$)Vs frequency plot (w)

Bode Plot…

Magnitude in dB

$$20\log_{10}|G(jw)|=20\log_{10}|K|+20\log_{10}|1+jwT_1|+20\log_{10}|1+jwT_2|\ldots+20\log_{10}|1+jwT_n|$$

$$+20\log_{10}|1+jwT_a|+20\log_{10}|1+jwT_b|\ldots+20\log_{10}|1+jwT_m|+etc$$

Phase Angle

$$\angle G(jw)=\angle(K)+\angle(1+jwT_1)+\angle(1+jwT_2)\ldots+\angle(1+jwT_n)$$

$$-\angle(1+jwT_a)-\angle(1+jwT_b)\ldots-(1+jwT_m)-etc$$

$$=90^0+\tan^{-1}(jwT_1)+\tan^{-1}(jwT_2)\ldots+\tan^{-1}(jwT_n)$$

$$-\tan^{-1}(jwT_a)-\tan^{-1}(jwT_b)\ldots-\tan^{-1}(jwT_m)-etc$$

Bode Plot…

Type of System	**Initial Slope**	**Intersection with 0 dB line**
0	0 dB/dec	Parallel to 0 axis
1	-20dB/dec	=K
2	-40dB/dec	$=K^{1/2}$
3	-60dB/dec	$=K^{1/3}$
.	.	.
.	.	.
.	.	.
N	-20NdB/dec	$=K^{1/N}$

Bode Plot Procedure

- **Steps to draw Bode Plot**

1. **Convert the TF in following standard form & put s=jw**

$$G(jw) = \frac{K1(1+jwT_1)(1+jwT_2)\ldots(1+jwT_m)}{(jw)^N(1+jwT_a)(1+jwT_b)\ \ldots\ (1+jwT_n)\left\{1+2\xi\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right\}}$$

2. **Find out corner frequencies by using**

$$\frac{1}{T_1},\frac{1}{T_2},\frac{1}{T_3}\cdots\frac{1}{T_a},\frac{1}{T_b},\frac{1}{T_c}\ Rad/\sec \qquad etc$$

Bode Plot Procedure …

3. **Draw the magnitude plot.** The slope will change at each corner frequency by +20dB/dec for zero and -20dB/dec for pole.
   - For complex conjugate zero and pole the slope will change by $\pm 40dB/dec$
4. **Starting plot**
   i. For type Z**ero (N=0)** system, draw a line up to first (lowest) corner frequency having 0dB/dec slope of magnitude (height) $20\log_{10}K$
   ii. For type **One (N=1)** system, draw a line having slope -20dB/dec from w=K and mark first (lowest) corner frequency.
   iii. For type **One (N=2)** system, draw a line having slope -40dB/dec from $w=K^{1/2}$ and mark first (lowest) corner frequency.

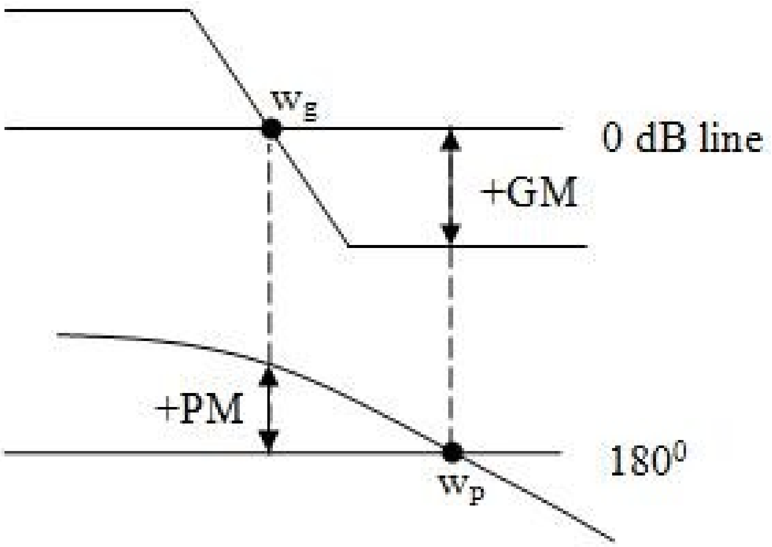
Bode Plot Procedure …

5. Draw a line up to second corner frequency by adding the slope of next pole or zero to the previous slope and so on….
   - i. Slope due to a zero = +20dB/dec
   - ii. Slope due to a pole = -20dB/dec
6. Calculate phase angle for different value of ‘w’ and draw phase angle Vs frequency curve

Bode Plot GM & PM

- **Gain Margin:** It is the amount of gain in db that can be added to the system before the system become unstable
  - GM in dB = $20\log_{10}(1/|G(jw|) = -20\log_{10}|G(jw|$
  - **Gain cross-over frequency:** Frequency where magnitude plot intersect the 0dB line (x-axis) denoted by w_g
- **Phase Margin:** It is the amount of phase lag in degree that can be added to the system before the system become unstable
  - PM in degree = 180^0+angle[G(jw)]
  - **Phase cross-over frequency:** Frequency where phase plot intersect the 180^0 dB line (x-axis) denoted by w_p
  - Less PM => More oscillating system

Bode Plot GM & PM

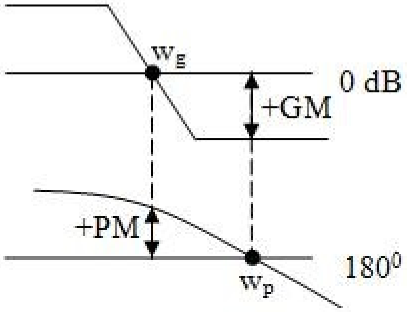


Bode Plot & Stability

Stability by Bode Plot

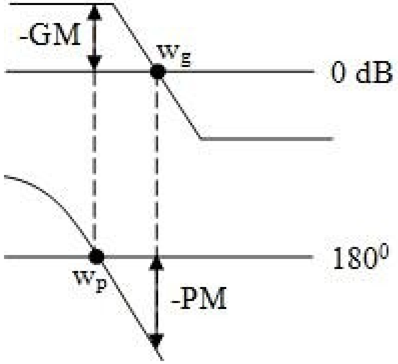
1. **Stable**

   If $w_g<w_p$ => GM & PM are +ve



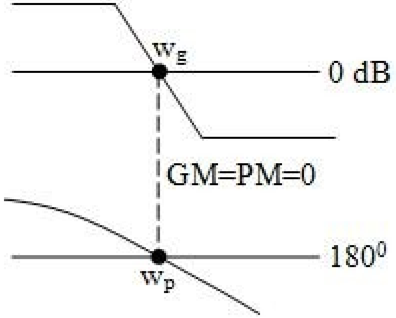
2. **Unstable**

   If $w_g>w_p$ => GM & PM are –ve



3. **Marginally stable**

   If $w_g=w_p$ => GM & PM are zero



Bode Plot Examples

Example 1: Sketech the Bode plot for the TF

$$G(s) = \frac{1000}{(1+0.1s)(1+0.001s)}$$

Determine

- (i) GM
- (ii) PM
- (iii) Stability

Bode Plot Examples…

Solution:

1. Convert the TF in following standard form & put s=jw

$$G(jw) = \frac{1000}{(1+0.1jw)(1+0.001jw)}$$

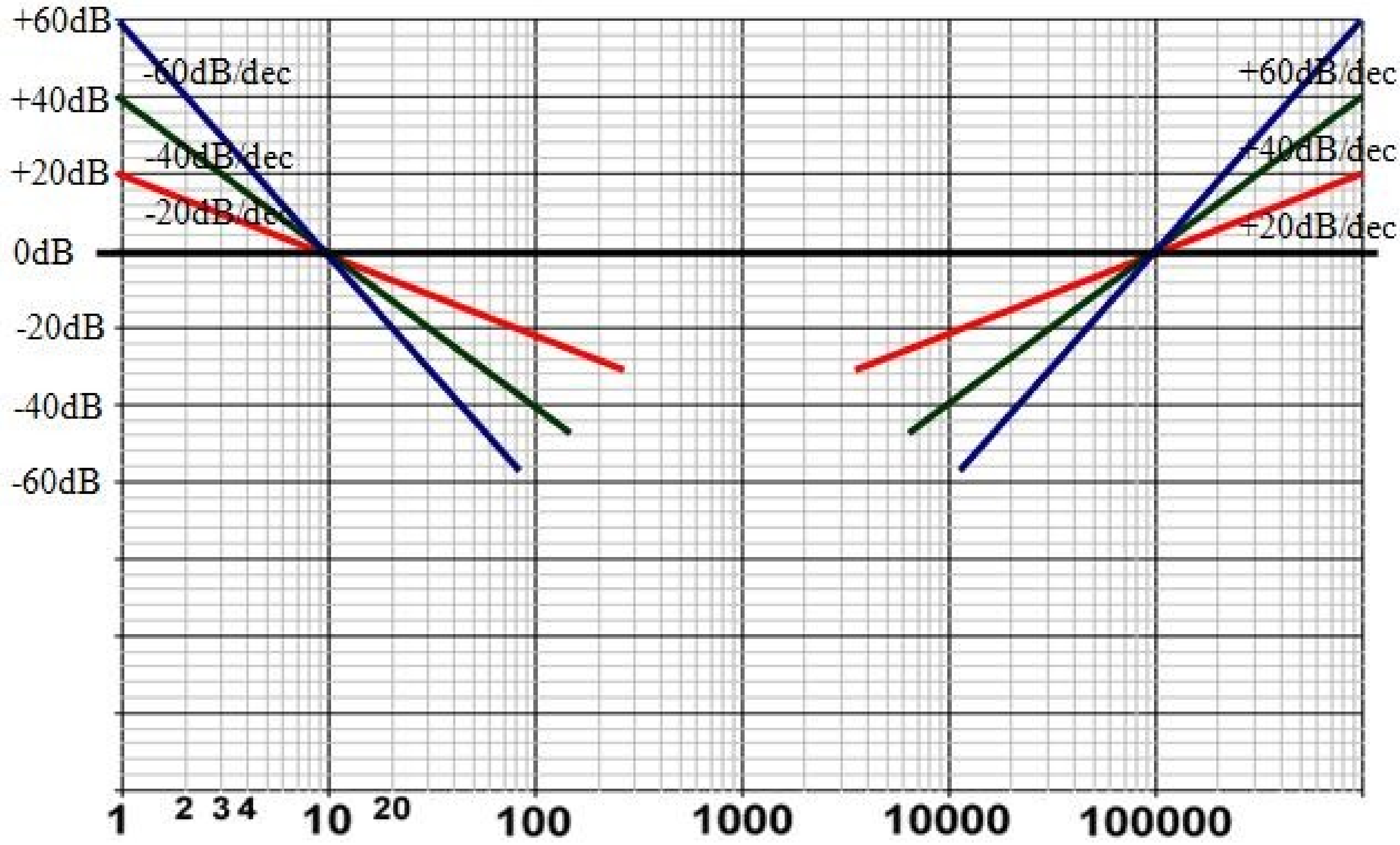
2. Find out corner frequencies

$$\frac{1}{0.1} = 10 \qquad \frac{1}{0.001} = 1000$$

So corner frequencies are 10, 1000 rad/sec

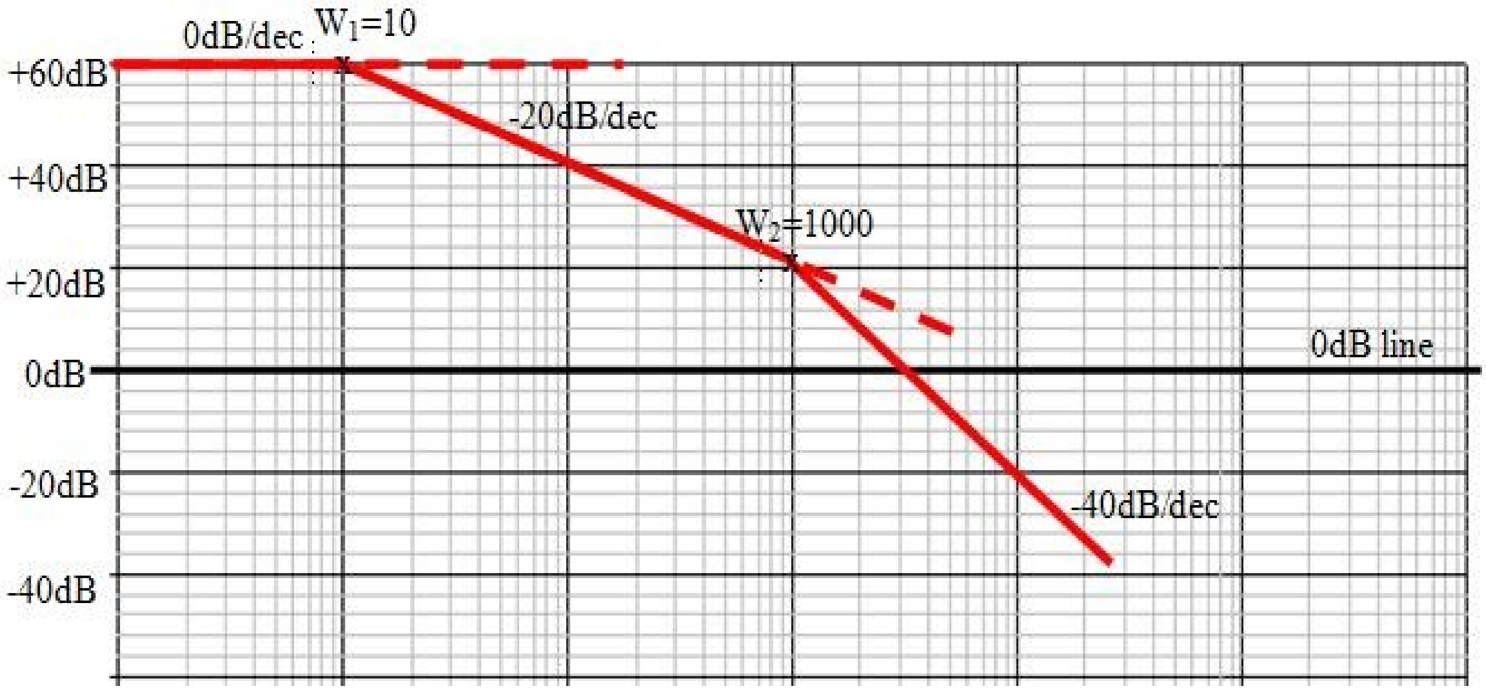
Bode Plot Examples…

- How to draw different slopes



Bode Plot Examples…

- **Magnitude Plot**



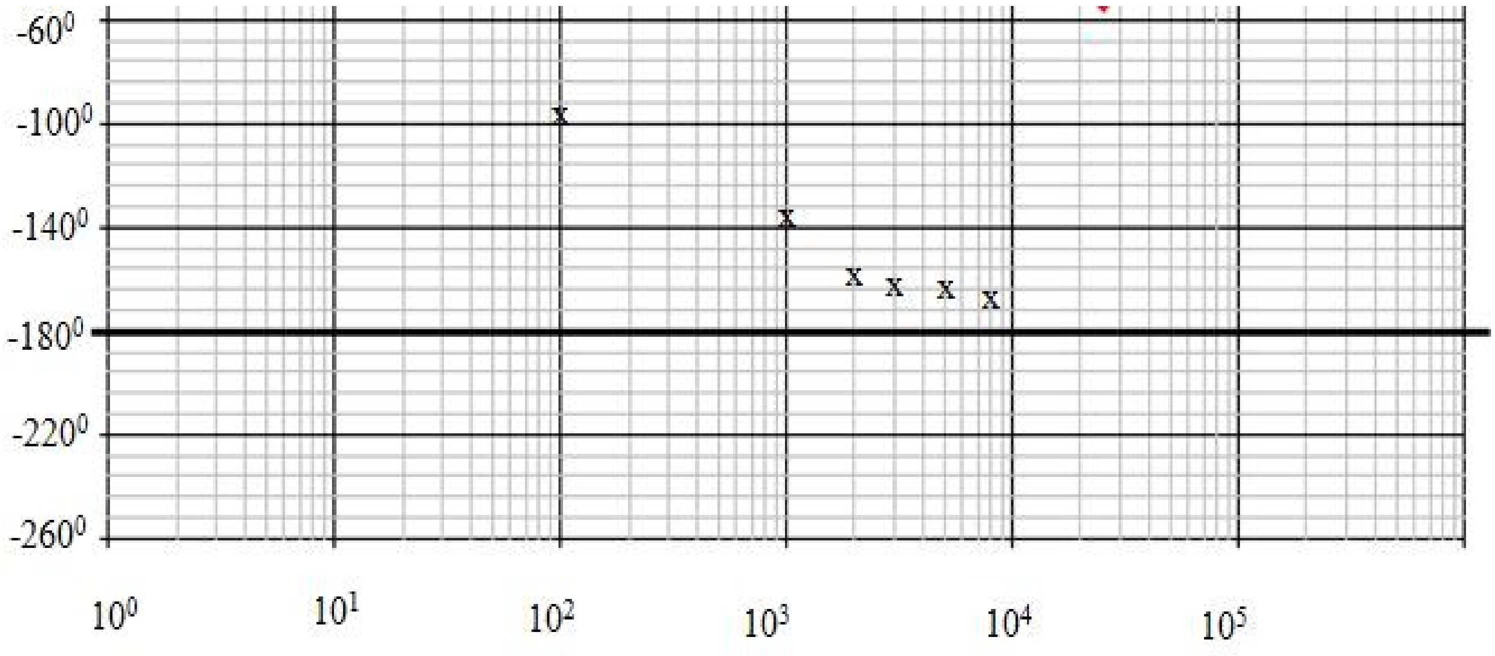
Bode Plot Examples…

- Phase Plot

S.No	W	Angle (G(jw)
1	1	----
2	100	-90^0
3	200	-98^0
4	1000	-134.42^0
5	2000	-153.15^0
6	3000	-161.36^0
7	5000	-168.57^0
8	8000	-172.79^0
9	Infi	-180^0

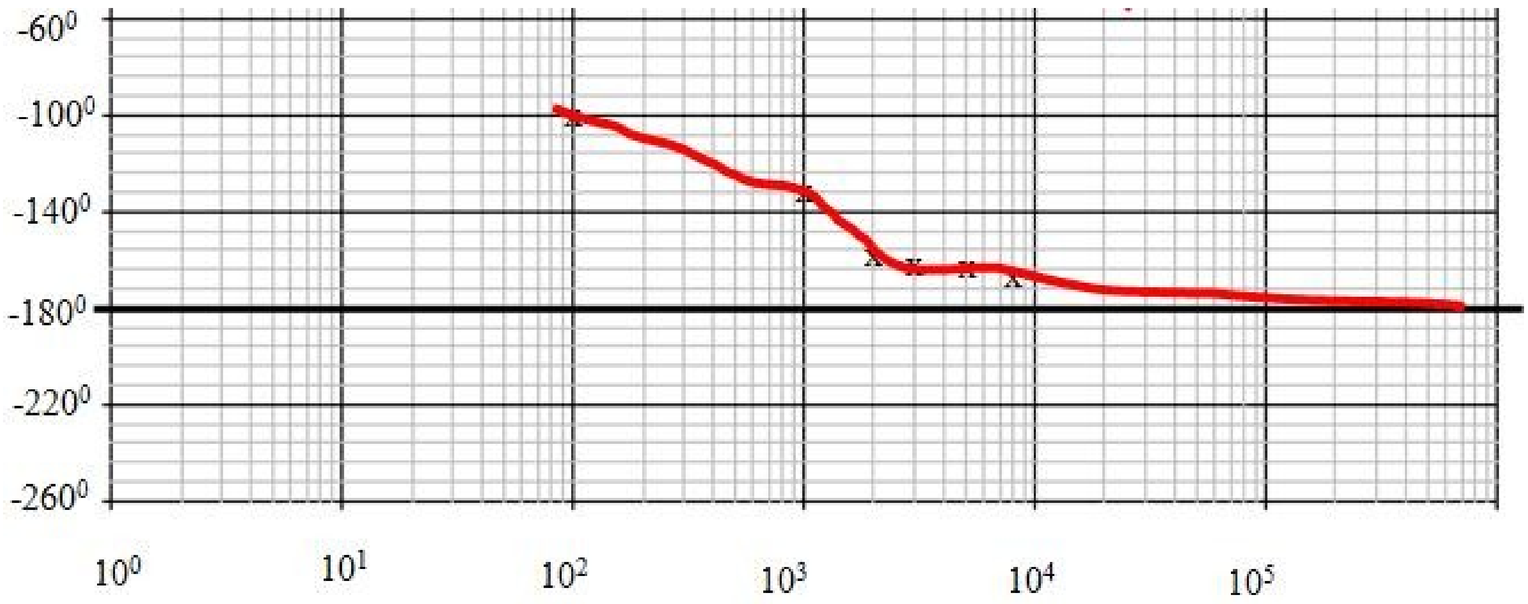
Bode Plot Examples…

- **Phase Plot**



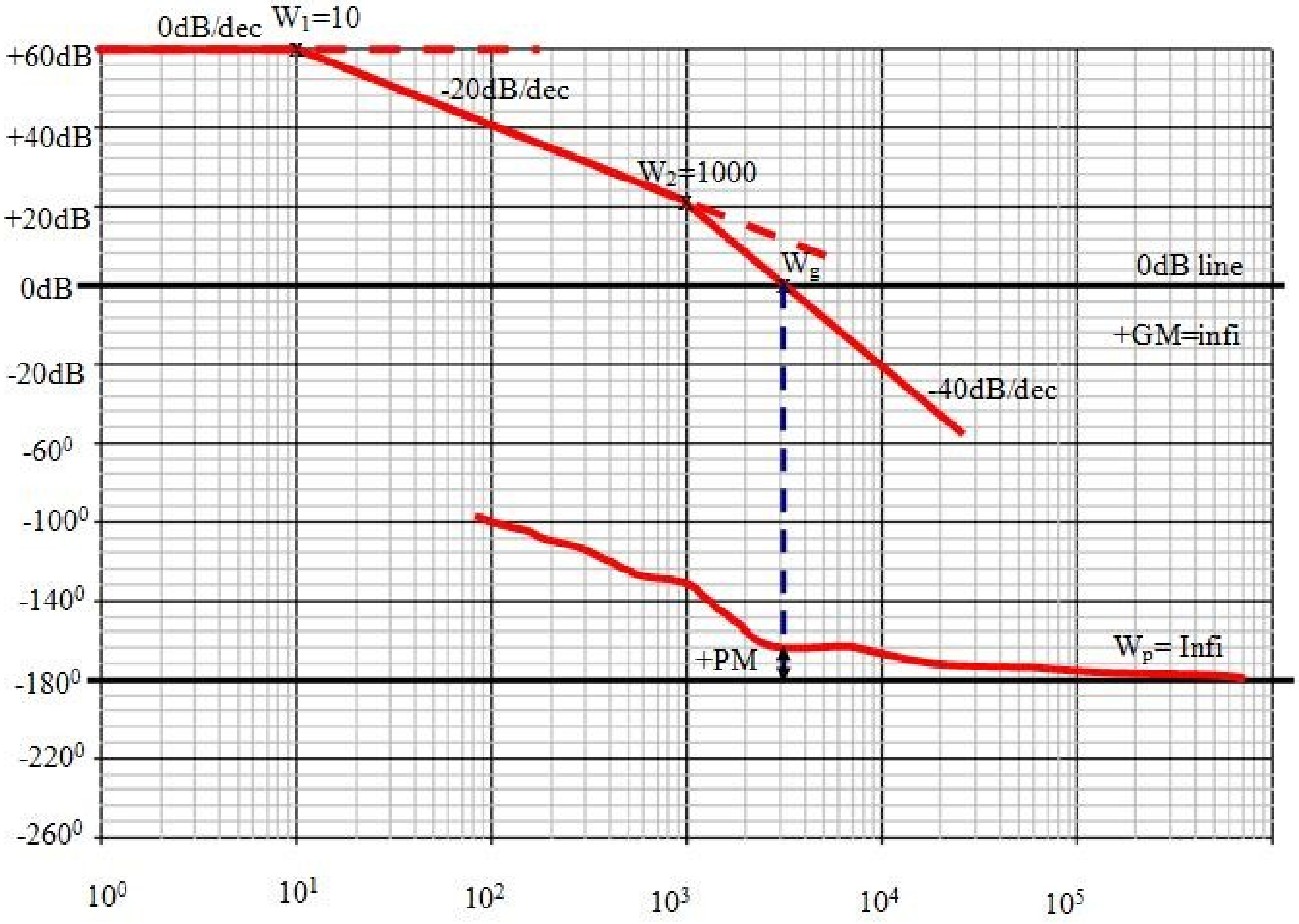
Bode Plot Examples…

- **Phase Plot …**



Bode Plot Examples…

- So Complete Bode Plot



References

- Automatic Control System By Hasan Saeed
  - Katson Publication

Thanks