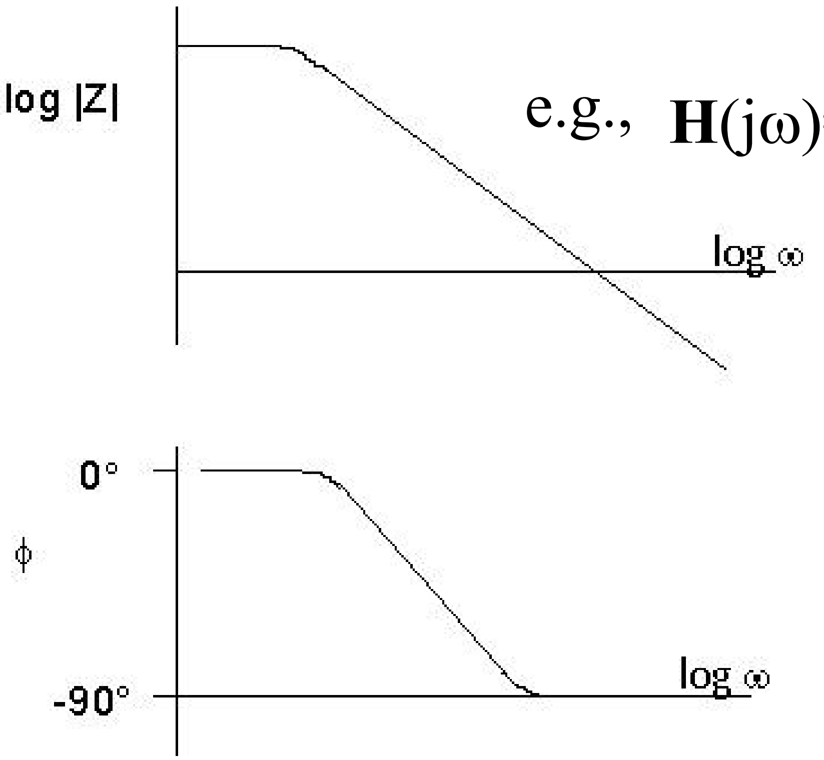
Bode Plots

- Frequency Response
- Bode plots
- Examples

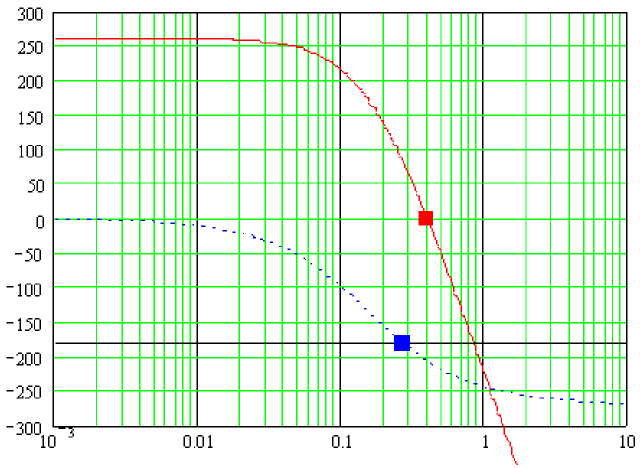
Frequency Response

- The transfer function can be separated into magnitude and phase angle information

$$\mathbf{H}(j\omega) = |\mathbf{H}(j\omega)| \angle \mathbf{\Phi}(j\omega)$$



e.g., $\mathbf{H}(j\omega)=\mathbf{Z}(j\omega)$



Bode Plots

- A **Bode plot** is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency
- The gain magnitude is many times expressed in terms of decibels (dB)

$$dB = 20 \log_{10} A$$

where A is the amplitude or gain
  - a *decade* is defined as any 10-to-1 frequency range
  - an *octave* is any 2-to-1 frequency range

20 dB/decade = 6 dB/octave

Bode Plots

- Straight-line approximations of the Bode plot may be drawn quickly from knowing the poles and zeros
  - response approaches a minimum near the zeros
  - response approaches a maximum near the poles
- The overall effect of constant, zero and pole terms

Term	Magnitude Break	Asymptotic Magnitude Slope	Asymptotic Phase Shift
Constant (K)	N/A	0	0°
Zero	upward	+20 dB/decade	+ 90°
Pole	downward	–20 dB/decade	– 90°

Bode Plots

- Express the transfer function in standard form

$$\mathbf{H}(j\omega) = \frac{K(j\omega)^{\pm N}(1+j\omega\tau_1)\left[1+2\varsigma_2(j\omega\tau_2)+(j\omega\tau_2)^2\right]\cdots}{(1+j\omega\tau_a)\left[1+2\varsigma_b(j\omega\tau_b)+(j\omega\tau_b)^2\right]\cdots}$$

- There are four different factors:
  - Constant gain term, *K*
  - Poles or zeros at the origin, $(j\omega)^{\pm N}$
  - Poles or zeros of the form $(1+ j\omega\tau)$
  - Quadratic poles or zeros of the form $1+2\zeta(j\omega\tau)+(j\omega\tau)^2$

Bode Plots

- We can combine the constant gain term (*K*) and the *N* pole(s) or zero(s) at the origin such that the magnitude crosses 0 dB at

$$Pole: \quad \frac{K}{(j\omega)^N} \qquad \omega_{0\,dB} = K^{1/N}$$

$$Zero: \quad K(j\omega)^N \quad \omega_{0\,dB} = (1/K)^{1/N}$$

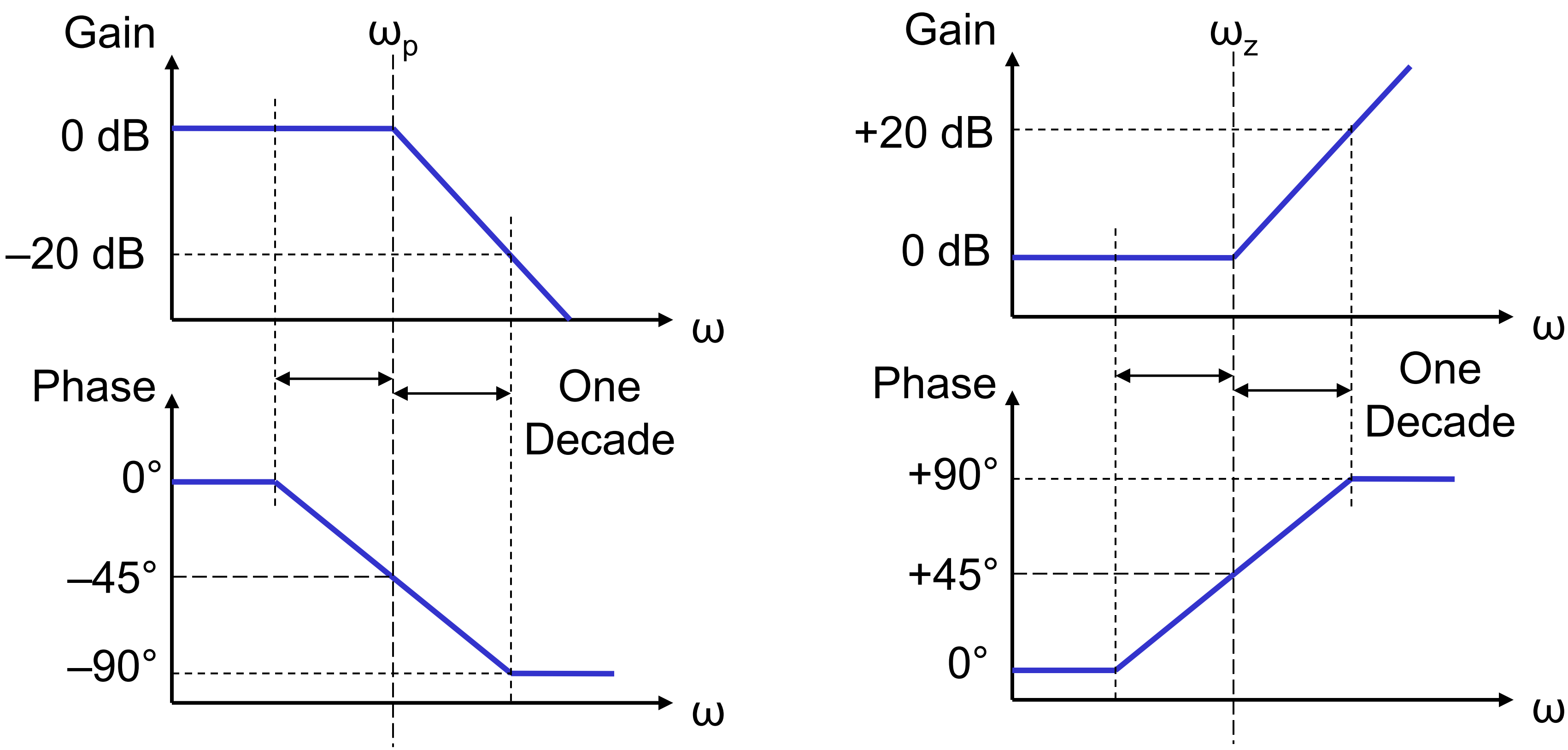
- Define the *break frequency* to be at $\omega=1/\tau$ with magnitude at ±3 dB and phase at ±45°

Bode Plot Summary

Factor	Magnitude Behavior			Phase Behavior		
	Low Freq	Break	Asymptotic	Low Freq	Break	Asymptotic
Constant	$20 \log_{10}(K)$ for all frequencies			0° for all frequencies		
Poles or zeros at origin	±20N dB/decade for all frequencies with a crossover of 0 dB at $\omega=1$			±90°(N) for all frequencies		
First order (simple) poles or zeros	0 dB	±3N dB at $\omega=1/\tau$	±20N dB/decade	0°	±45°(N) with slope ±45°(N) per decade	±90°(N)
Quadratic poles or zeros	0 dB	see ζ at $\omega=1/\tau$	±40N dB/decade	0°	±90°(N)	±180°(N)

where N is the number of roots of value τ

Single Pole & Zero Bode Plots



Pole at
$\omega_p = 1/\tau$

Assume K=1
$20 \log_{10}(K) = 0$ dB

Zero at
$\omega_z = 1/\tau$

Bode Plot Refinements

- Further refinement of the magnitude characteristic for first order poles and zeros is possible since
  - Magnitude at half break frequency: $|\mathbf{H}(\frac{1}{2}\omega_b)| = \pm 1$ dB
  - Magnitude at break frequency: $|\mathbf{H}(\omega_b)| = \pm 3$ dB
  - Magnitude at twice break frequency: $|\mathbf{H}(2\omega_b)| = \pm 7$ dB
- Second order poles (and zeros) require that the *damping ratio* (ζ value) be taken into account; see Fig. 9-30 in textbook

Bode Plots to Transfer Function

- We can also take the Bode plot and extract the transfer function from it (although in reality there will be error associated with our extracting information from the graph)
- First, determine the constant gain factor, K
- Next, move from lowest to highest frequency noting the appearance and order of the poles and zeros

Class Examples

- Drill Problems P9-3, P9-4, P9-5, P9-6 (hand-drawn Bode plots)
- Determine the system transfer function, given the Bode magnitude plot below