Frequency Response Methods and Stability

In previous chapters we examined the use of test signals such as a step and a ramp signal. In this chapter we consider the steady-state response of a system to a sinusoidal input test signal. We will see that the response of a linear constant coefficient system to a sinusoidal input signal is an output sinusoidal signal at the same frequency as the input. However, the magnitude and phase of the output signal differ from those of the input sinusoidal signal, and the amount of difference is a function of the input frequency. Thus we will be investigating the steady-state response of the system to a sinusoidal input as the frequency varies.

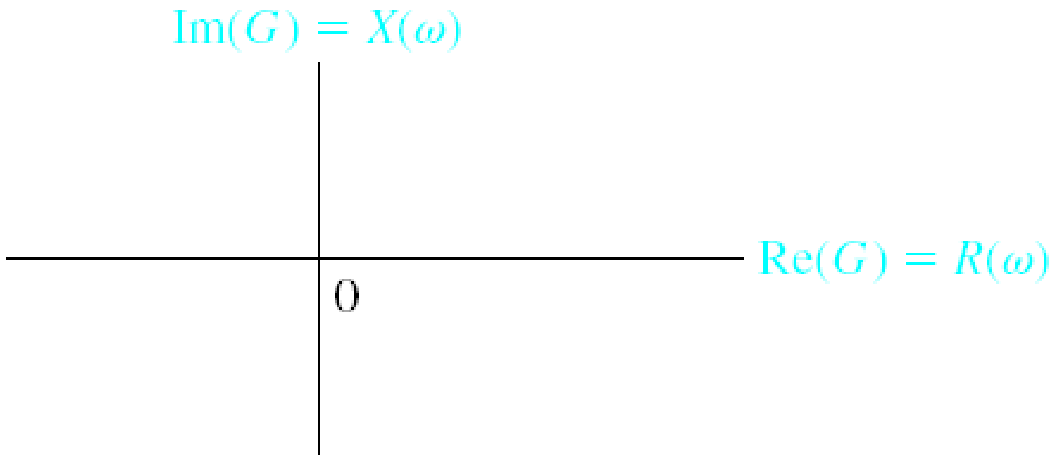
We will examine the transfer function G(s) when s =jw and develop methods for graphically displaying the complex number G(j)as w varies. The Bode plot is one of the most powerful graphical tools for analyzing and designing control systems, and we will cover that subject in this chapter. We will also consider polar plots and log magnitude and phase diagrams. We will develop several time-domain performance measures in terms of the frequency response of the system as well as introduce the concept of system bandwidth.

Introduction

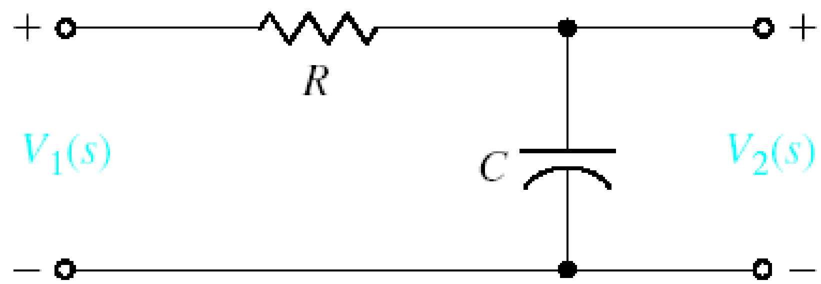
The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal. The sinusoid is a unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady-state; it differs form the input waveform only in amplitude and phase.

Frequency Response Plots

Polar Plots



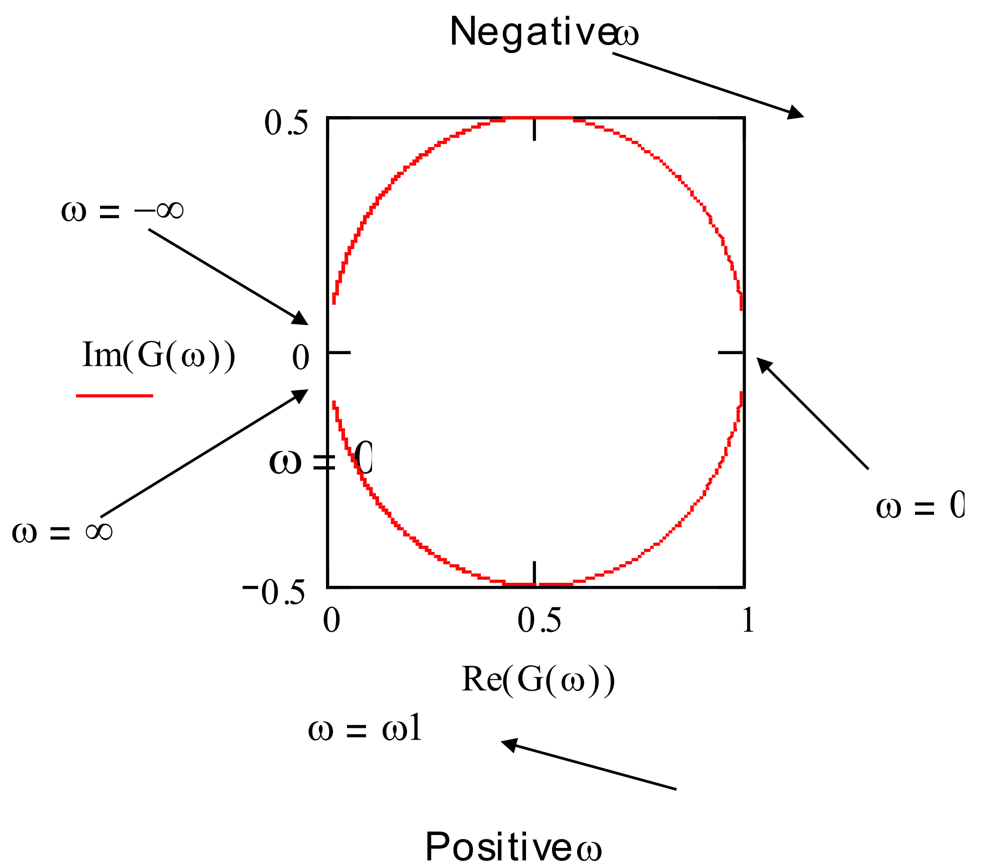
Frequency Response Plots



Polar Plots

$\omega := -1000, -999 .. 1000 \qquad j := \sqrt{-1} \qquad R := 1 \qquad C := 0.01 \qquad \omega 1 := \frac{1}{R \cdot C}$

$$G(\omega) := \frac{1}{\left(j \cdot \frac{\omega}{\omega 1}\right) + 1}$$

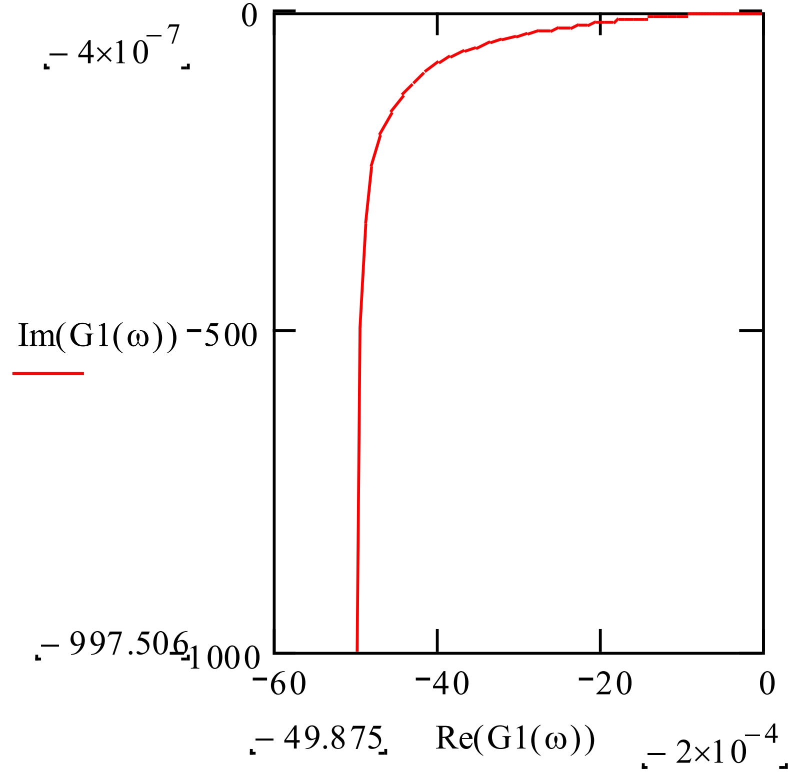


Frequency Response Plots

Polar Plots

$\omega := 0, .1 .. 1000 \qquad \tau := 0.5 \qquad K := 10($

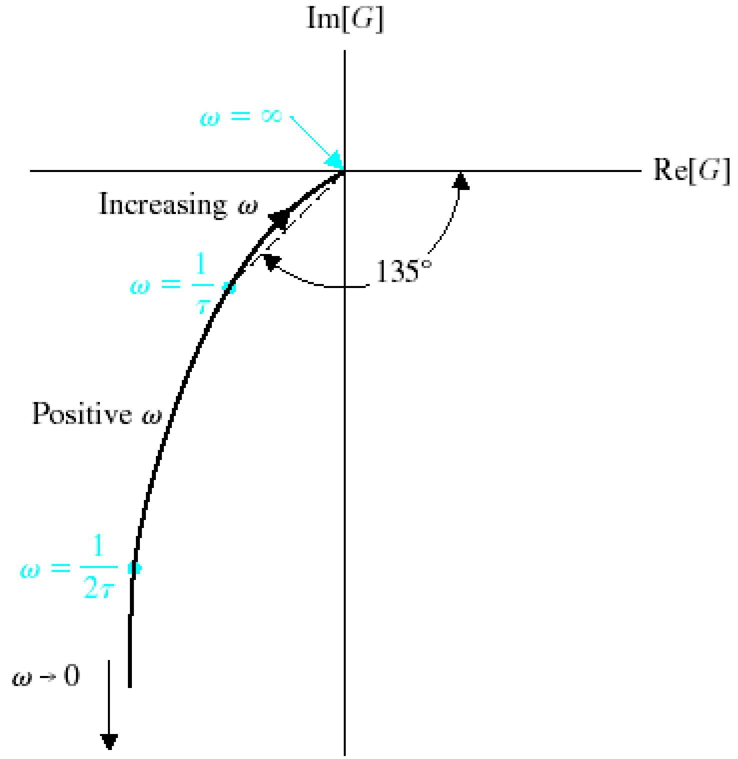
$$G1(\omega) := \frac{\frac{K}{\tau}}{j \cdot \omega \cdot \left(j \cdot \omega + \frac{1}{\tau} \right)}$$



Polar plot for $G(j\omega) = K/j\omega(j\omega\tau + 1)$. Note that $\omega = \infty$ at the origin.

Frequency Response Plots

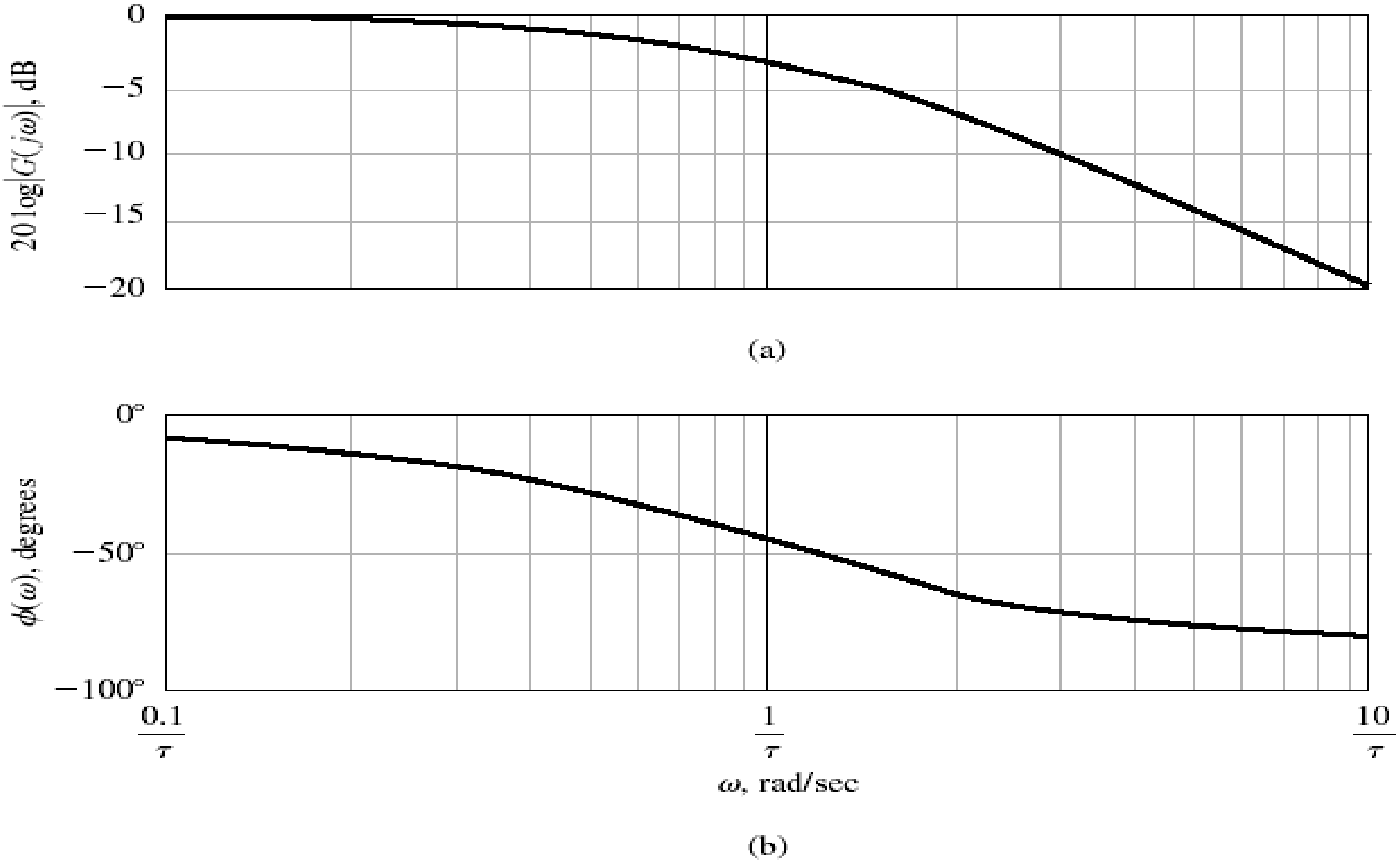
Polar Plots



Polar plot for $G(j\omega) = K/j\omega(j\omega\tau + 1)$. Note that $\omega = \infty$ at the origin.

Frequency Response Plots

Bode Plots – Real Poles



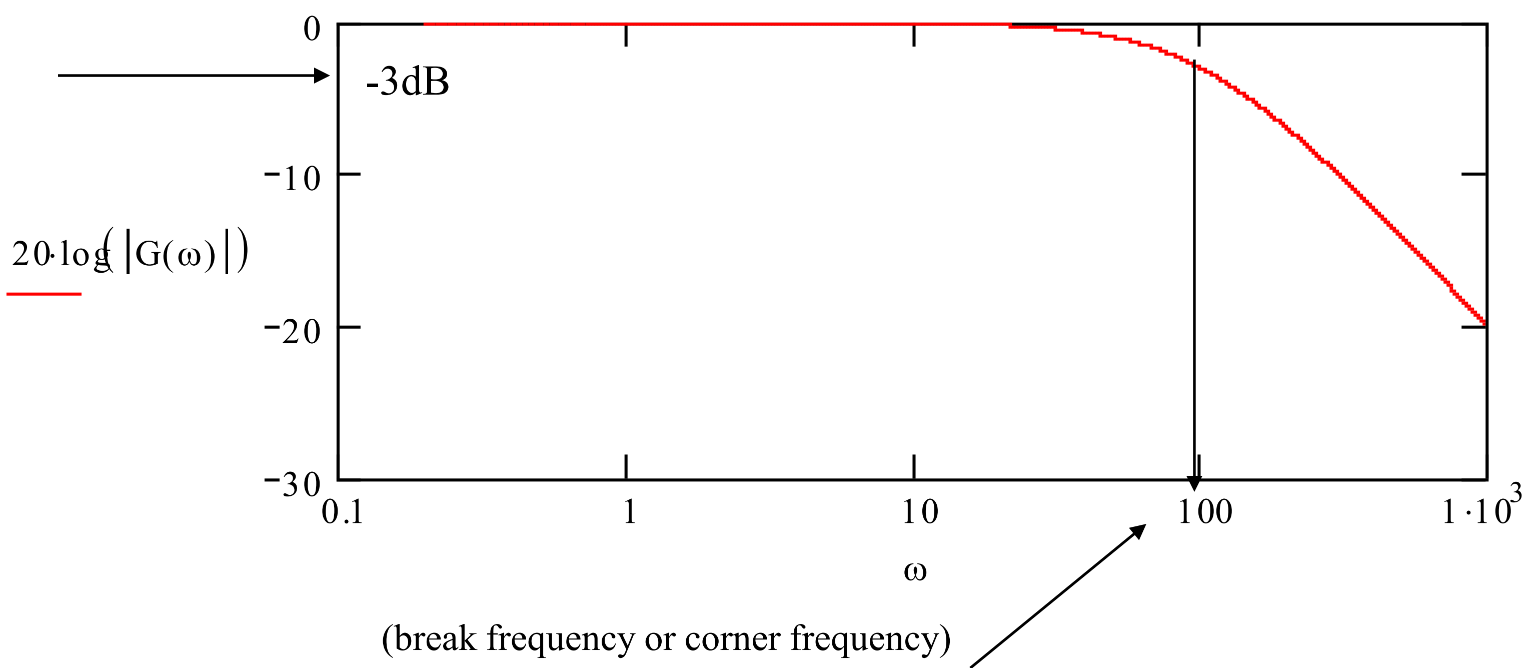
Bode diagram for $G(j\omega) = 1/(j\omega\tau + 1)$: (a) magnitude plot and (b) phase plot.

Frequency Response Plots

Bode Plots – Real Poles

$$\omega := \frac{0.1}{\tau}, \frac{0.11}{\tau} .. 1000 \qquad j := \sqrt{-1} \qquad R := 1 \qquad C := 0.01 \qquad \tau := R \cdot C$$

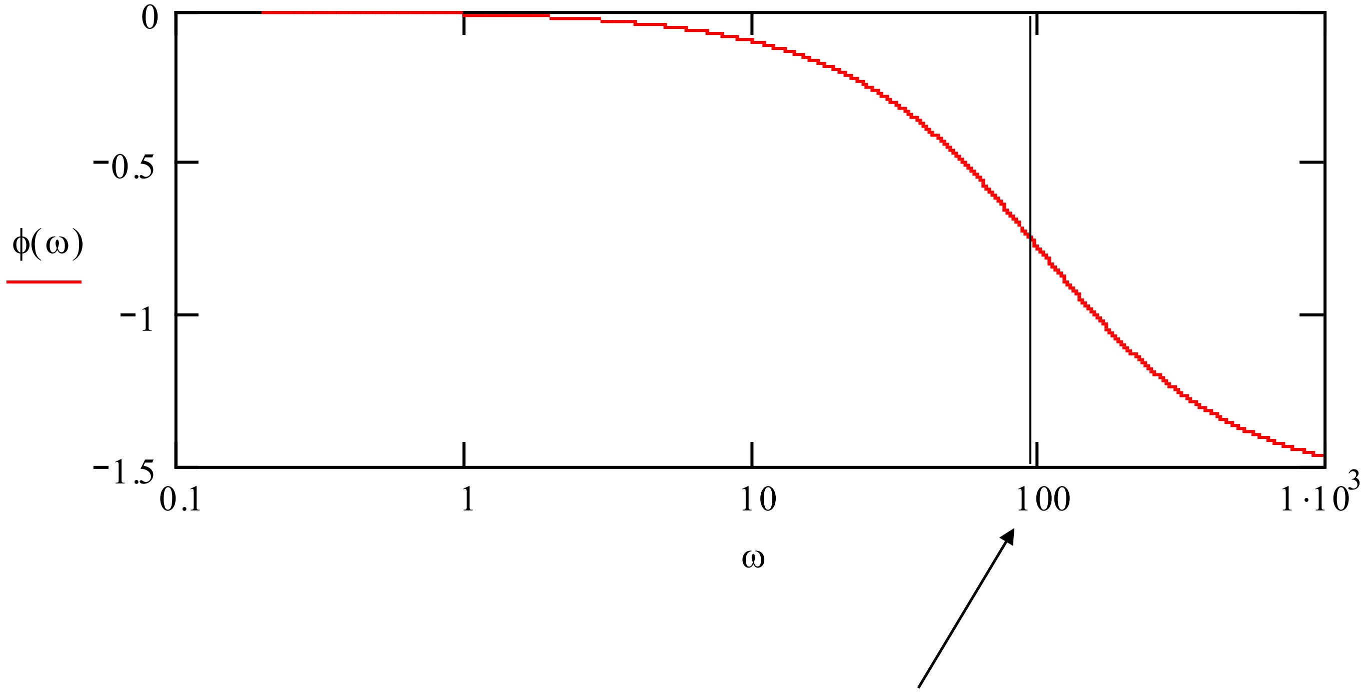
$$G(\omega) := \frac{1}{j \cdot \omega \cdot \tau + 1} \qquad \omega 1 := \frac{1}{\tau} \qquad \omega 1 = 100$$ (break frequency or corner frequency)



Frequency Response Plots

Bode Plots – Real Poles

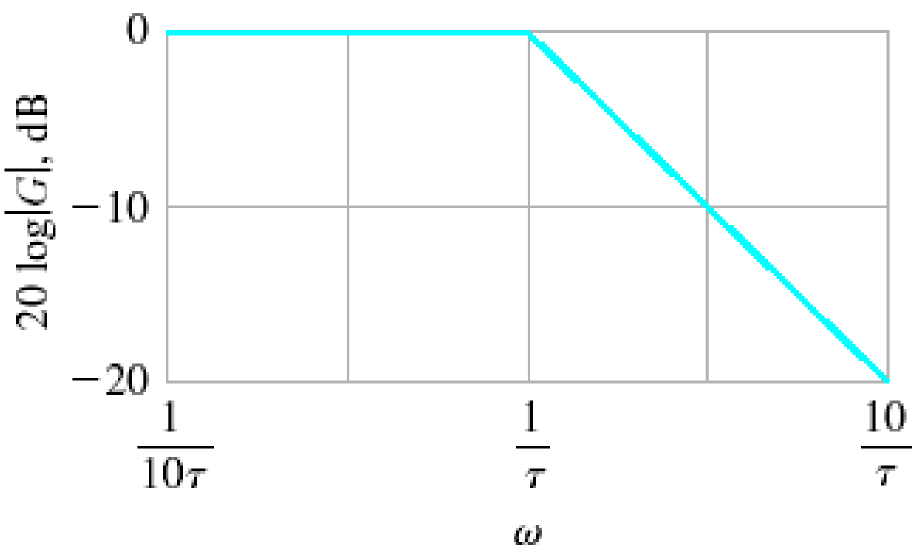
$$\phi(\omega) := -\text{atan}(\omega \cdot \tau)$$



(break frequency or corner frequency)

Frequency Response Plots

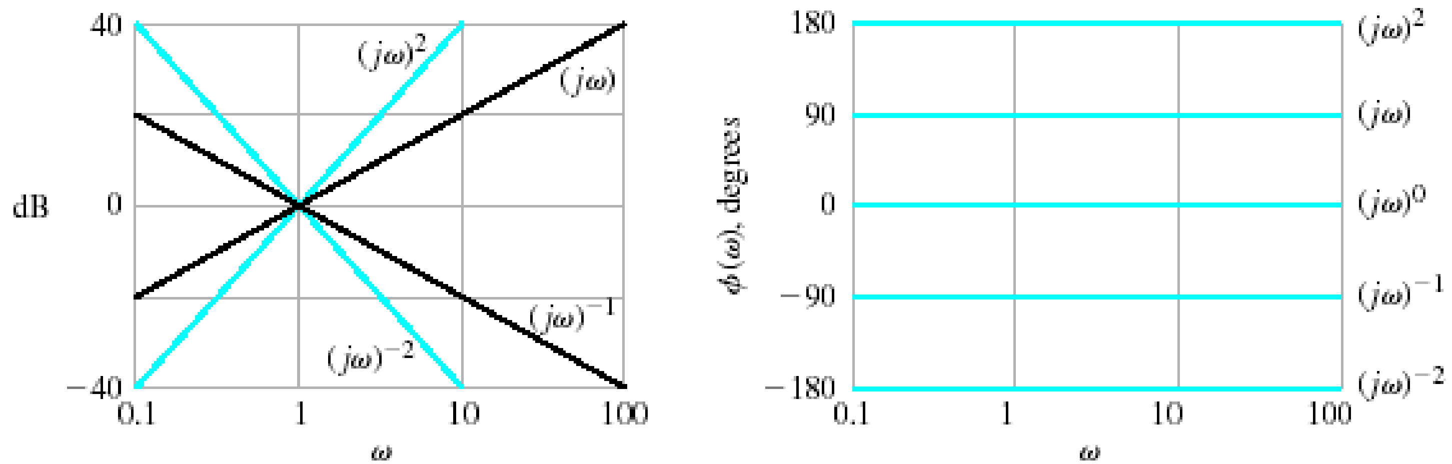
Bode Plots – Real Poles (Graphical Construction)



Asymptotic curve for $(j\omega\tau + 1)^{-1}$.

Frequency Response Plots

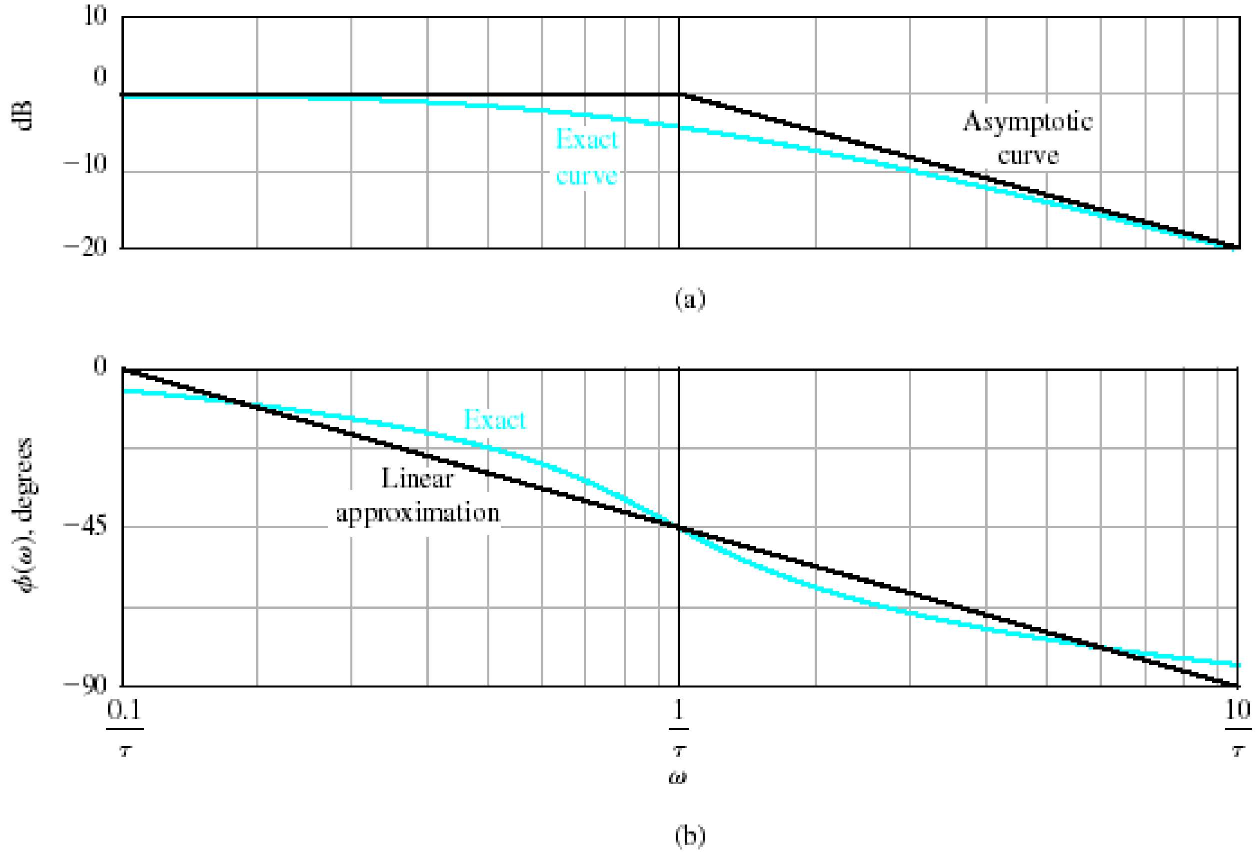
Bode Plots – Real Poles



Bode diagram for $(j\omega)^{\pm N}$.

Frequency Response Plots

Bode Plots – Real Poles



Bode diagram for $(1 + j\omega\tau)^{-1}$.

Frequency Response Plots

Bode Plots – Real Poles

Magnitude:

$$db(G,\omega) := 20\cdot\log(|G(j\cdot\omega)|)$$

Phase shift:

$$ps(G,\omega) := \frac{180}{\pi}\cdot\arg(G(j\cdot\omega)) - 360\cdot(\text{if}(\arg(G(j\cdot\omega)) \geq 0,1,0))$$

Assume

$$K := 2 \qquad G(s) := \frac{K}{s\cdot(1+s)\cdot\left(1+\frac{s}{3}\right)}$$

Next, choose a frequency range for the plots (use powers of 10 for convenient plotting):

lowest frequency (in Hz): $\omega_{start} := .01$ number of points: $N := 50$

highest frequency (in Hz): $\omega_{end} := 100$

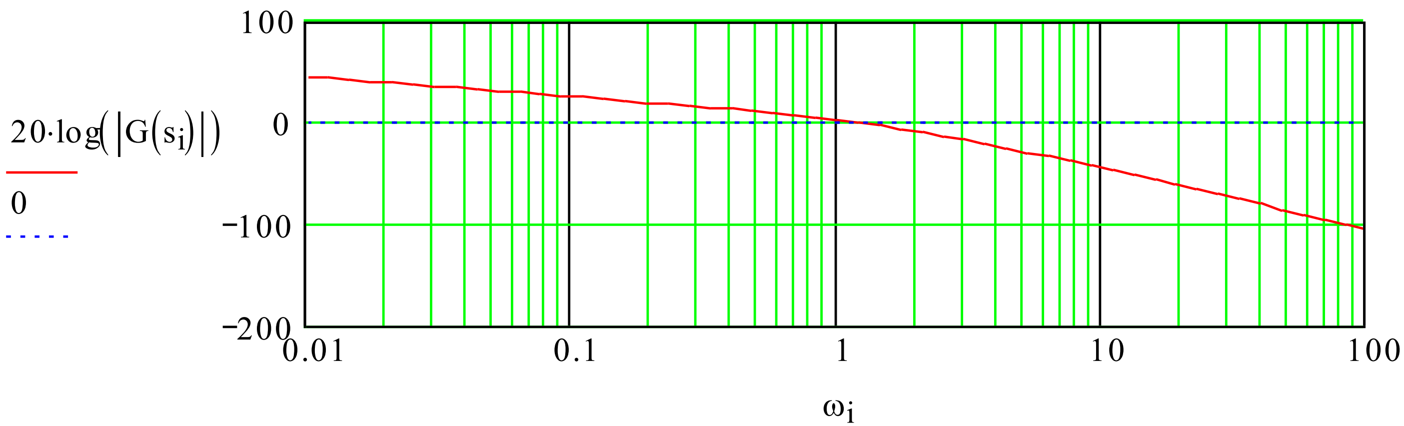
step size: $r := \log\left(\frac{\omega_{start}}{\omega_{end}}\right)\cdot\frac{1}{N}$

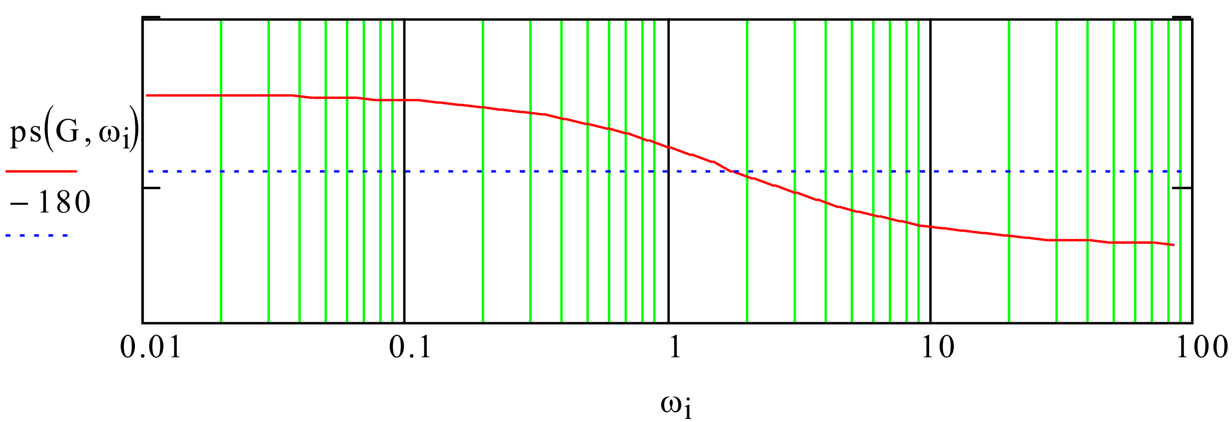
range for plot: $i := 0..N$ range variable: $\omega_i := \omega_{end}\cdot 10^{i\cdot r}$ $s_i := j\cdot\omega_i$

Frequency Response Plots

Bode Plots – Real Poles

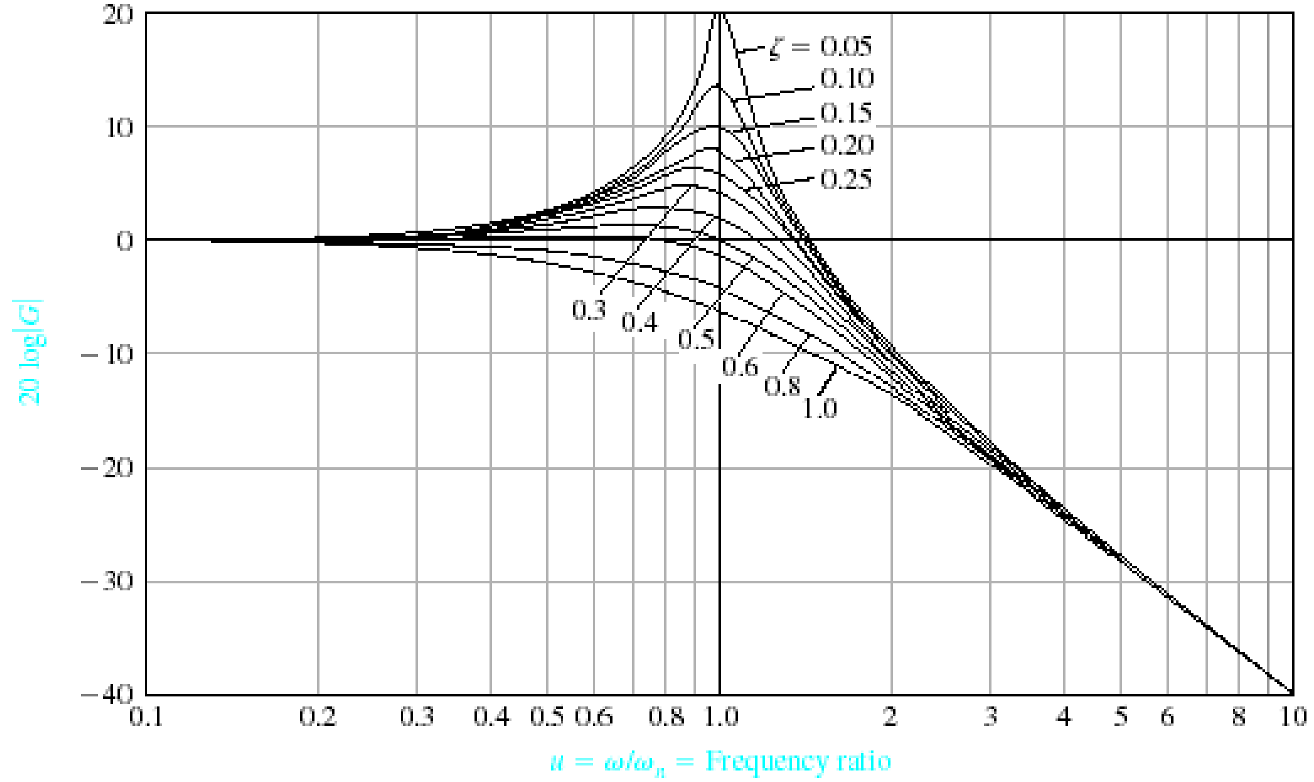
range for plot: $i := 0..N$ range variable: $\omega_i := \omega_{end} \cdot 10^{i \cdot r}$ $s_i := j \cdot \omega_i$





Frequency Response Plots

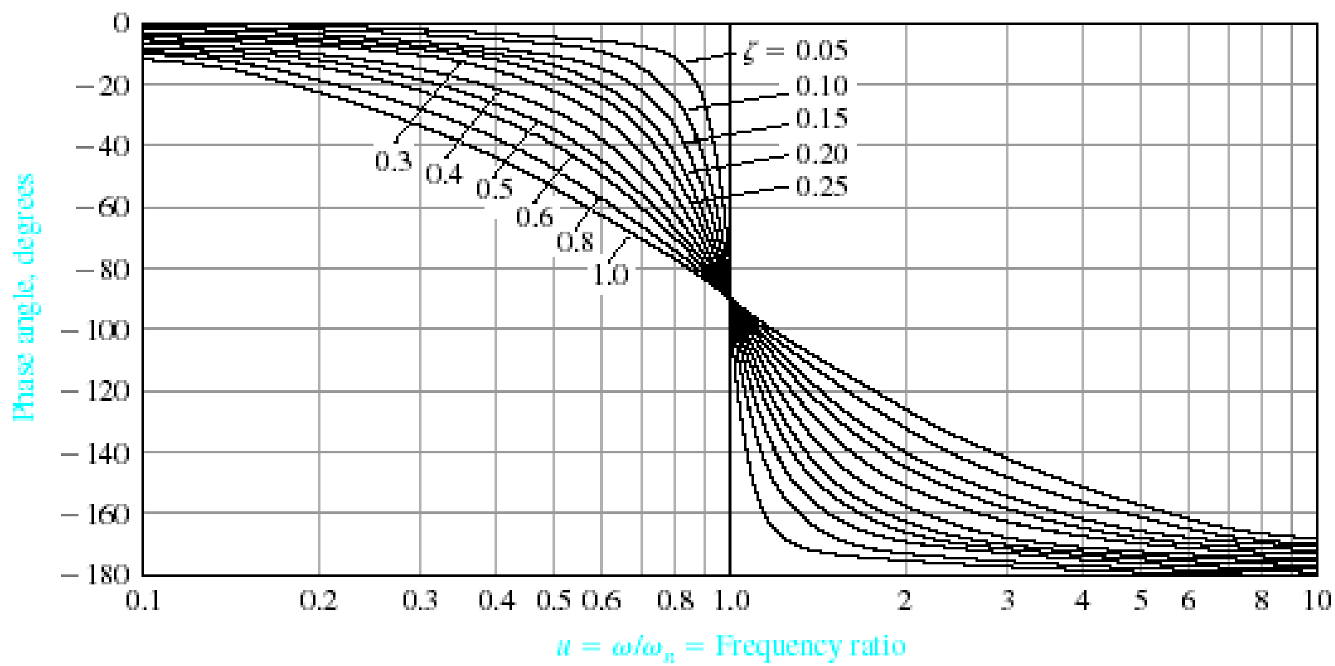
Bode Plots – Complex Poles



Bode diagram for $G(j\omega) = [1 + (2\zeta/\omega_n)j\omega + (j\omega/\omega_n)^2]^{-1}$.

Frequency Response Plots

Bode Plots – Complex Poles



Bode diagram for $G(j\omega) = [1 + (2\zeta/\omega_n)\, j\omega + (j\omega/\omega_n)^2]^{-1}$.

Frequency Response Plots

Bode Plots – Complex Poles

$$\omega_r = \omega_n \cdot \sqrt{1 - 2\cdot\zeta^2} \qquad \zeta < 0.707$$

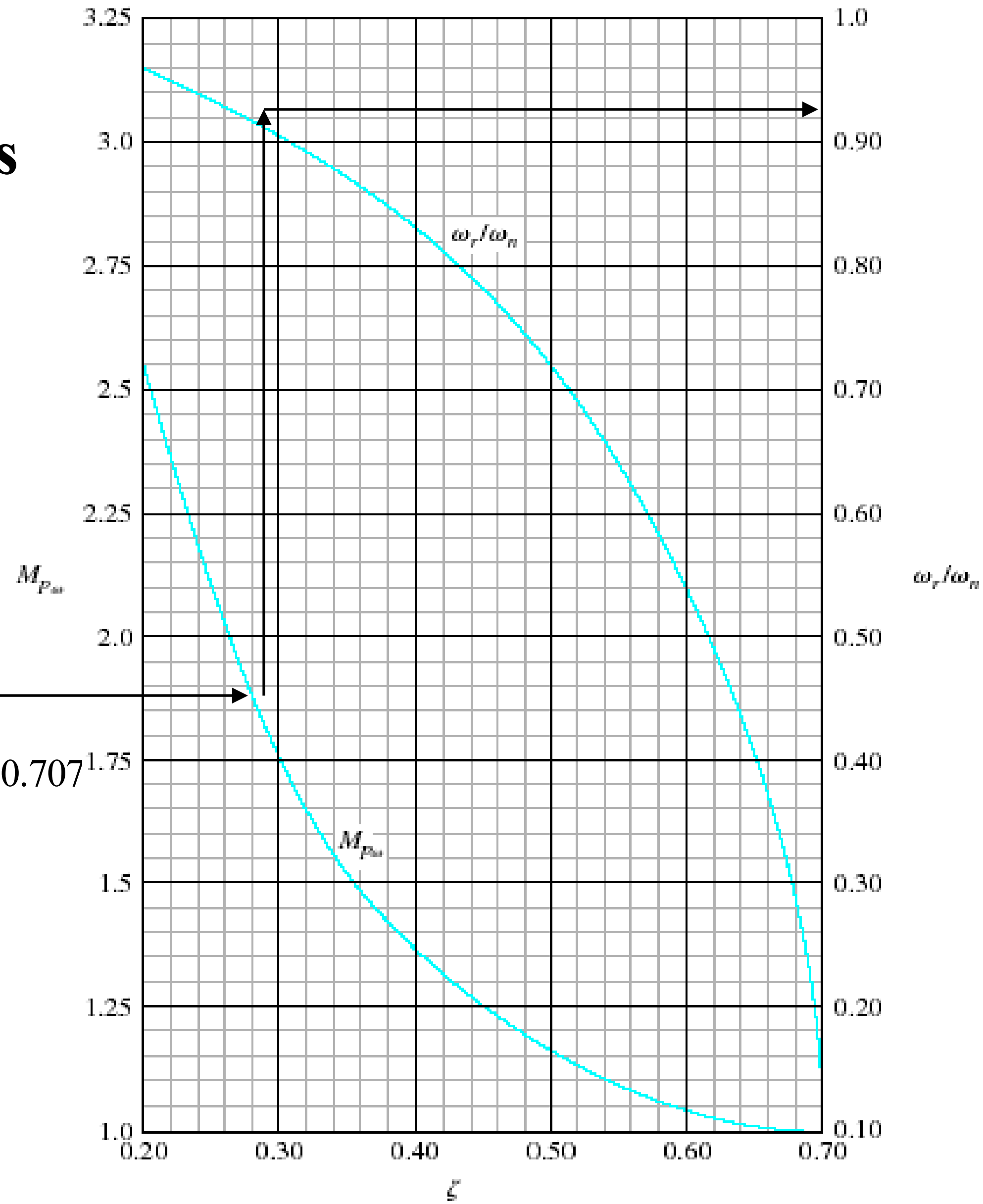
$$M_{p\omega} = \left|G\left(\omega_r\right)\right| = \frac{1}{\left(2\cdot\zeta\cdot\sqrt{1 - \zeta^2}\right)} \qquad \zeta < 0.707$$

Frequency Response Plots

Bode Plots – Complex Poles

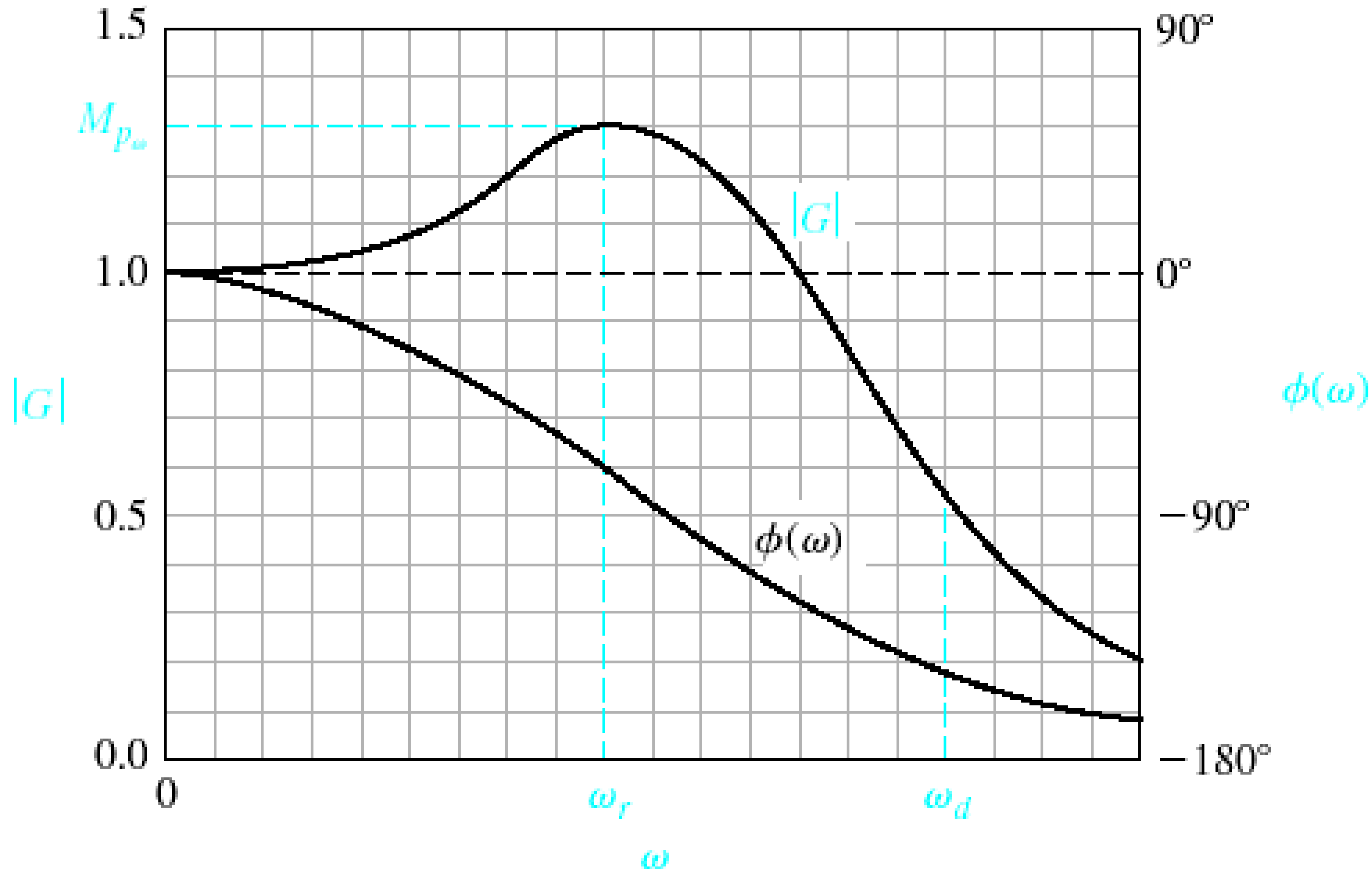
$$\omega_r = \omega_n \cdot \sqrt{1 - 2\cdot\zeta^2} \qquad \zeta < 0.707$$

$$M_{p\omega} = \left|G(\omega_r)\right| = \frac{1}{\left(2\cdot\zeta\cdot\sqrt{1-\zeta^2}\right)} \qquad \zeta < 0.707$$



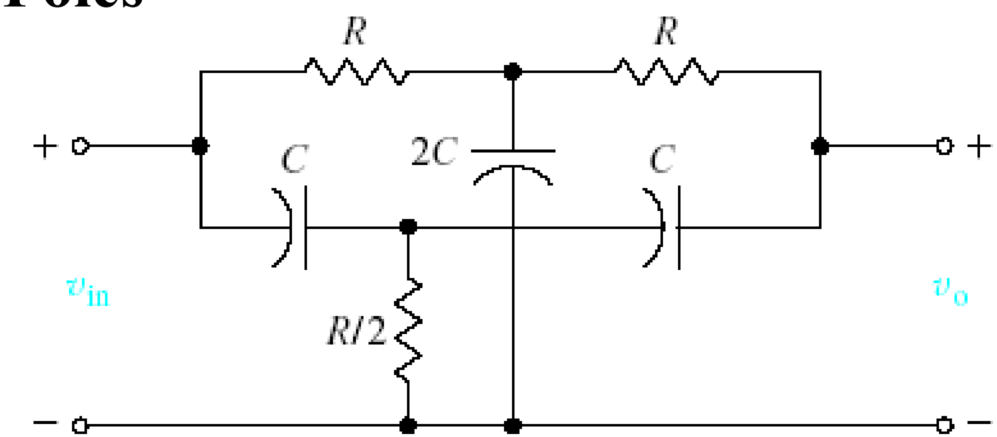
Frequency Response Plots

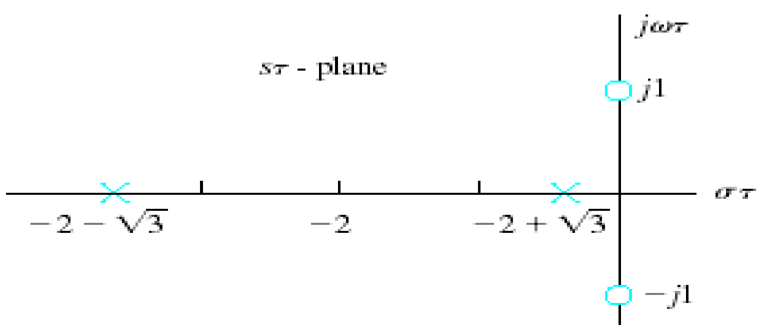
Bode Plots – Complex Poles



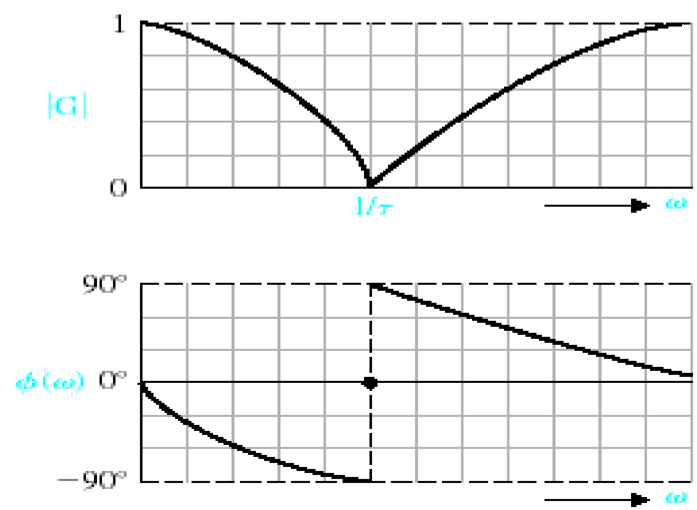
Frequency Response Plots

Bode Plots – Complex Poles



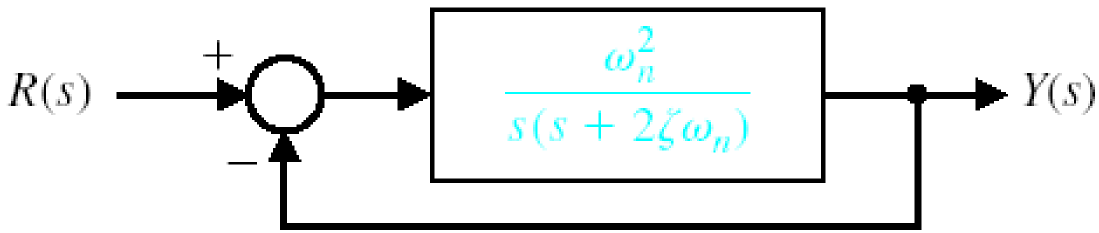


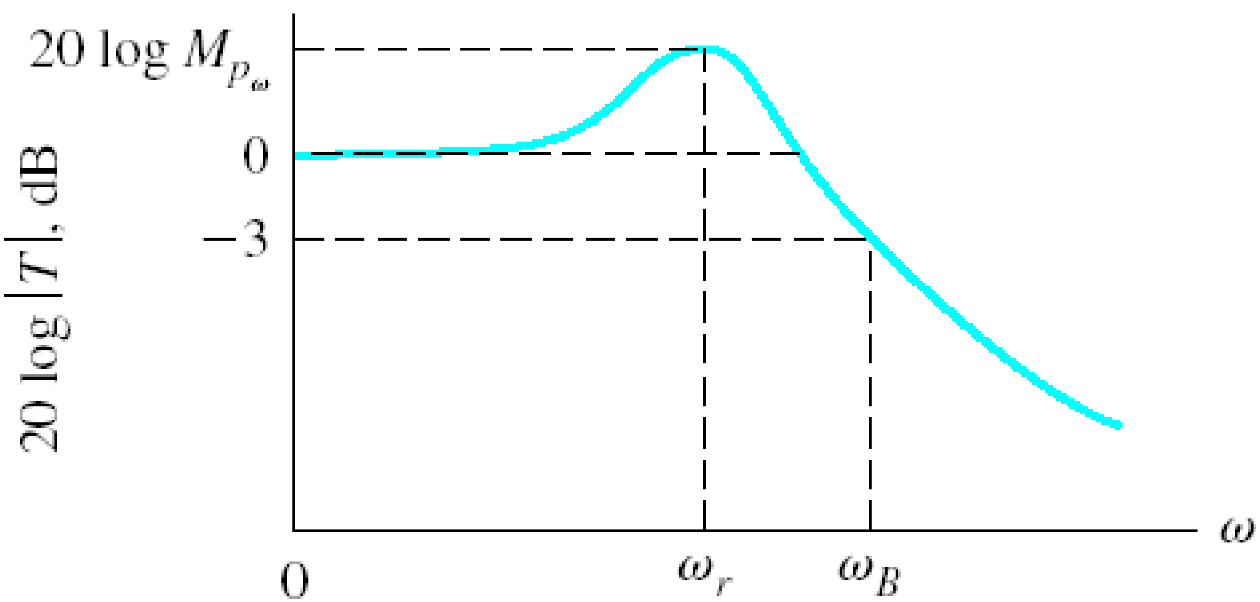
(a)



(b)

Performance Specification In the Frequency Domain



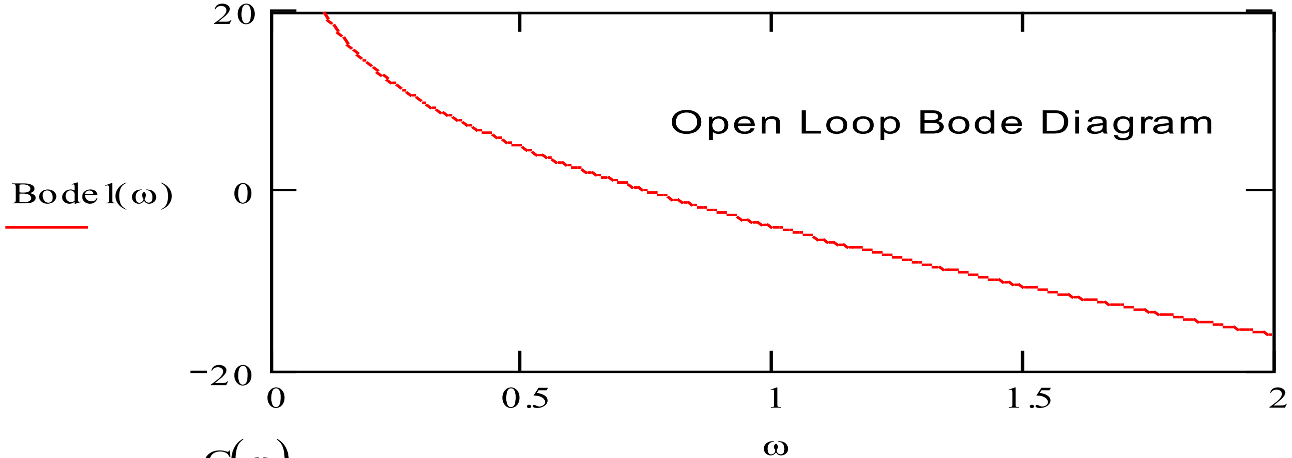


Performance Specification In the Frequency Domain

$\omega := .1, .11 .. 2$ $\quad K := 2$ $\quad j := \sqrt{-1}$

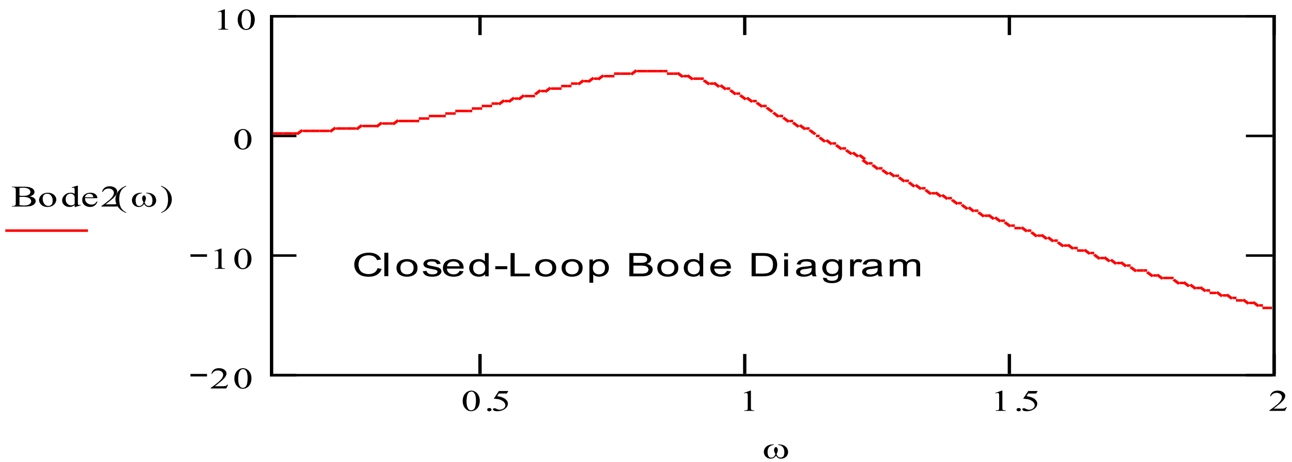
$$G(\omega) := \frac{K}{j\cdot\omega\cdot(j\cdot\omega + 1)\cdot(j\cdot\omega + 2)}$$

$$\text{Bode1}(\omega) := 20\cdot\log(|G(\omega)|)$$



$$T(\omega) := \frac{G(\omega)}{1 + G(\omega)}$$

$$\text{Bode2}(\omega) := 20\cdot\log(|T(\omega)|)$$



Performance Specification In the Frequency Domain

$w := 4$

Finding the Resonance Frequency

Given

$20 \cdot \log(|T(w)|) = 5.282$

$wr := \text{Find}(w) \qquad wr = 0.813$

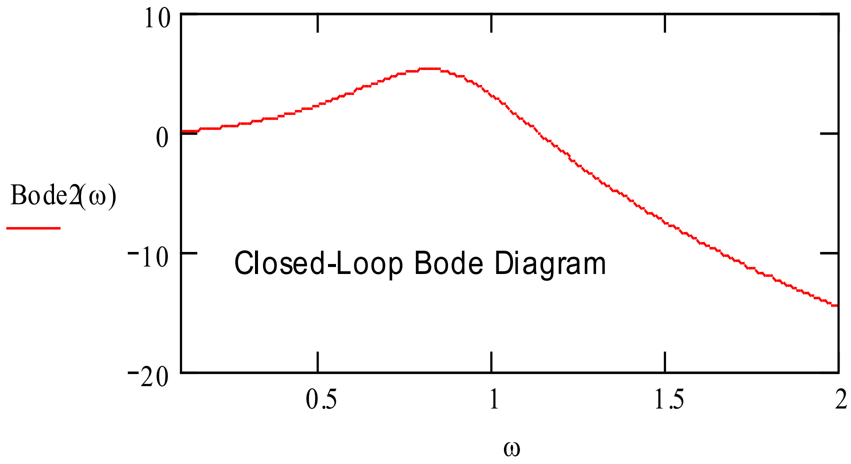
$Mpw := 1$

Given

$20 \cdot \log(Mpw) = 5.282$

Finding Maximum value of the frequency resp

$Mpw := \text{Find}(Mpw) \qquad Mpw = 1.837$



Performance Specification In the Frequency Domain

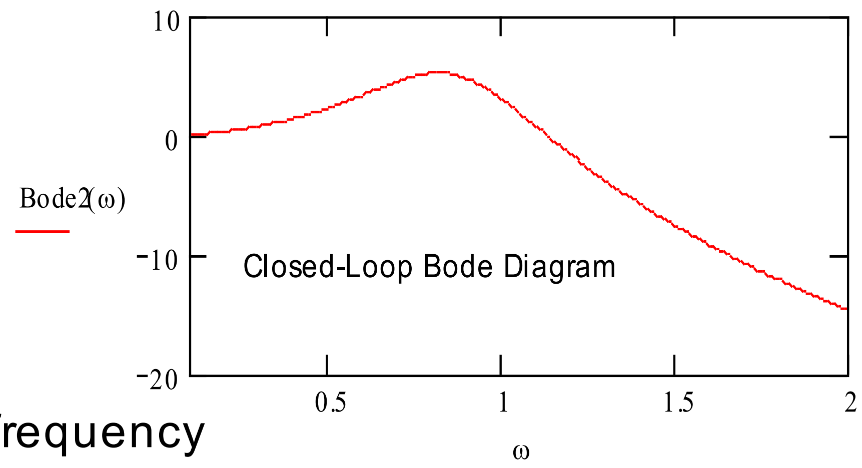
Assume that the system has dominant second-order roo

$\zeta := .1$

Finding the damping factor

Given

$$Mpw = \left[2\cdot\zeta\cdot\left(\sqrt{1-\zeta^2}\right)\right]^{-1}$$



$\zeta := \text{Find}(\zeta)$ $\zeta = 0.284$

$wn := .1$

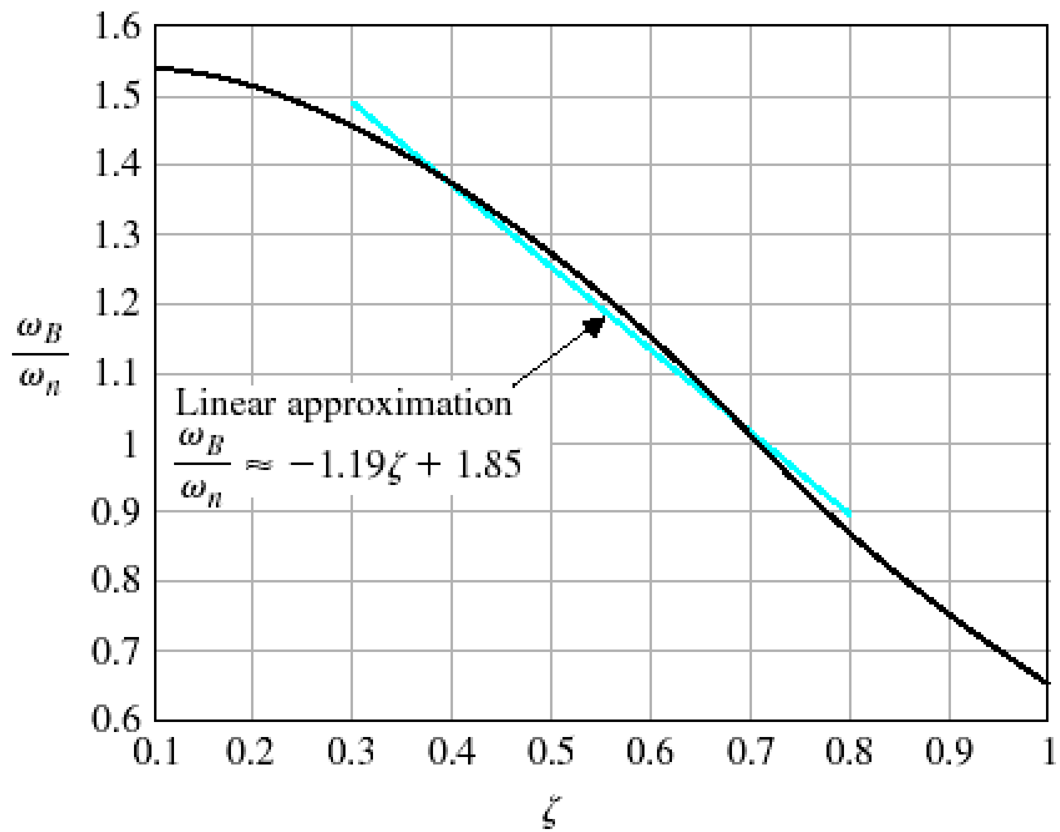
Finding the natural frequency

Given

$$wr = wn\cdot\sqrt{1-2\cdot\zeta^2}$$

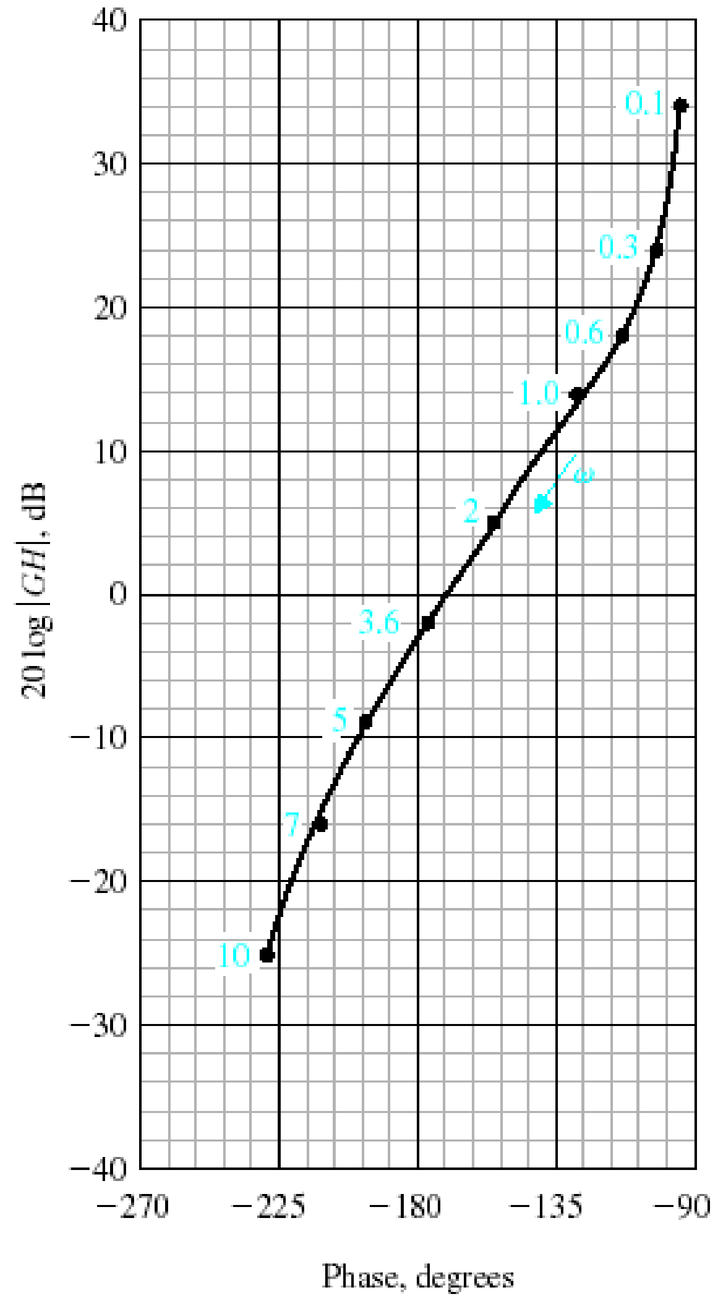
$wn := \text{Find}(wn)$ $wn = 0.888$

Performance Specification In the Frequency Domain

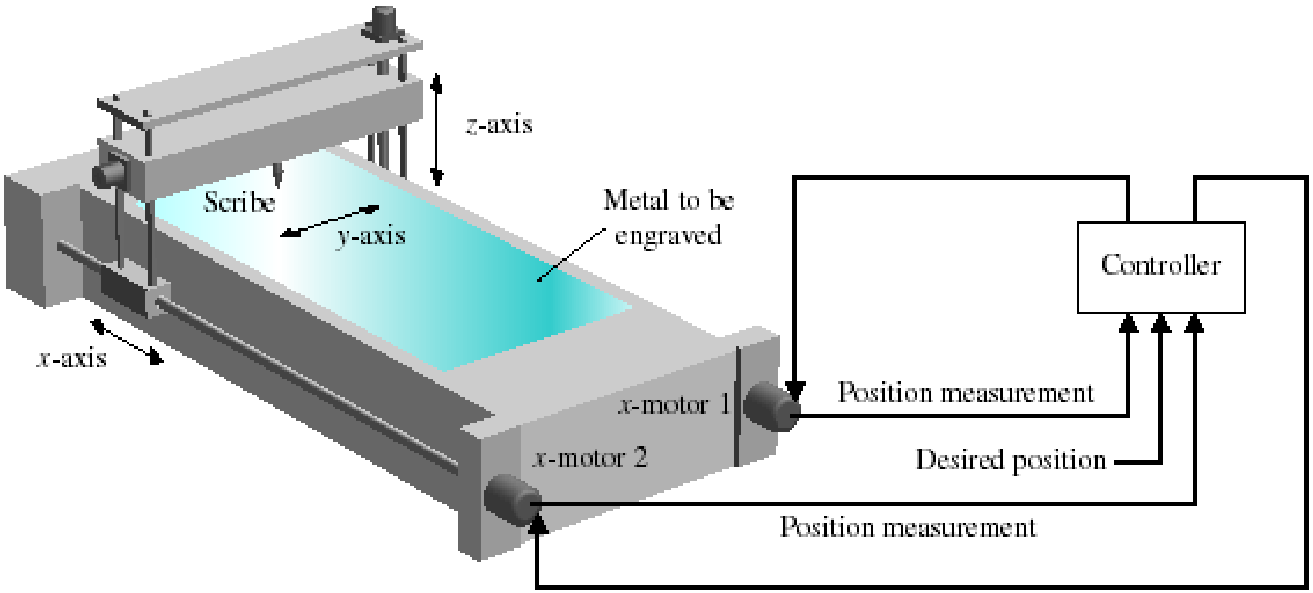


Performance Specification In the Frequency Domain

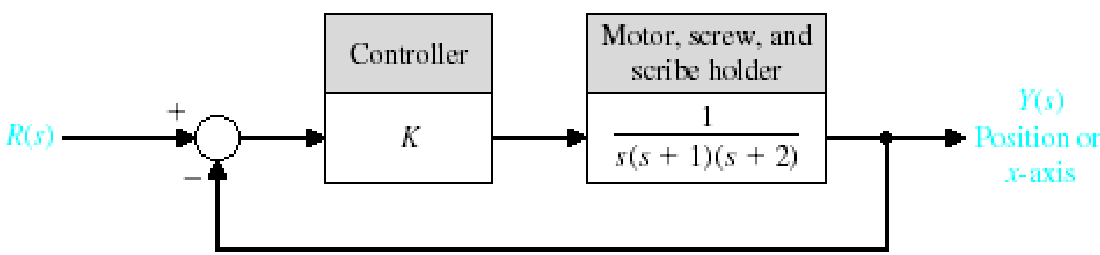
$$GH1(\omega) = \frac{5}{j\cdot\omega\cdot(0.5\,j\cdot\omega + 1)\cdot\left(j\cdot\frac{\omega}{6} + 1\right)}$$



Performance Specification In the Frequency Domain Example

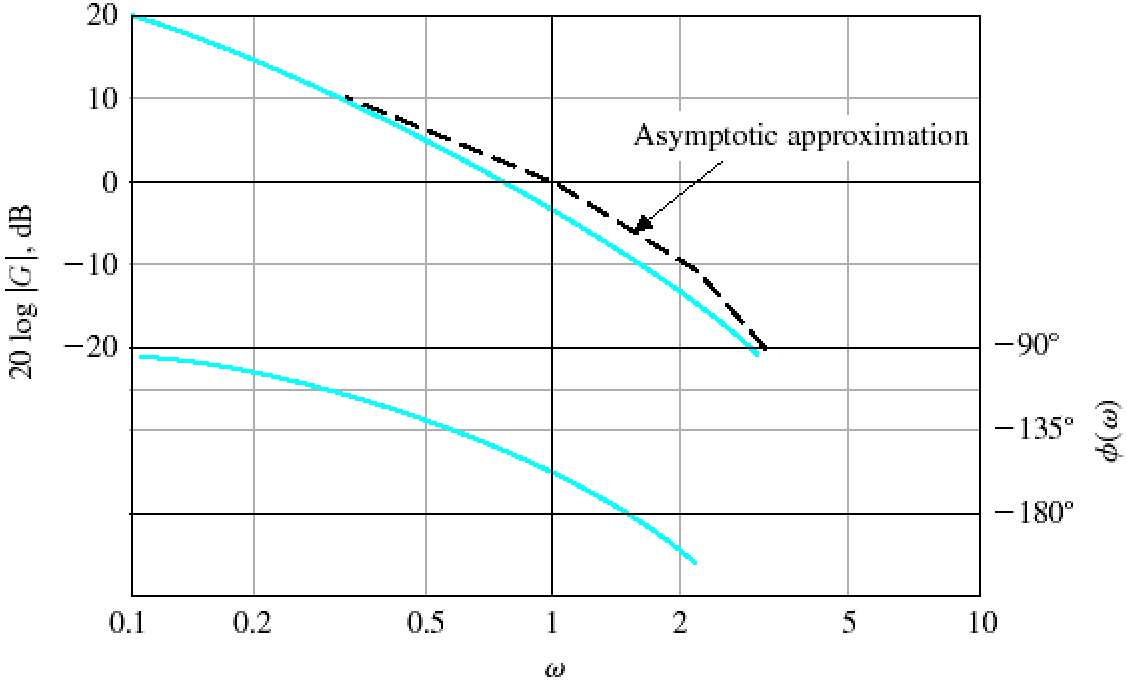


(a)



(b)

Performance Specification In the Frequency Domain Example



Performance Specification In the Frequency Domain Example