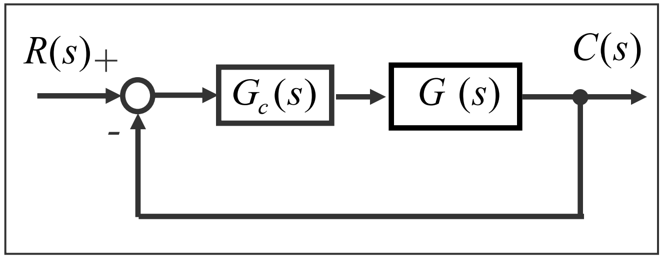
M & N CIRCLES

Frequency domain charts

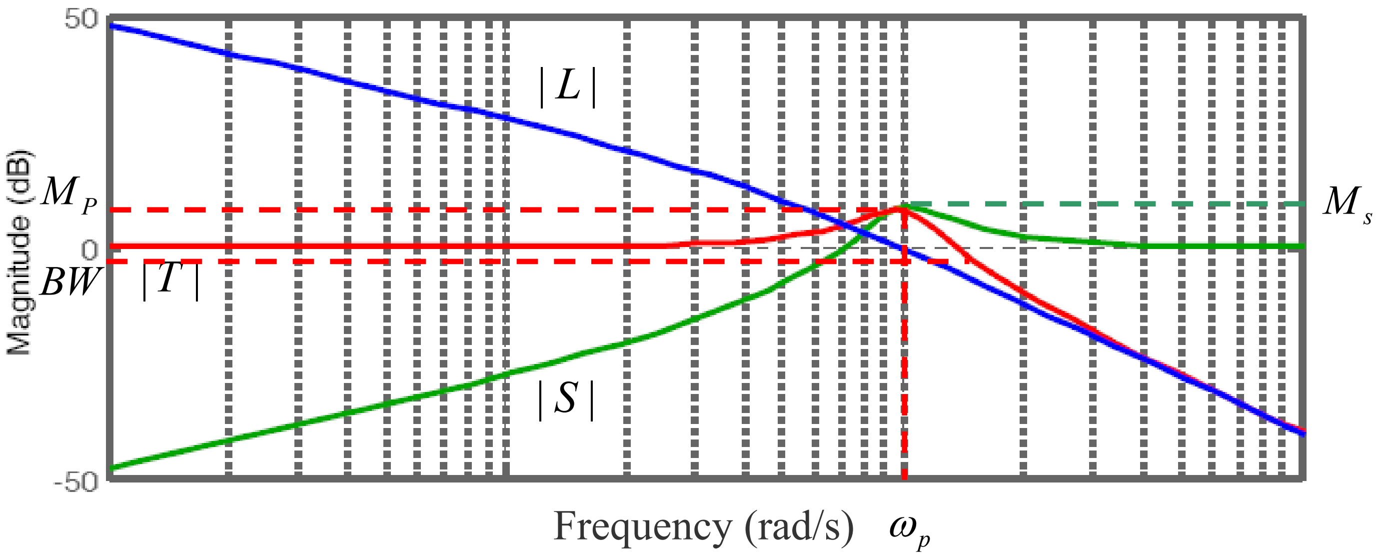
Topics to be covered include:

- Nyquist chart.
  - Constant M loci.
  - Constant N loci.
- Nichols chart.
  - Constant gain loci.
  - Constant phase loci.
  - Nichols chart specification.
- Effect of adding poles and zeros on loop transfer function.

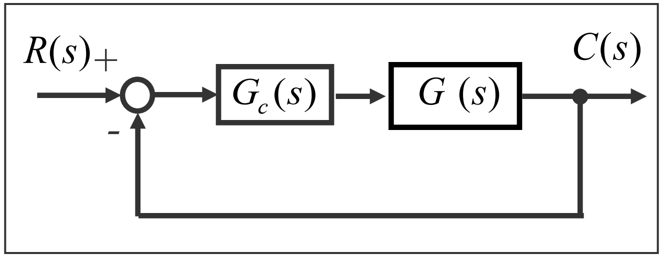
Closed loop transfer functions



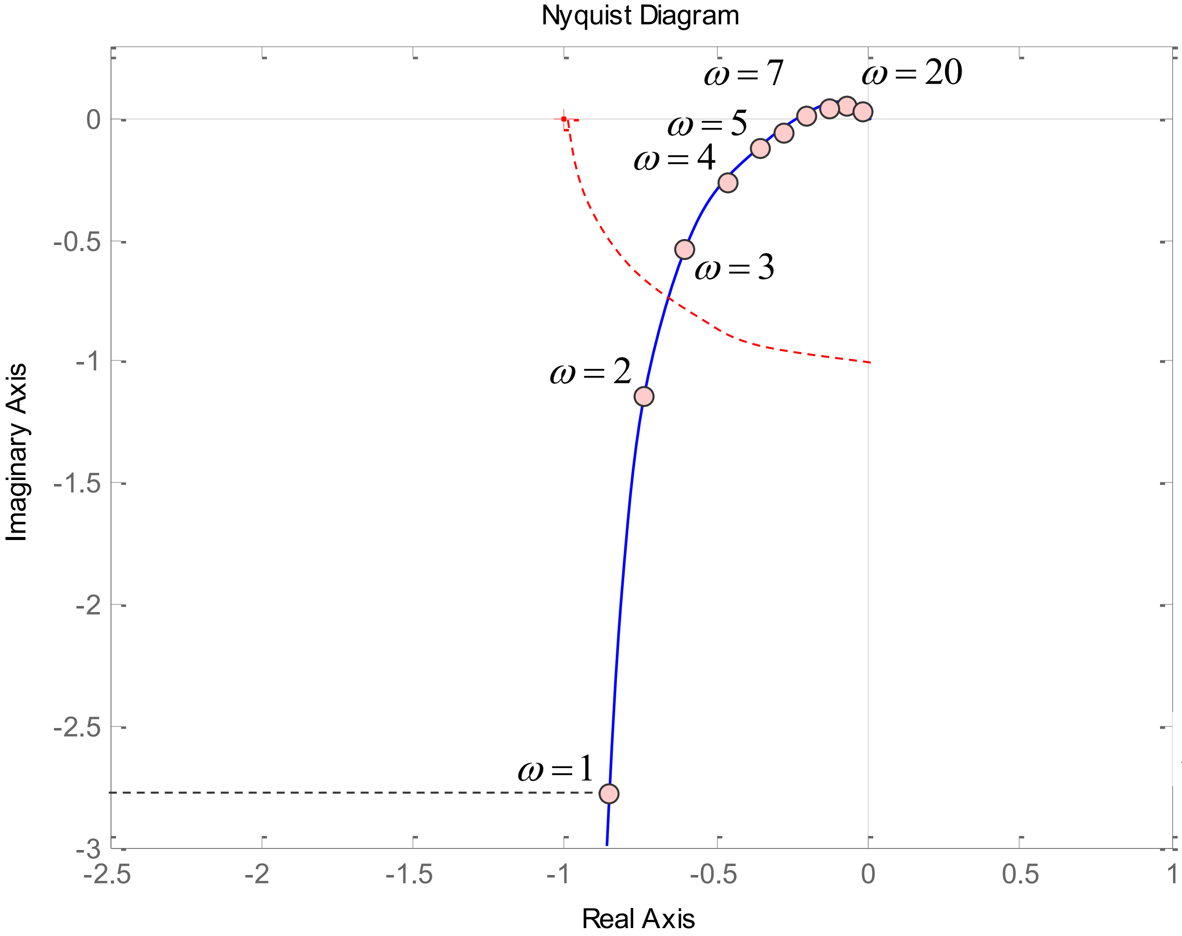
$$L(s) = G_c(s)G(s) \qquad T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{L(s)}{1+L(s)} \qquad S(s) = \frac{1}{1+G_c(s)G(s)} = \frac{1}{1+L(s)}$$



Closed loop values from Nyquist chart



Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$



$$T(j1) = ? \qquad \omega = 1$$

$$\Rightarrow \quad G(j1)G_c(j1) = -0.86 - 2.8j$$

$$T(j1) = ?$$

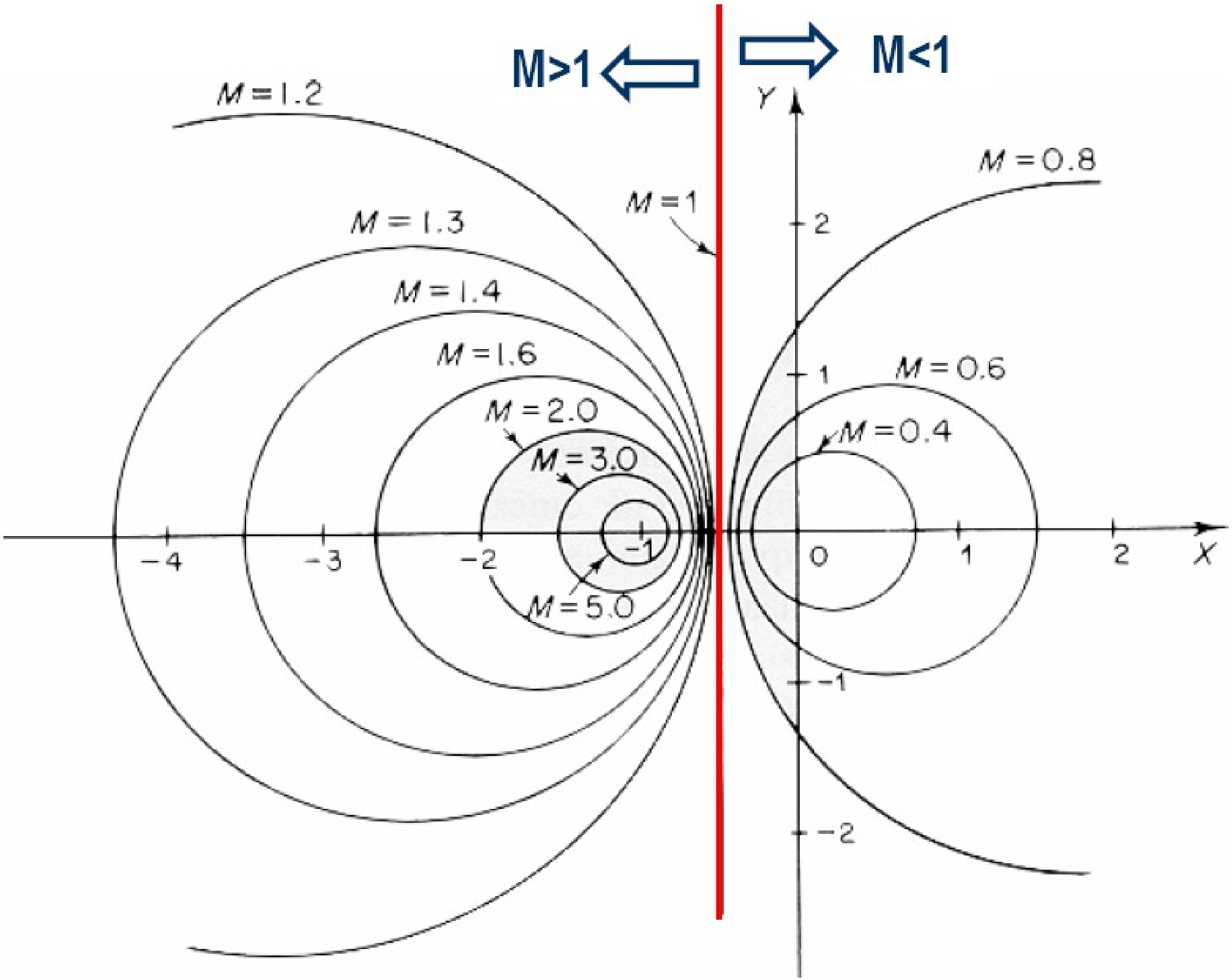
$$T(j1) = \frac{L(j1)}{1+L(j1)} = \frac{-0.86-2.8j}{1-0.86-2.8j}$$

$$T(j1) = 1.04\angle -20$$

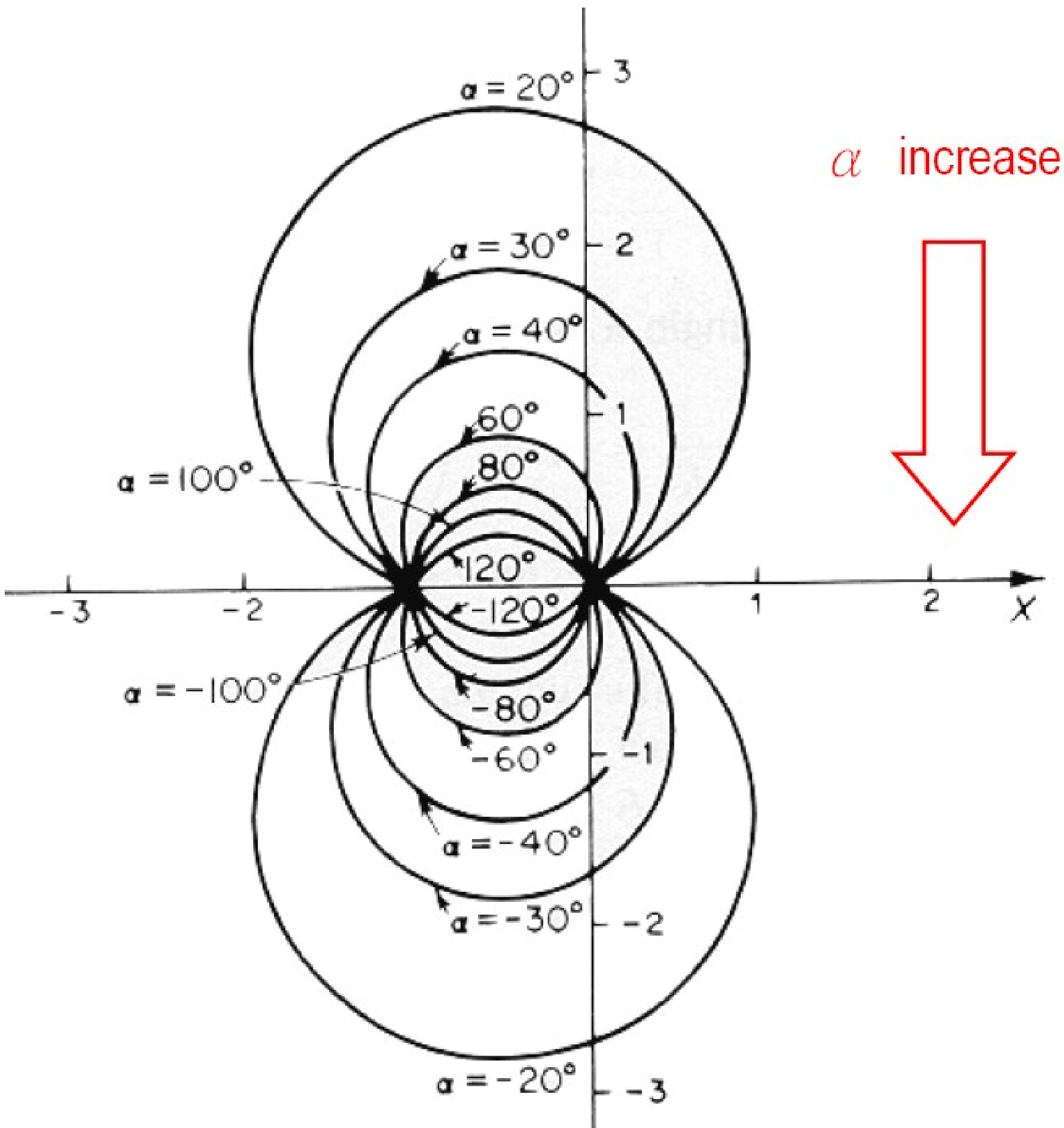
$$T(j2) = ? \quad T(j3) = ? \quad T(j\omega) = ?$$

$$M_p = ?$$

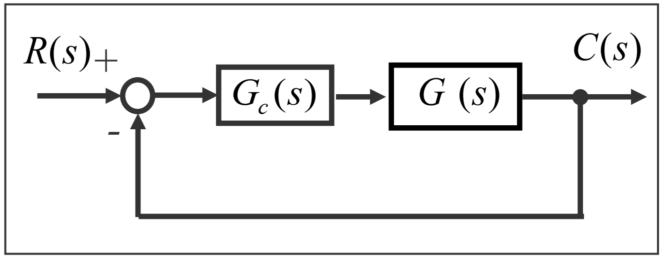
M circles (constant magnitude of T)



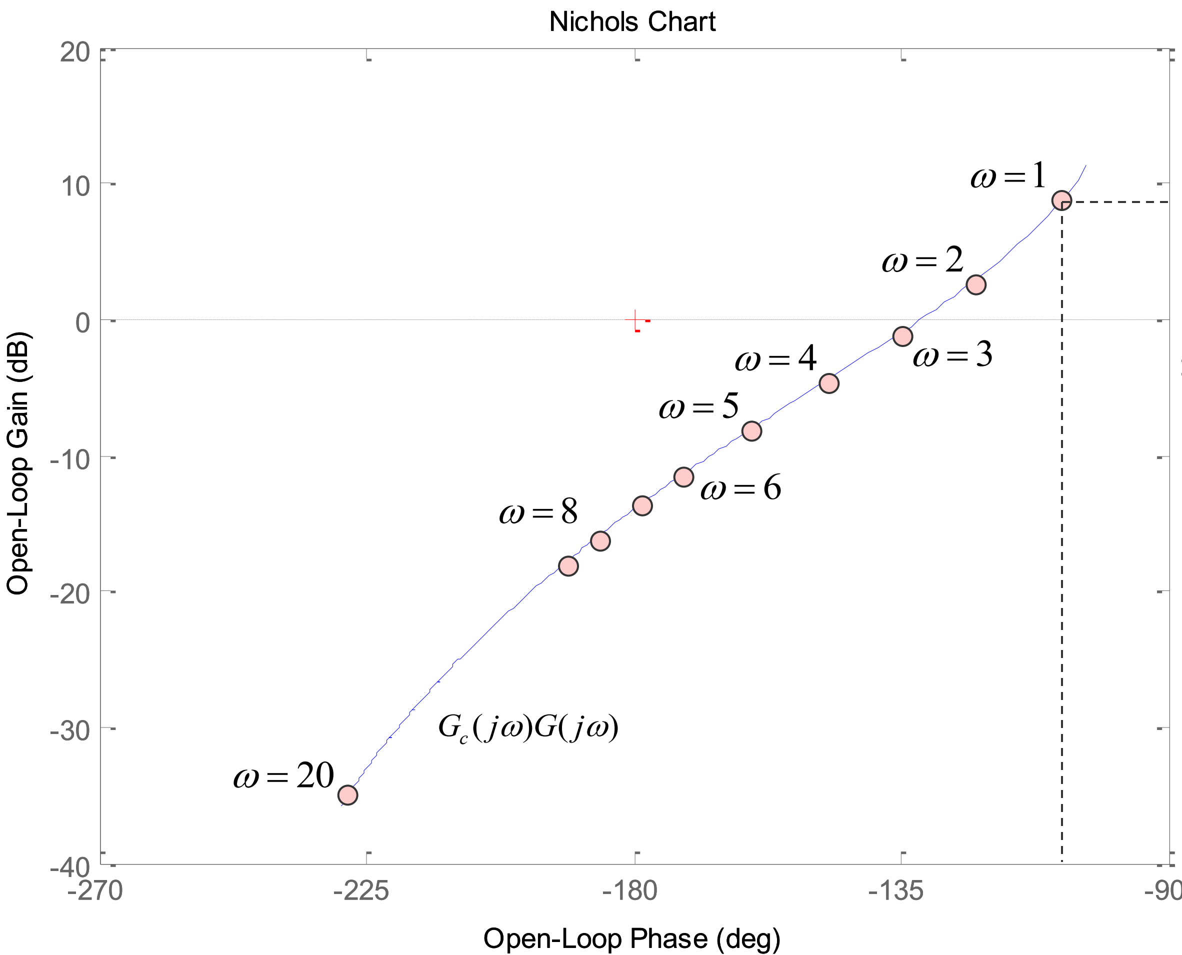
N circles (constant phase of T)



Closed loop values from Nichols chart



Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$



$$T(j1) = ? \qquad \omega = 1$$

$$20\log\left|G_c(j\omega)G(j\omega)\right| = 9.43$$

$$\angle G_c(j\omega)G(j\omega) = -107$$

$$\Rightarrow \quad G(j1)G_c(j1) = 2.93\angle -107^\circ$$

$$T(j1) = ?$$

$$T(j1) = \frac{L(j1)}{1 + L(j1)} = \frac{2.93\angle -107}{1 + 2.93\angle -107}$$

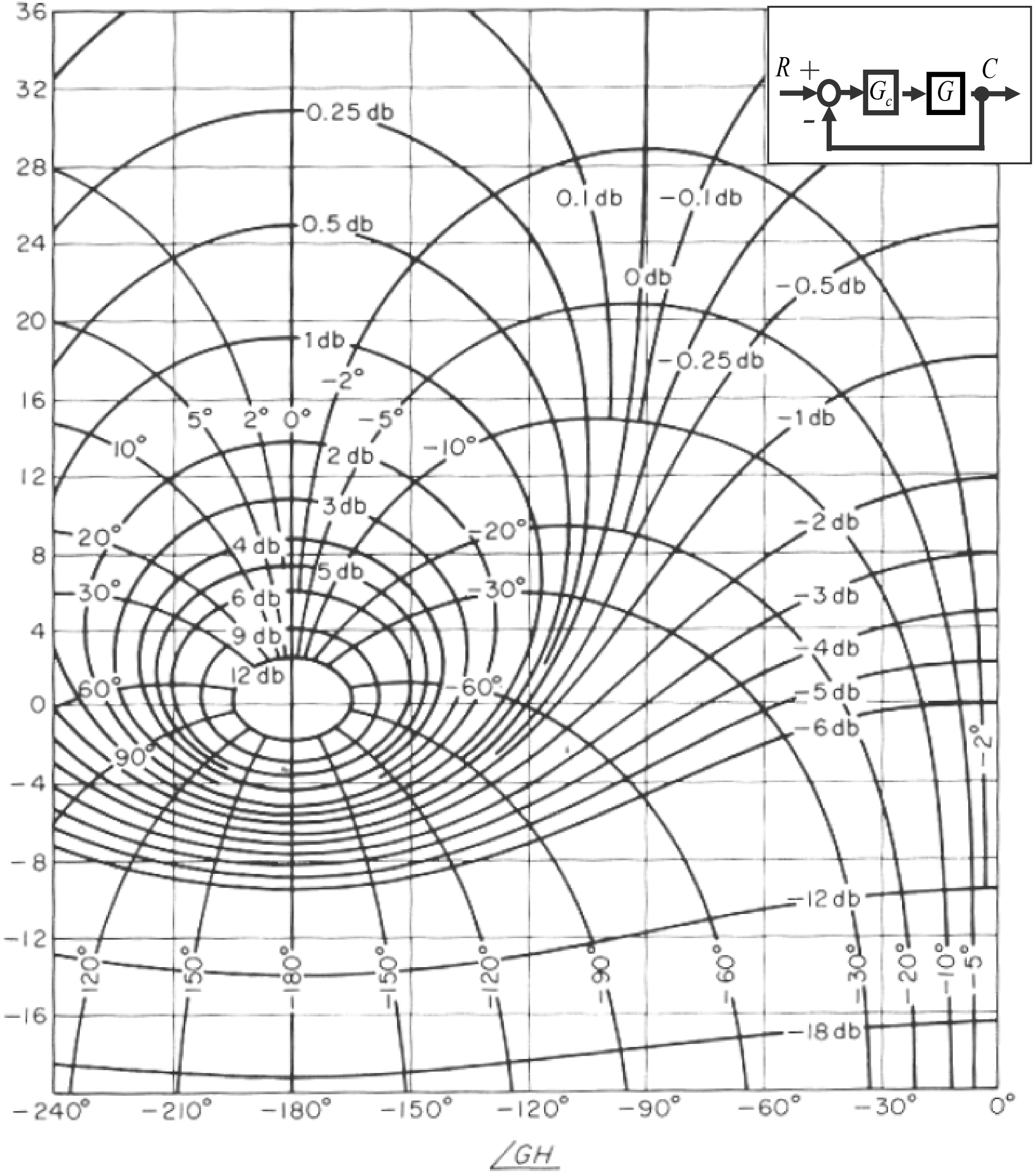
$$T(j1) = 1.04\angle -20$$

$$T(j2) = ? \quad T(j3) = ? \quad T(j\omega) = ?$$

$$M_p = ?$$

Constant gain and phase loci in Nichols chart

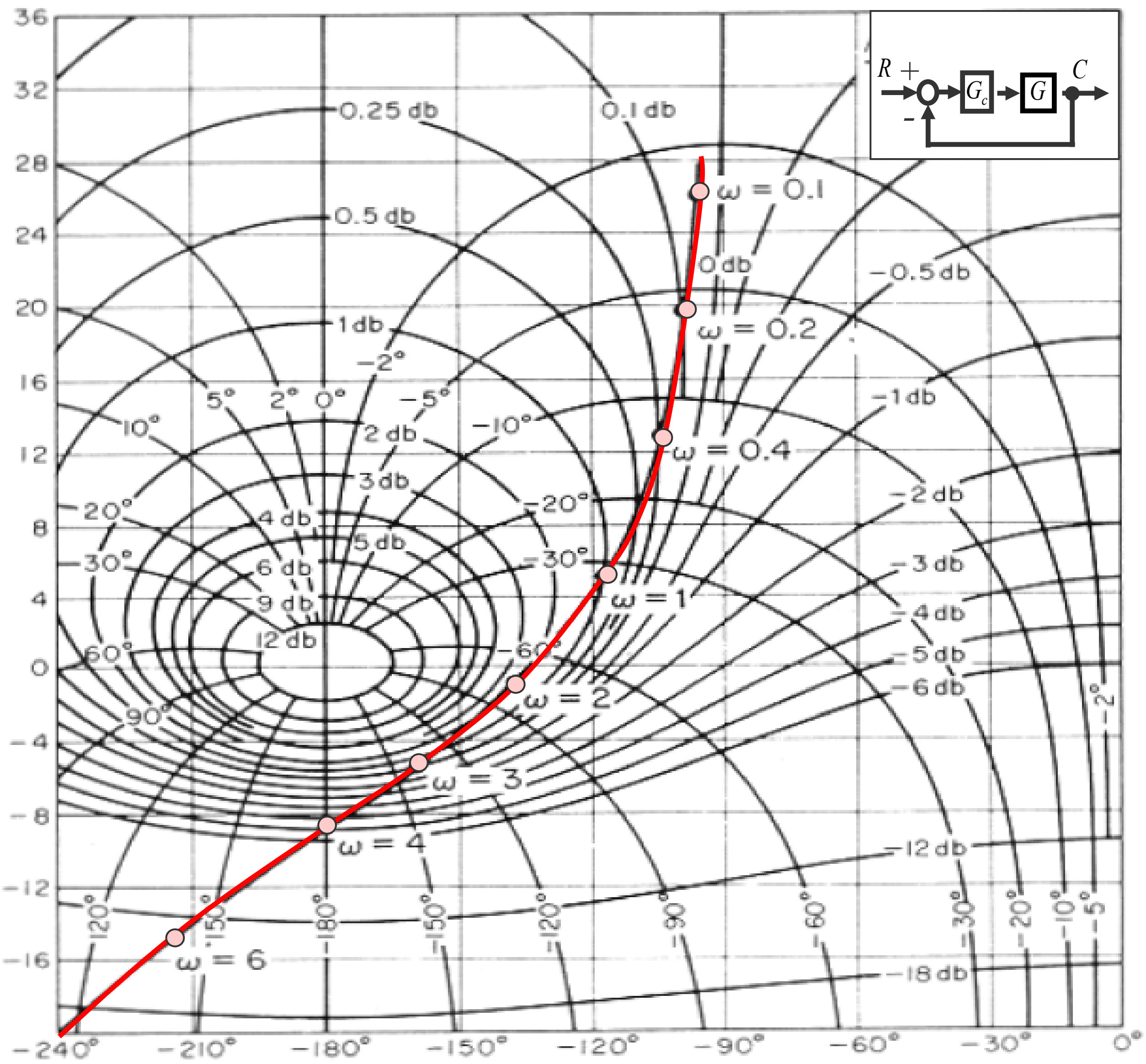
$|L|_{dB}$



$\angle L$(deg)

M circles and N circles on Nichols chart

Nichols chart specification



How to plot |T| versus frequency?

How to plot <T versus frequency?

How to derive cross over frequencies?

How to derive φ_m and GM?

How to derive open loop bandwidth?

How to derive closed loop bandwidth?

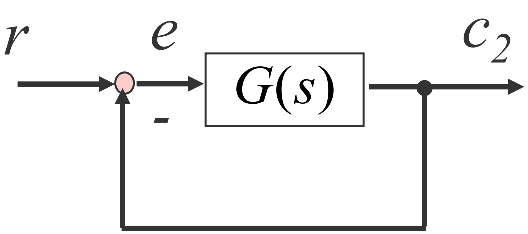
How to derive M_p?

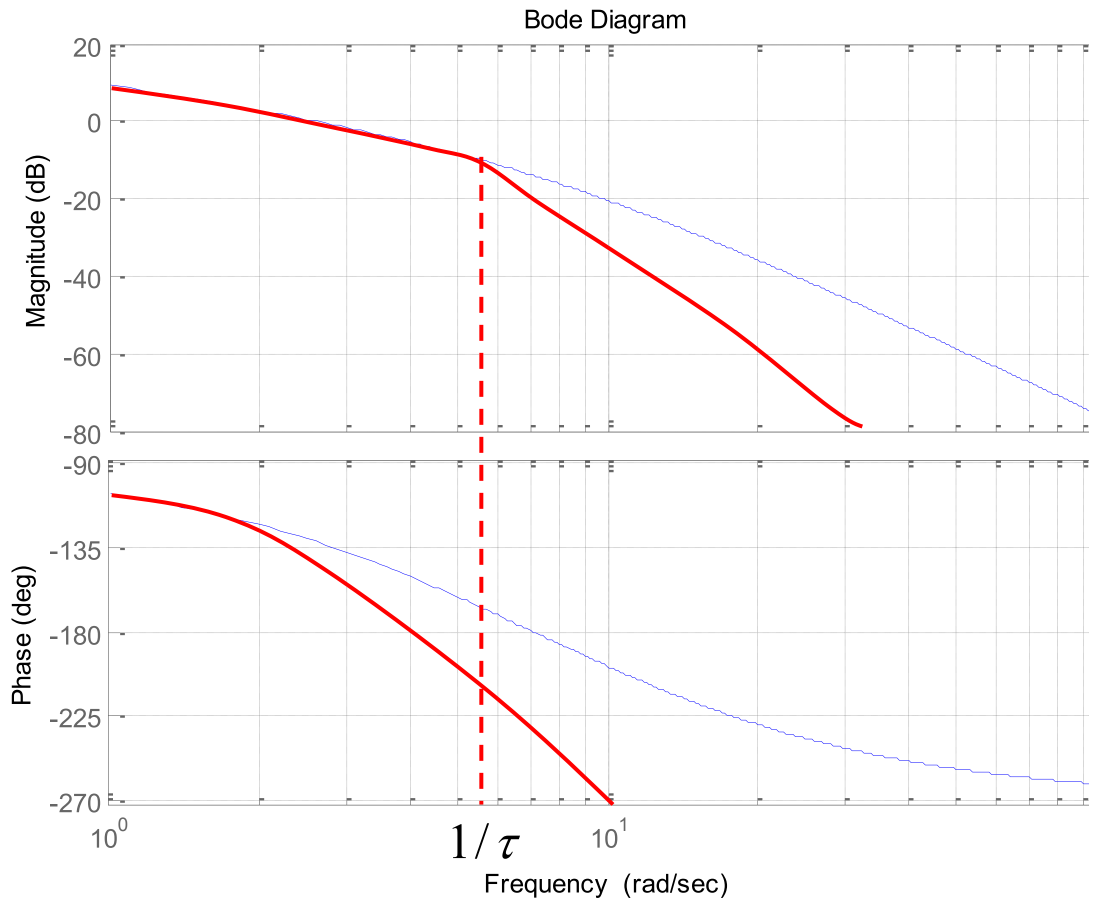
How to derive ω_p?

How to derive type of system?

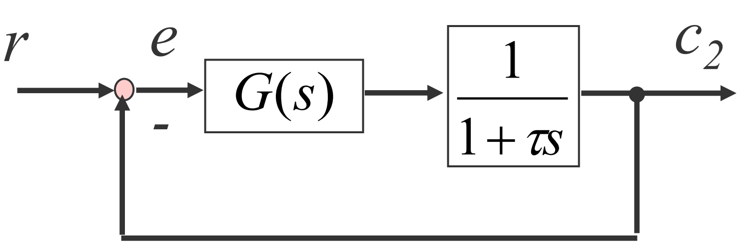
How to derive error coefficient?

Effect of adding poles on Bode plot.





Adding poles

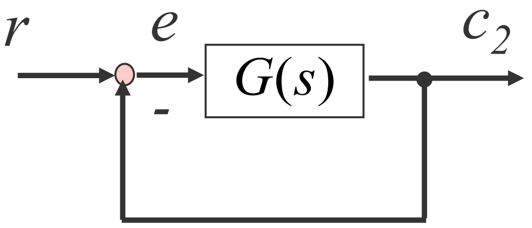


BW ↓

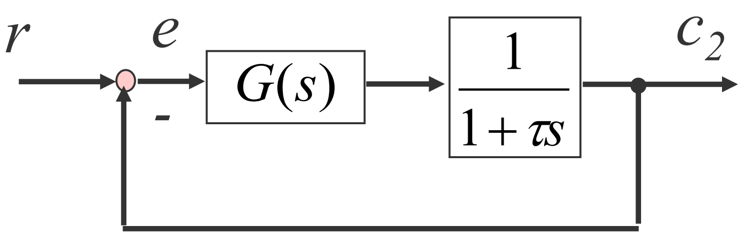
System speed ↓

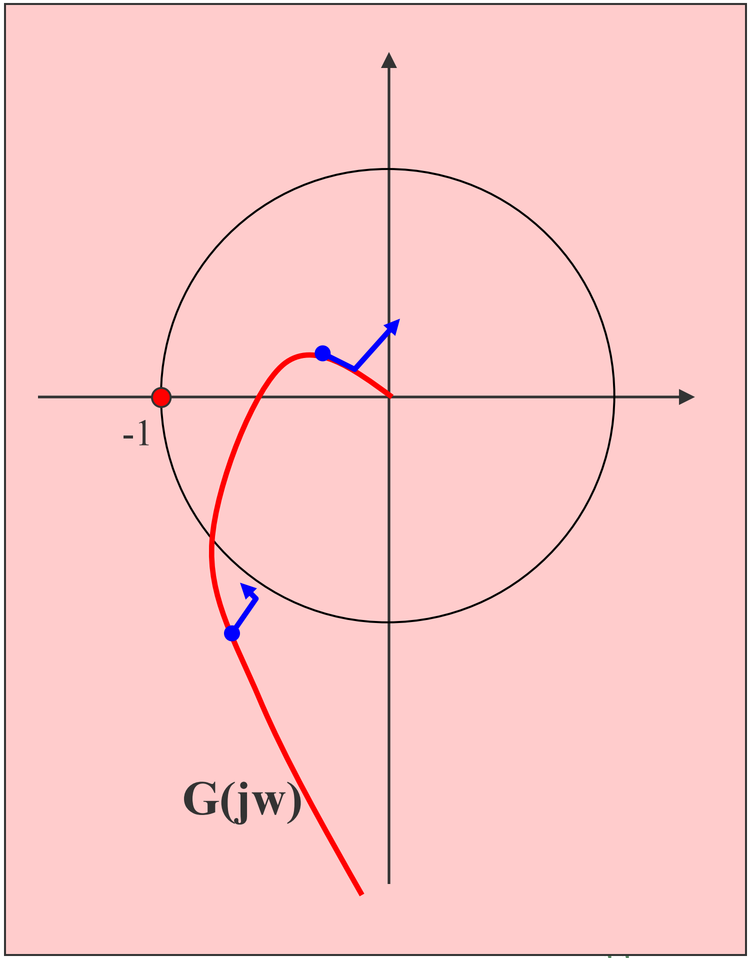
t_r ↑

Effect of adding poles on Nyquist plot.



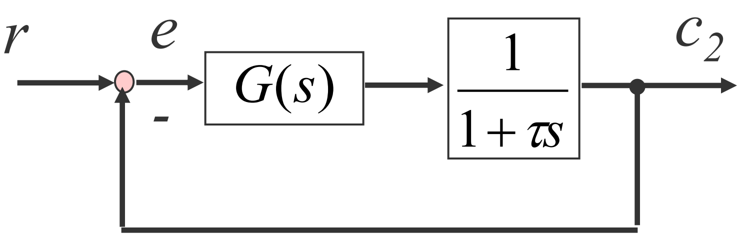
Adding poles





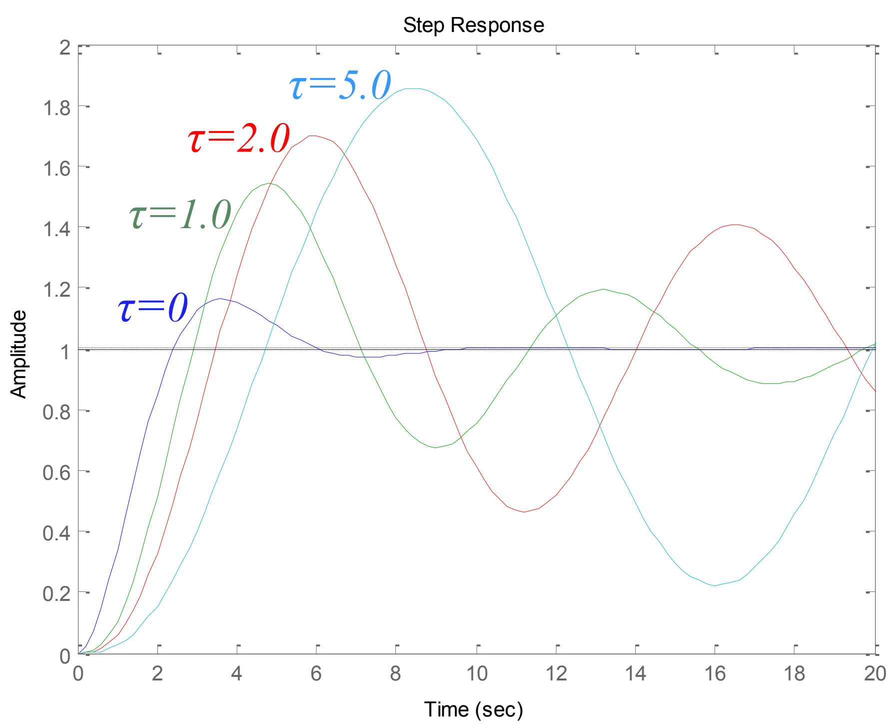
Adding poles to open loop transfer functions

اضافه کردن قطب به تابع انتقال حلقه باز



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{\omega_n^2}{\tau s^3+(1+2\zeta\omega_n\tau)s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n=1 \quad \zeta=0.5 \quad \tau=0,1,2,5$$



P.O. ↑

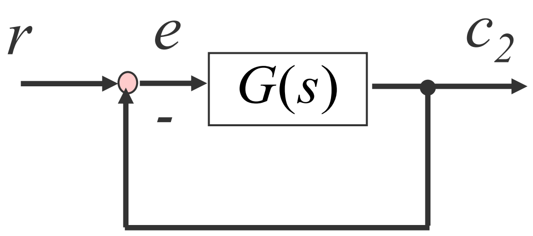
t_r ↑

System speed ↓

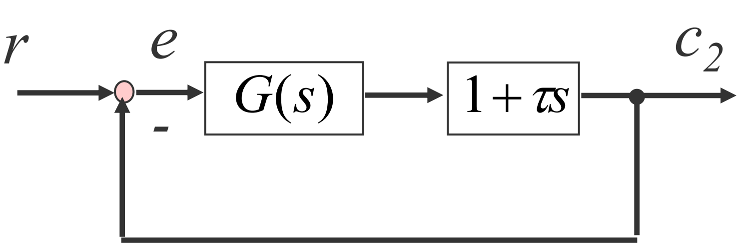
BW ↓

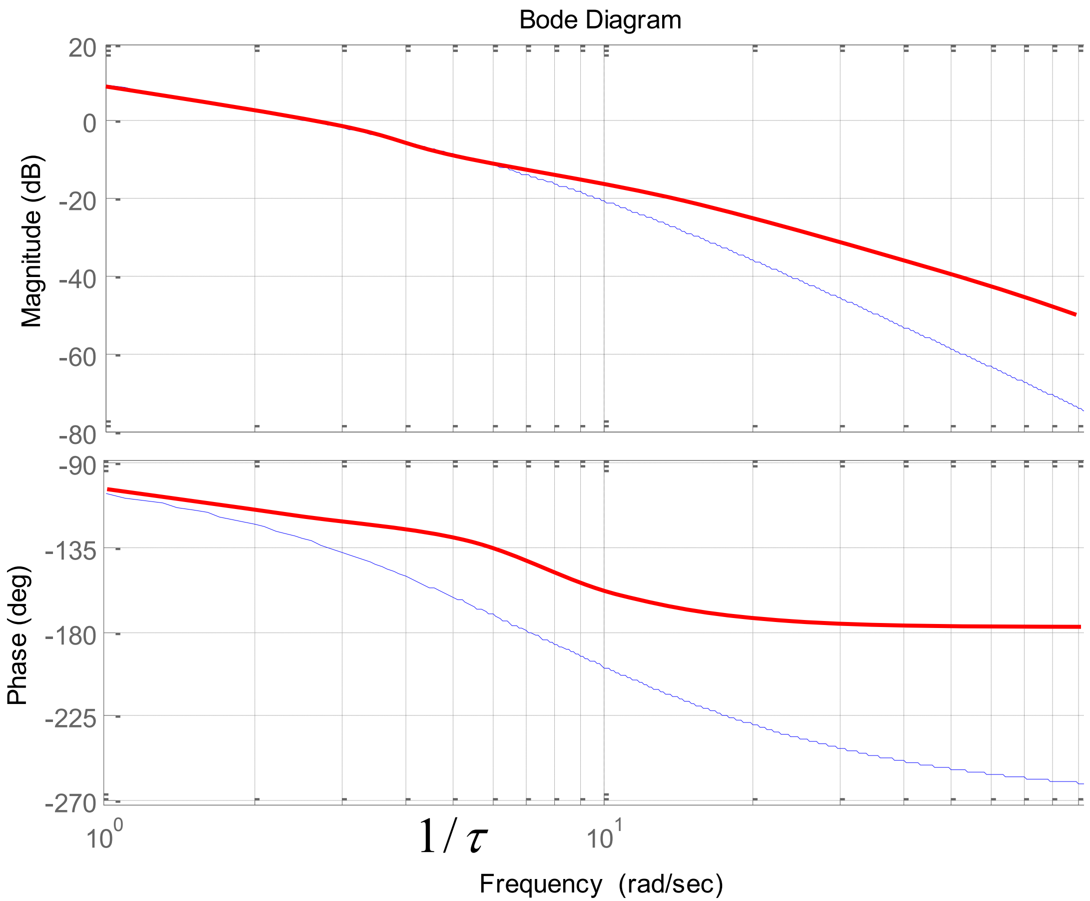
More problem as poles go to ??

Effect of adding zeros on Bode plot.



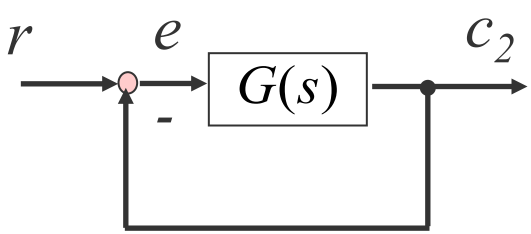
Adding zeros



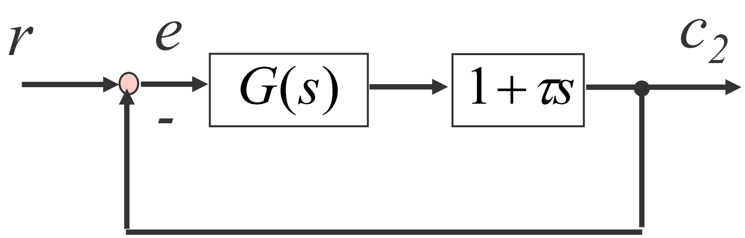


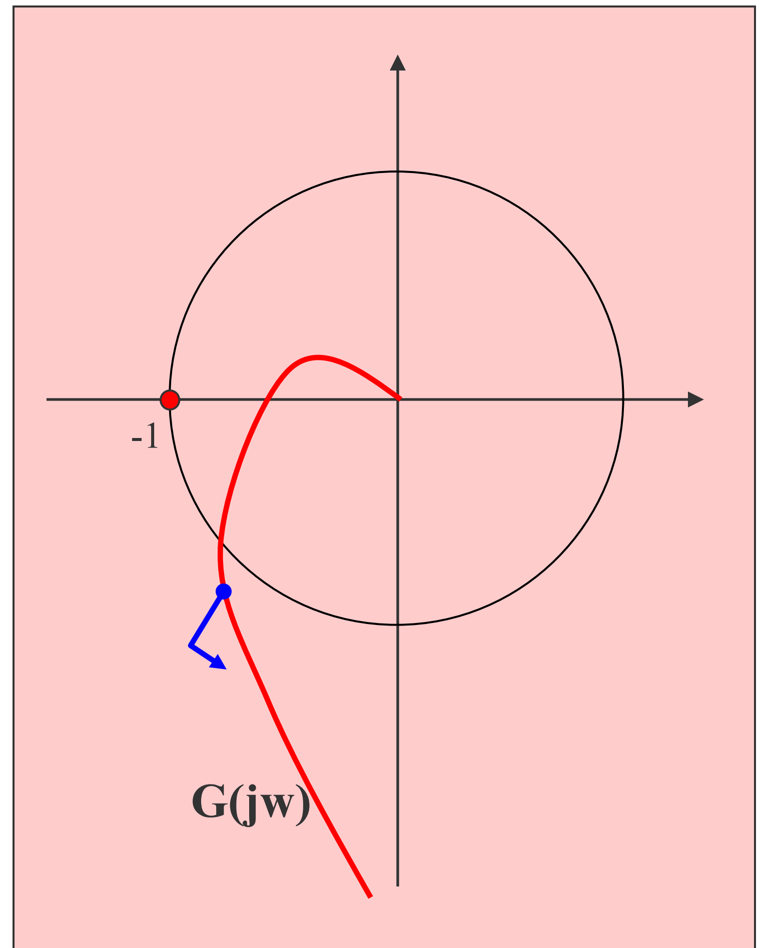
BW ↑ System speed ↑ t_r ↓

Effect of adding zeros on Nyquist plot.



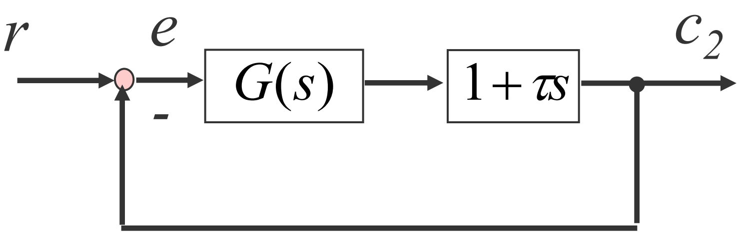
Adding zeros





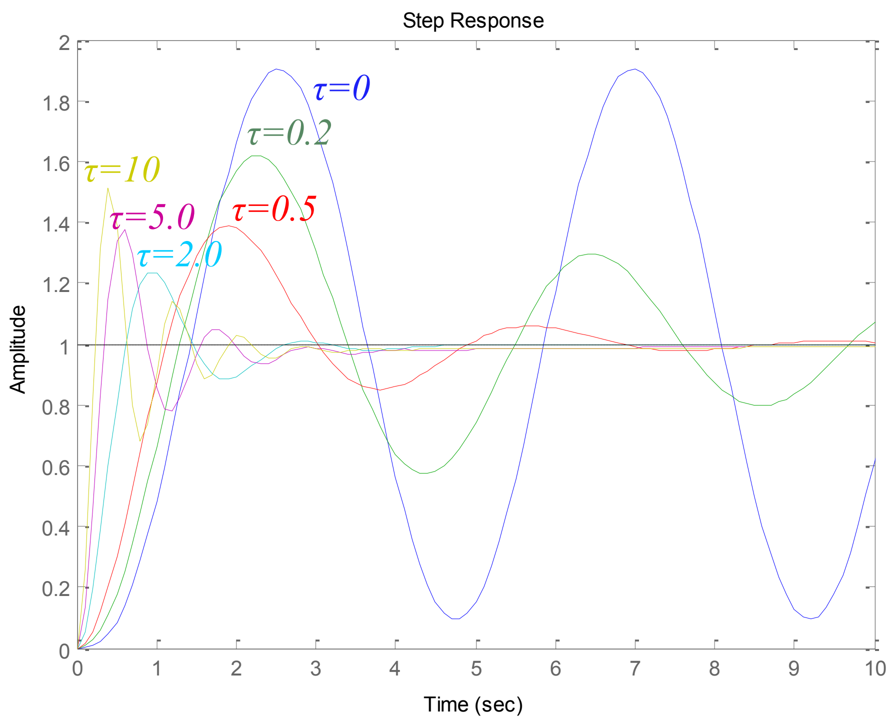
Adding zeros to open loop transfer functions

اضافه کردن صفر به تابع انتقال حلقه باز



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{6(1+\tau s)}{s^3+3s^2+(2+6\tau)s+6}$$

$$\tau=0,0.2,0.5,2,5,10$$



P.O. ↓ ↑

t_r ↓

System speed ↑

BW ↑

Note: For $\tau<0$ system is unstable. Why?

Example 1: Derive the Bode plot of following system.

$$G(s) = \frac{a\tau s+1}{\tau s+1}$$

$Let \quad a>1$

$$20\log|G(j\omega)| =$$

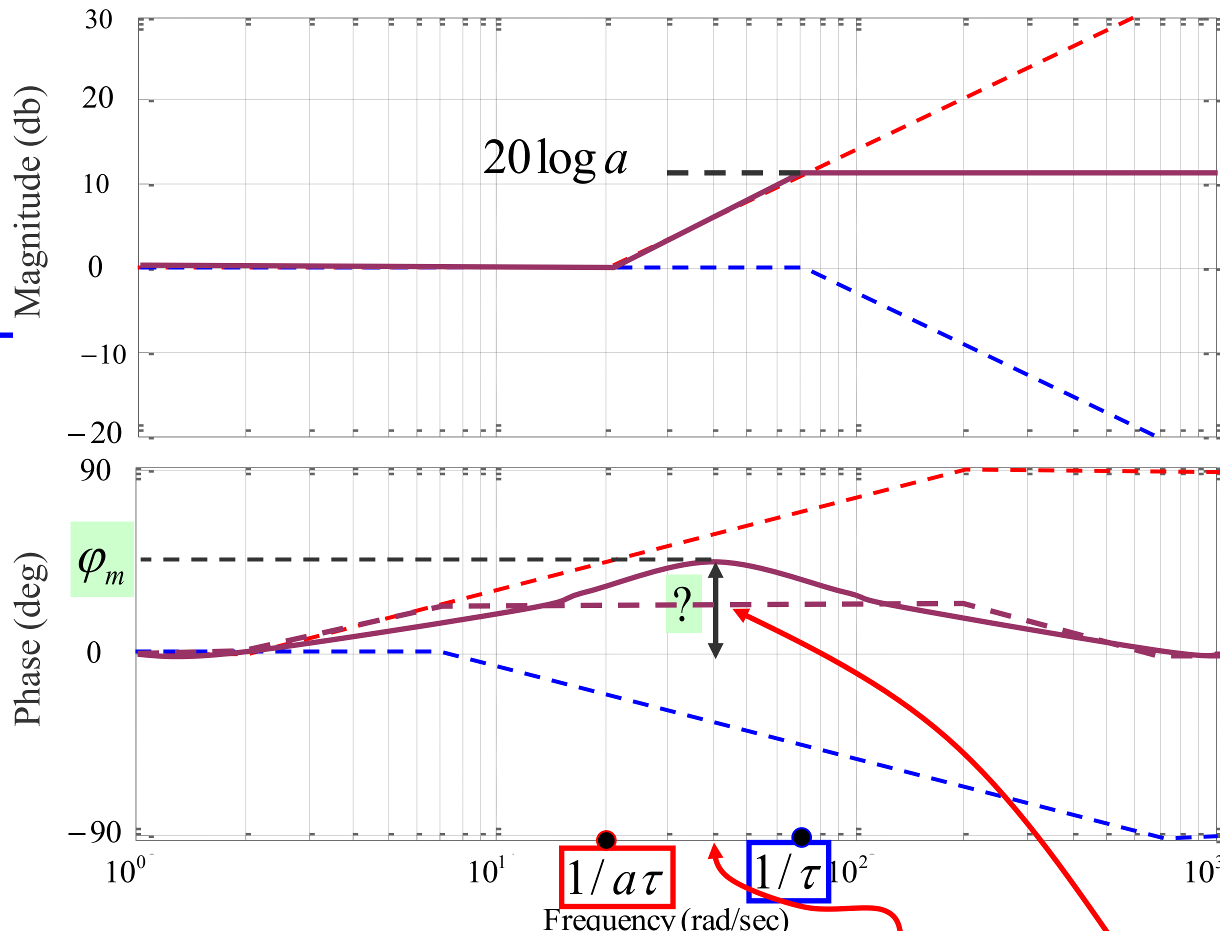
$$= 20\log|a\tau j\omega+1| + 20\log\left|\frac{1}{\tau j\omega+1}\right|$$

$$\angle G(j\omega) =$$

$$= \angle(a\tau j\omega+1) + \angle\left(\frac{1}{\tau j\omega+1}\right)$$

$$\varphi_m = \tan^{-1}(a\tau\omega) - \tan^{-1}(\tau\omega)$$

$$\tan(\varphi_m) = \frac{a\tau\omega - \tau\omega}{1+a\tau^2\omega^2}$$



$$\frac{\partial \tan(\varphi_m)}{\partial\omega} = \frac{(a\tau-\tau)(1+a\tau^2\omega^2) - 2a\tau^2\omega(a\tau\omega-\tau\omega)}{(1+a\tau^2\omega^2)^2}$$

$$(1+a\tau^2\omega^2) = 2a\tau^2\omega^2 \qquad \omega = \frac{1}{\tau\sqrt{a}} \qquad \sin\varphi_m = \frac{a-1}{a+1}$$

Example 1: Derive the Bode plot of the following system.

$$G(s) = \frac{a\tau s + 1}{\tau s + 1}$$

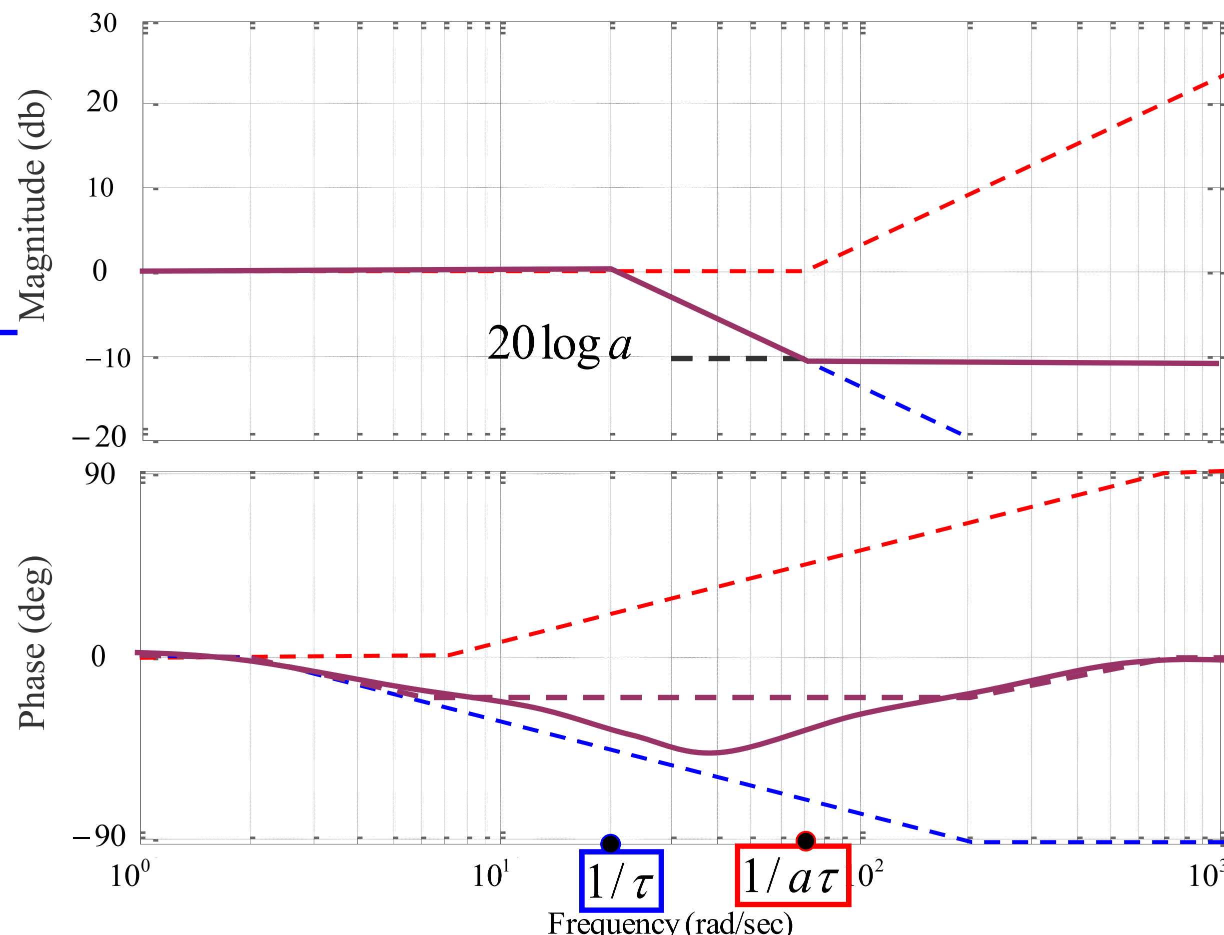
$$Let \quad a < 1$$

$$20\log|G(j\omega)| =$$

$$= 20\log|a\tau\, j\omega + 1| + 20\log\left|\frac{1}{\tau\, j\omega + 1}\right|$$

$$\angle G(j\omega) =$$

$$= \angle(a\tau\, j\omega + 1) + \angle\left(\frac{1}{\tau\, j\omega + 1}\right)$$

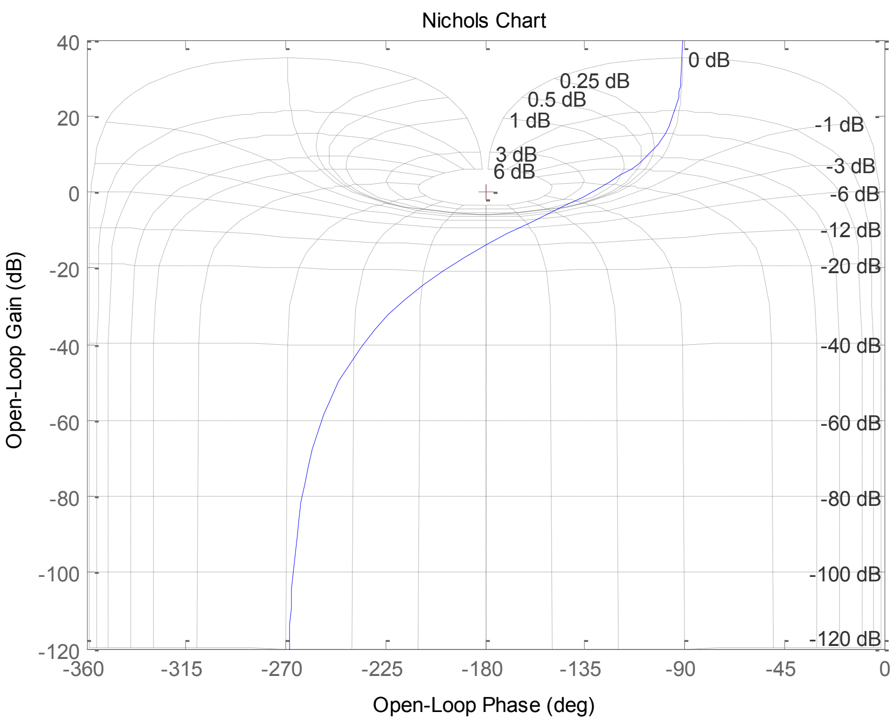


Exercises

1 The Nichols chart of an open loop system with negative unit feedback is shown.

a) Find the GM and PM.

b) Find M_P.



$answer\ a: GM = 14\,db, PM = 45^\circ \quad b: M_p = 1.8\,db$

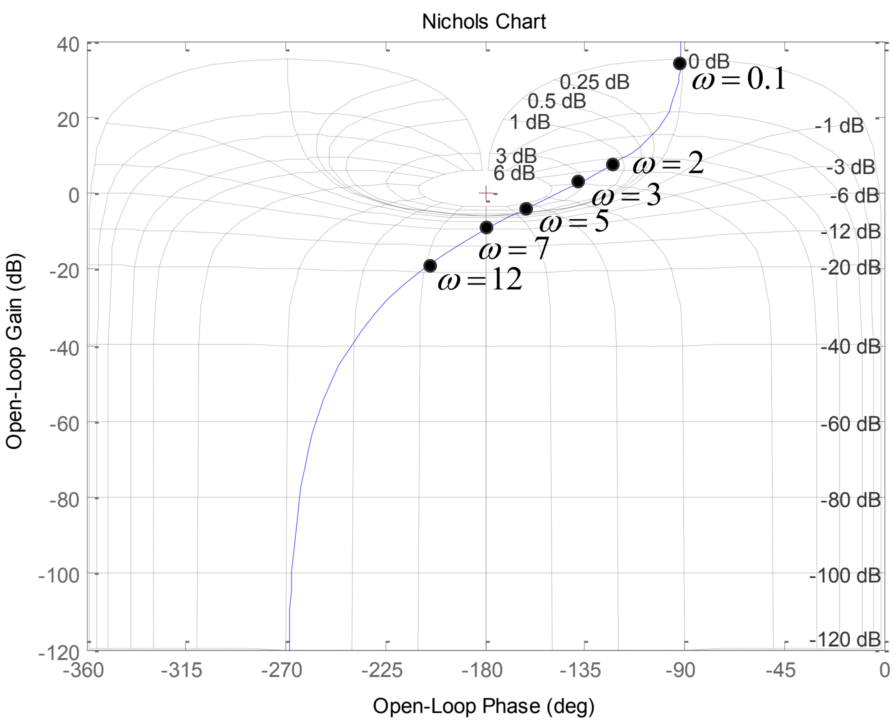
Exercises

2 The Nichols chart of a open loop system with negative unit feedback is shown.

a) Find the error constants

b) Find the GM and PM and gain crossover frequency and phase crossover frequency.

c) Find M_P, open loop bandwidth and closed loop bandwidth.



$answer\ a: k_p = \infty, k_v = 5, k_a = 0$

$b: GM = 10\,db, PM = 32^\circ, \omega_c = 3.75\,rad/\sec, \omega_{180} = 7\,rad/\sec$

$c: M_p = 5.3\,db, BW_{openloop} = 4.7\,rad/\sec, BW_{closedloop} = 6.3\,rad/\sec$