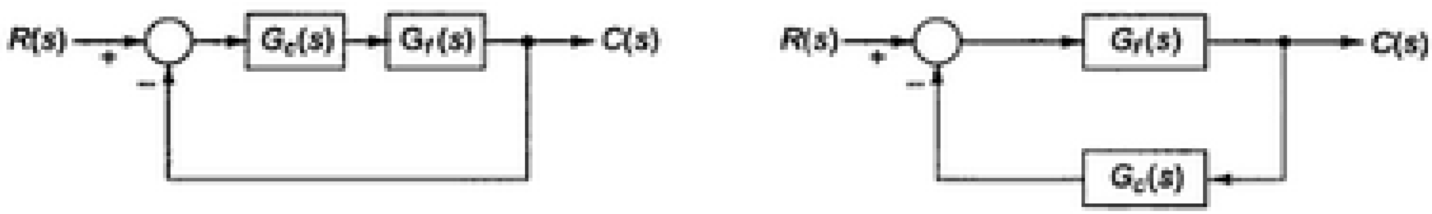
COMPENSATOR NETWORK DESIGN USING TIME DOMAIN ANALYSIS

TIME-DOMAIN DESIGN USING COMPENSATOR NETWORKS

Most of the design procedures of an automatic control system are based on trial and error approach. Each control system has to meet some specifications, i.e., transient response and steady state error, already discussed in Chapter 3 on Time Response Analysis. When system is unstable, a compensating network having poles and zeros could be used to stabilize it as well as to meet all the specification to the best possible extent. If the system is stable, a compensator could be designed to meet the set specifications to the maximum possible extent as well as keeping the stability status intact.

Compensator can be put in cascade or in series with the plant or put in feedback path or parallel, as shown in Fig 6.18 (a) and Fig 6.18 (b).



(a) Compensator in cascade *(b) Compensator in feedback path*

Fig. 6.18

Compensator can be electronic network, electrical, mechanical or hydraulic type. Lead and lag compensators are used quite extensively in control systems. For a sinusoidal input if the sinusoidal output leads the input, it is known as Lead Compensator and if it lags the input, it is known as Lag Network. For Lag–Lead Network, for the small range of frequencies, the network behaves as Lag Network and for higher range of frequencies it acts as Lead Network.

A lead compensator can increase the stability or speed of response of a system; a lag compensator can reduce (but not eliminate) the steady state error. Depending on the effect desired, one or more lead and lag compensators may be used in various combinations. Lead, lag, and lead–lag compensators are usually designed for a system in transfer function form. In general the compensator network can have the simplest transfer function of the form

$$G_c(s) = \frac{(s+z_c)}{(s+p_c)} = \frac{(s+1/\tau)}{(s+1/\alpha\tau)}, \quad \alpha = \frac{z_c}{p_c} < 1, \quad \tau > 0 \qquad \ldots(6.14)$$

It may be noted that

(i) The **Compensator Edit** *C* option in the SISO Design window of MATLAB © *can be used for designing appropriate compensator.*

(ii) *By right clicking on the root-locus curve we can have options of design and representation such as –* Add pole/zero; Delete pole/zero; Edit Compensator; Design Constraints; Grid; Zoom, etc.

6.4.1 Lead Compensator

Note that $\alpha < 1$, ensures that the pole is located to the left of the zero, such a compensator is referred as *first-order phase lead compensator*. The transfer function of a phase lead compensator may be expressed as

$$G_c(s) = \frac{(s+z_c)}{(s+p_c)} \text{ with } |z_c| < |p_c| \qquad \ldots(6.15)$$

This compensator tends to shift the root locus toward the left half *s*-plane and, therefore, results in improvement in the system's stability and an increase in the response speed.

How is this accomplished? Let us recall—for finding the asymptotes of the root locus that lead to the zeros at infinity, the equation to determine intersection of the asymptotes (centroid) along the real axis is:

$$-\sigma_A = \frac{\sum_p p_i - \sum_z z_i}{n-m}$$

When a lead compensator is added to a system, the value of this intersection will be a larger negative number than it was before. The net number of zeros and poles will be the same (one zero and one pole are added), but the added pole is a larger negative number than the added zero. Thus, the result of a lead compensator is that the asymptotes' intersection is moved further into the left-half plane, and the entire root locus will be shifted to the left. This can increase the region of stability as well as the response speed.

In MATLAB a phase lead compensator in root locus is implemented by using the transfer function in the form

```
numlead = kc*[1 z];
denlead = [1 p];
```

and using the conv ( ) function to implement it with the numerator and denominator of the plant

```
newnum = conv (num, numlead);
newden = conv (den, denlead);
```

Realisation of a Phase Lead Compensator A first-order phase lead compensator can be realised using *R C* Network of Fig. 6.19 as

$$G_c(s) = \frac{V_0(s)}{V_i(s)} = \frac{R_2}{R_2 + \dfrac{R_1/sC}{R_1 + 1/sC}} = \frac{(s + 1/R_1C)}{s + \dfrac{1}{[R_2/(R_1+R_2)]R_1C}} \qquad ...(6.16)$$

with $\tau = R_1C$ and $\alpha = R_2/(R_1 + R_2) < 1$ in Eq. 6.14.

For a given or desired value of τ and α, we can choose appropriate (practical) values of R_1, R_2 and C. The first step in the design of Lead Compensator, using Root locus Technique is to convert time domain specifi-cations in to dominant set of desired close loop pole s_d, with real and imaginary part $-\zeta\omega_n$ and $j\omega_n$ as shown in Fig. 6.20(a) and then check whether this desired pole lies on the rootlocus of the unity feedback system with forward path transfer function

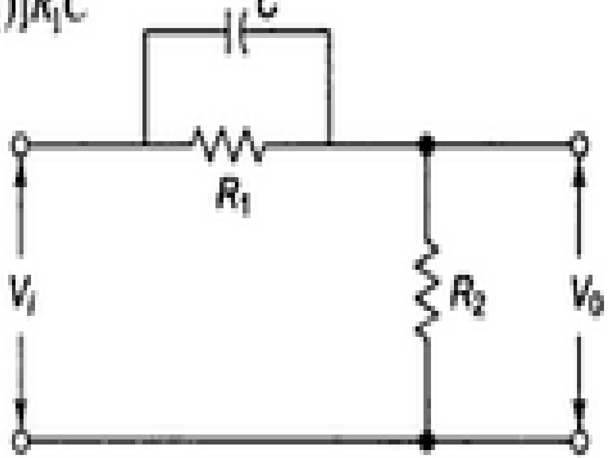


Fig. 6.19 *First-order lead Compensator*

$G_f(s)$ or not. For checking this, angle at $G_f(s_d)$ is calculated and if it is found to be $\pm$ 180°, then s_d is lying on $G_f(s)$ and there is no need of designing the compensator. But if angle criterion is not met then compensator is designed such that

$$\angle G_c(s_d) = \phi = \pm 180° - \angle G_f(s_d)$$

where ϕ is the angle contribution by the compensator at point s_d. For this calculated ϕ, we can place the pole-zero pair of compensator anywhere on negative real axis or in other words, there is no unique position for this pole pair. The angle contribution of lead compensator is shown on Fig. 6.20(b).

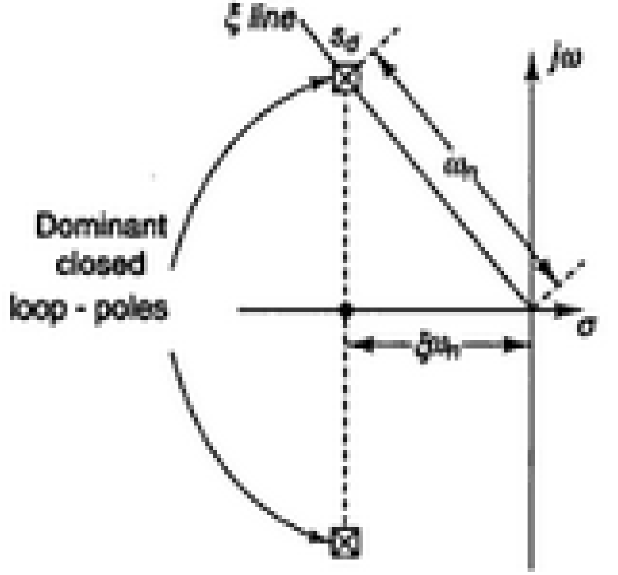


Fig. 6.20(a) *Dominant closed loop-poles*

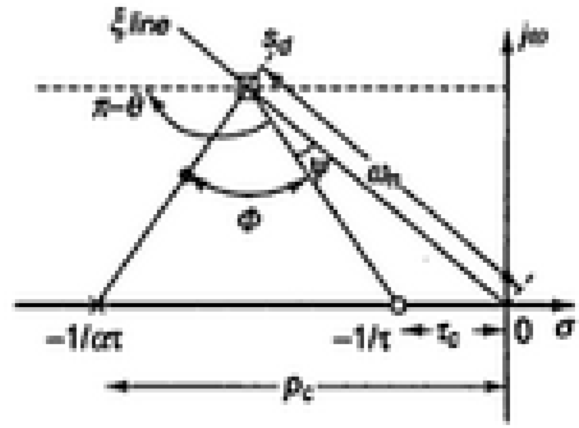


Fig. 6.20(b) *Angle contribution of lead compensator*

If we consider the attenuation of lead network, then the compensator pole zero location should be selected in such a way that it gives largest value of α. The compensator zero is selected by drawing a line from s_d making an angle ψ with ⊠s_d. The compensator pole can then be selected by drawing a line making an angle $\Phi + \psi$ with ⊠s_d.

Now from the geometrical concept

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$$z_c = \omega_n \left[\frac{\sin\psi}{\sin(\pi - \theta - \psi)}\right],\ p_c = \omega_n \left[\frac{\sin(\psi + \phi)}{\sin(\pi - \theta - \psi - \phi)}\right]$$

and

$$\alpha = \left[\frac{\sin(\psi)\sin(\pi - \theta - \psi - \phi)}{\sin(\pi - \theta - \psi)\sin(\psi - \phi)}\right]$$

The angle ψ for the greatest α is obtained by solving $d\alpha/d\psi = 0$

$$\psi = (1/2)(\pi - \theta - \Phi)$$

But this value of α does not guarantee the dominance of the desired closed loop poles in the compensated root locus and it must be checked. The steady state error is also checked and if it is not satisfactory, the fixation of pole zero location is repeated at some other location but keeping the angle fixed.

If the compensator poles lies on the positive real-axis, then a single lead network cannot help and more than one lead network in cascade can be used for final compensation.

Example 6.14 Design a lead compensation for the system with feed forward transfer function

$$G_f(s) = \frac{1}{s(s+1)},$$

to provide a closed-loop damping > 0.5 and natural frequency of oscillation $\omega_n > 7$ rad/s. The general transfer function of a lead compensator is given as

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} = \frac{(s + 1/\tau)}{(s + 1/\alpha\tau)},\ \alpha = \frac{z_c}{p_c}\langle 1,\ \tau \rangle 0$$

Let us design for the limiting condition of the damping ratio and natural frequency of oscillation. Therefore, let us choose $\xi = 0.5$ and $\omega_n = 7$ rad/s.

Hence the closed loop poles of the system are given by, $-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$, or desired dominant closed loop complex conjugate poles are $s = -3.5 \pm j\,6.062$.

Let us choose $z = -2$. The angle subtended by all the poles and zeros of the feed forward transfer function, with the closed loop pole at $-3.5 + j\,6.062$ is $-\theta p - 112.41 - 120 + 103.898°$.

For this angle to be equal to $-180°$, the angle subtended by the compensator pole with the closed loop pole must be 51.488°. We can obtain graphically the pole as follows:

With tan (51.488) = $6.062/x$, $x = 4.824$ and hence $p_c = x + 3.5 = 8.324$.

The parameter k of the compensated network is calculated by satisfying the magnitude criterion at dominant pole location $s = -3.5 + j\,6.062$. The parameter k comes out to 60.

The root locus of the compensated system with lead network is obtained, as shown in Fig. 6.21 by using the following MATLAB© script:

```
n = [0   0   1]
d = [1   1   0]
g = tf(n, d)
SISO tool (g)
```

The transfer function of the lead compensator is obtained as

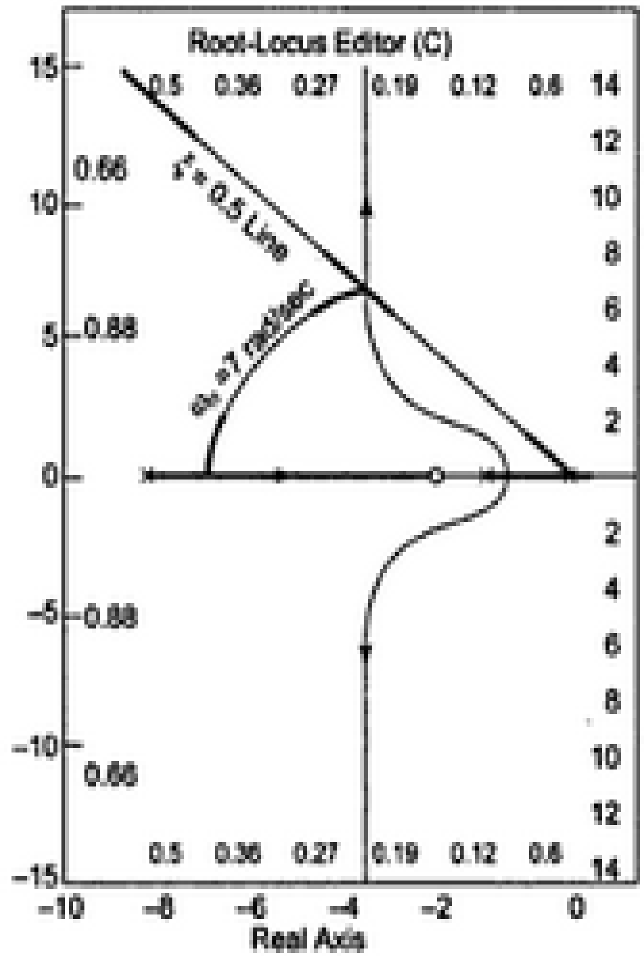

Fig. 6.21 *Root locus of compensated system with lead network (Example 6.14)*

$$G_c(s) = 60\,\frac{(s+2)}{(s+9)}$$

The block diagram of the compensated system is shown in Fig. 6.22.

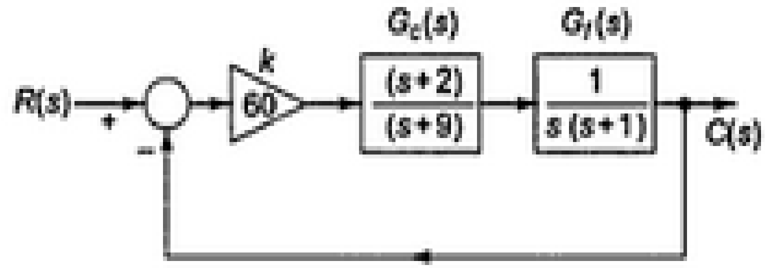

Fig. 6.22 *Block diagram of the compensated system*

6.4.2 Lag Compensator

A first order phase lag *compensator* is

$$G_c(s) = \frac{(s+z_c)}{(s+p_c)} = \frac{(s+1/\tau)}{(s+1/\alpha\tau)},\ \alpha = \frac{z_c}{p_c} > 1 \qquad \text{...(6.17)}$$

Since the magnitude of z_c is greater than the magnitude of p_c, a *phase-lag compensator* tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system.

How does the lag controller shift the rootlocus to the right? We recall—for finding the asymptotes of the root locus that lead to the zeros at infinity, the equation to determine the intersection of the asymptotes along the real-axis is:

$$-\sigma_A = \frac{\sum_p p_i - \sum_z z_i}{n-m}$$

When a lag compensator is added to a system, the value of this intersection will be a smaller negative number than it was before. The net number of zeros and poles will again be the same (one zero and one pole are added), but the added pole is a smaller negative number than the added zero of compensator zero. Thus, the result of a lag compensator is that the asymptotes' intersection is moved closer to the right-half plane, and the entire root locus will be shifted to the right.

It was previously stated that that lag controller should only minimally change the transient response because of its negative effect. If the phase-lag compensator is not supposed to change the transient response noticeably, what is it good for?. The answer is that a phase-lag compensator can improve the system's steady-state response. It works in the following manner. At high frequencies, the lag controller will have unity gain. At low frequencies, the gain will be z_c/p_c, which is greater than 1. This factor z_c/p_c will multiply the position, velocity, or acceleration constant (K_p, K_v, or K_a), and the steady state error will thus decrease by the factor z_c/p_c.

In MATLAB a *phase lag compensator* in root locus is implemented by using the transfer function in the form

```
numlag = [1 z];
denlag = [1 p];
```

and using the conv ( ) function to implement it with the numerator and denominator of the plant

```
newnum = conv (num, numlag);
newden = conv (den, denlag);
```

Realisation of a Phase Lag Compensator A first-order lag compensator can be realised using $R\,C$ Network of the Fig. 6.23 as

$$G_c(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC}{R_1 + R_2 + 1/sC} = \left[\frac{R_2}{R_1 + R_2}\right]\left[\frac{(s + 1/R_2C)}{s + \dfrac{1}{[(R_1 + R_2)/R_2]R_2C}}\right] \quad \ldots(6.18)$$

with $\tau = R_2C$ and $\beta = (R_1 + R_2)/R_2 > 1$ in Eq. (6.17).

with $\tau = R_2C$ and $\beta = (R_1 + R_2)/R_2 > 1$ in Eq. (6.17).

(a) Lag network (b) Location of pole-zero

Fig. 6.23 *First-order lag compensator*

Appropriate (practical) values of R_1, R_2 and C can be chosen for given or desired value of τ and α. For time-domain design of Lag Compensator using Root locus Technique the steps are:

1. Plot the root locus of the uncompensated system.
2. Change the time domain specification in to dominant set of desired close loop poles s_d (real and imaginary part with ξ and ω_n) and then check whether this desired pole lie on the root locus of $G_f(s)$ or not. If the system response satisfies the transient specification, the dominant roots of the closed loop system will lie on or be close to the root locus of the uncompensated system.
3. Calculate the gain k of the uncompensated system at the dominant root s_d and calculate the corresponding error constant.
4. Determine the factor by which the error constant of the uncompensated network should be increased to meet the specified value. Select the α parameter of the lag compensator to be somewhat greater than this factor.

5. Choose a zero of the compensator sufficiently close to the origin. For simplicity, construct a line making an angle of 10° with the desired ζ line from s_d. The intersection of this line with real-axis gives location of compensator zero.
6. Compensator pole can then be located at $-p_c = -z_c/\alpha$. In order to ensure that transient specifications are still met, the pole-zero pair should contribute an angle very small nearly 5° or less at s_d so that the root locus plot in the region of s_d is not changed appreciably.

Lag-Lead Compensator

We recall that lead compensator is suitable for systems having unsatisfactory transient response but it provides only a limited improvement in steady state response. Whereas a lag compensator is a good choice if the transient response is satisfactory but the steady state response is unsatisfactory.

When both the transient and steady state responses are unsatisfactory we use combination of lag and lead compensators. In such a case we first design a lead compensator to meet the transient response specifications. If the error constant requirements are also met satisfactorily, the design is complete and we do not use lag compensator. If the error constant is much higher than that obtained with lead compensator, then we design a lag network to compensate for the error constant. Usually acceptable design may be obtained after several trials.