Design Of Closed Loop Systems Using Compensation Techniques In Frequency Domain

Outline

- **Introduction to compensation design**
- **Phase Lead Compensation**
- **Phase Lag Compensation**
- **Phase Lead-lag Compensation**
- **PID Control**

Question: What is system compensation?

Given the control plant, the procedure of controller design to satisfy the requirement is called system compensation.

Question: Why to compensate?

The closed-loop system has the function of self-tunning. By selecting a particular value of the gain *K*, some single performance requirement may be met.

Is it possible to meet more than one performance requirement?

Sometimes, it is not possible.

Something new has to be done to the system in order to make it perform as required.

1.Control system design and compensation

- **Design：** **Need to design the whole controller to satisfy the system requirement.**
- **Compensation：** **Only need to design part of the controller with known structure.**

2. Three elements for compensation

Original part of the system

Performance requirement

Compensation device

7.1 Introduction to Compensation Design

7.1.1 Performance Requirement

1. Time domain criteria（step response）

- **Overshoot, settling time, rising time, steady-state error**

2. Frequency domain criteria

- **Open-loop frequency domain criteria:**
  crossover frequency, phase margin, gain margin
- **Closed-loop frequency domain criteria :**
  maximum value M_r , resonant frequency, bandwidth

Frequency domain and time domain criteria

Resonant peak $$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Resonant frequency $$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

Bandwidth $$\omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{(1-2\xi^2)^2+1}}$$

Gain crossover frequency $$\omega_c = \omega_n\sqrt{\sqrt{4\xi^4+1}-2\xi^2}$$

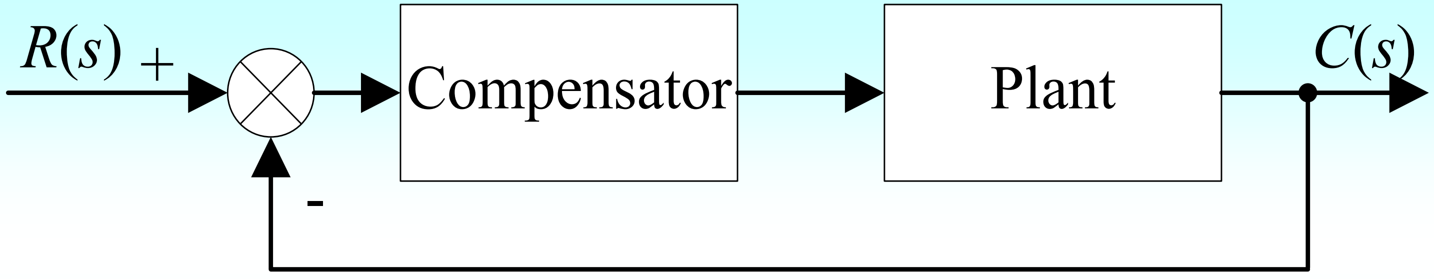
Phase margin $$\gamma = arctg\frac{2\xi}{\sqrt{\sqrt{4\xi^4+1}-2\xi^2}}$$

Percentage overshoot $$\sigma\% = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\times 100\%$$

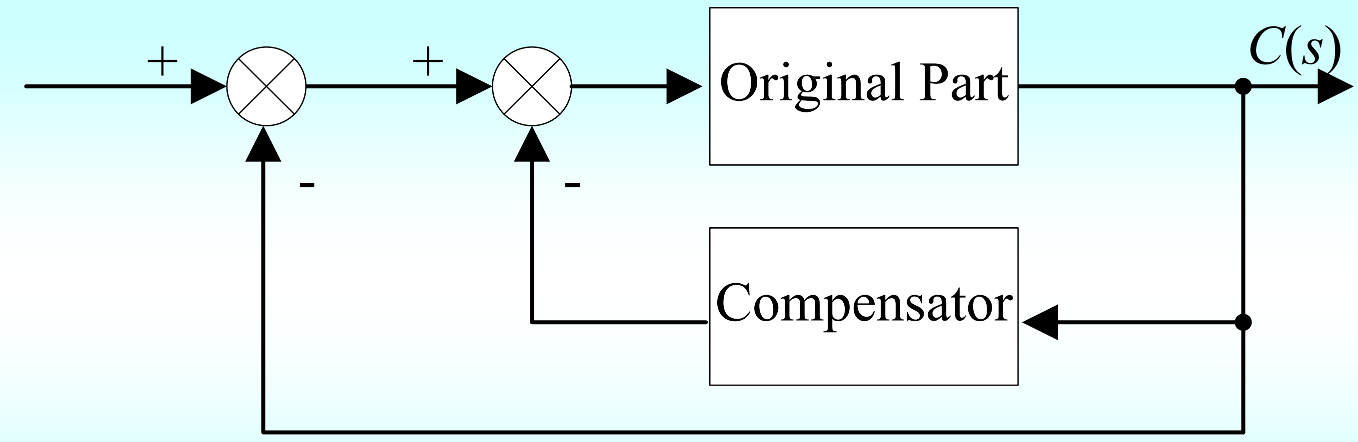
Settling time $$t_S = \frac{3}{\xi\omega_n}, \frac{4}{\xi\omega_n}$$

7.1.2 Structure of Compensator

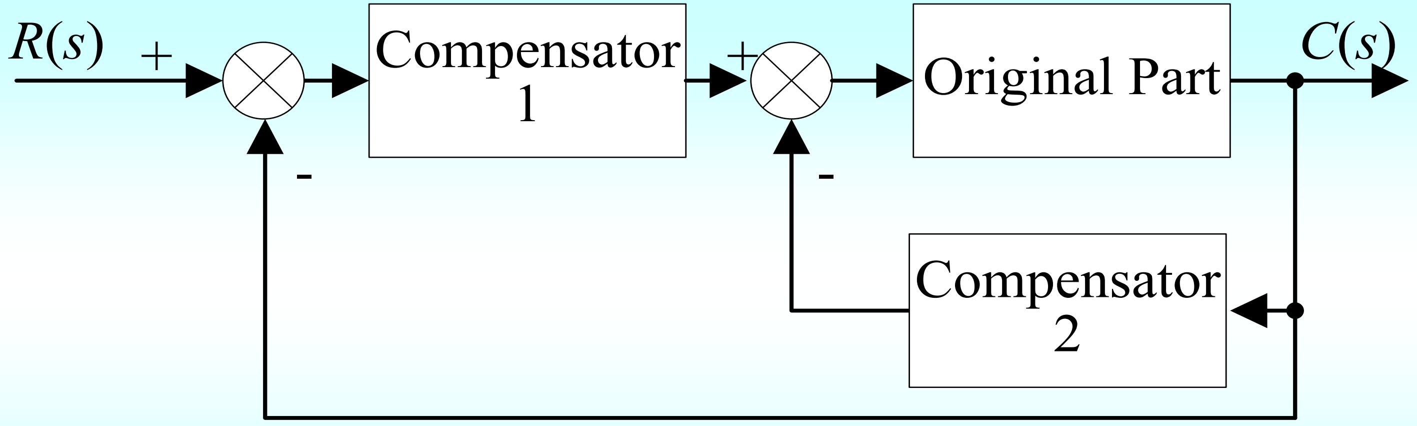
- **According to the way of compensation, the compensator can be classified into following categories:**



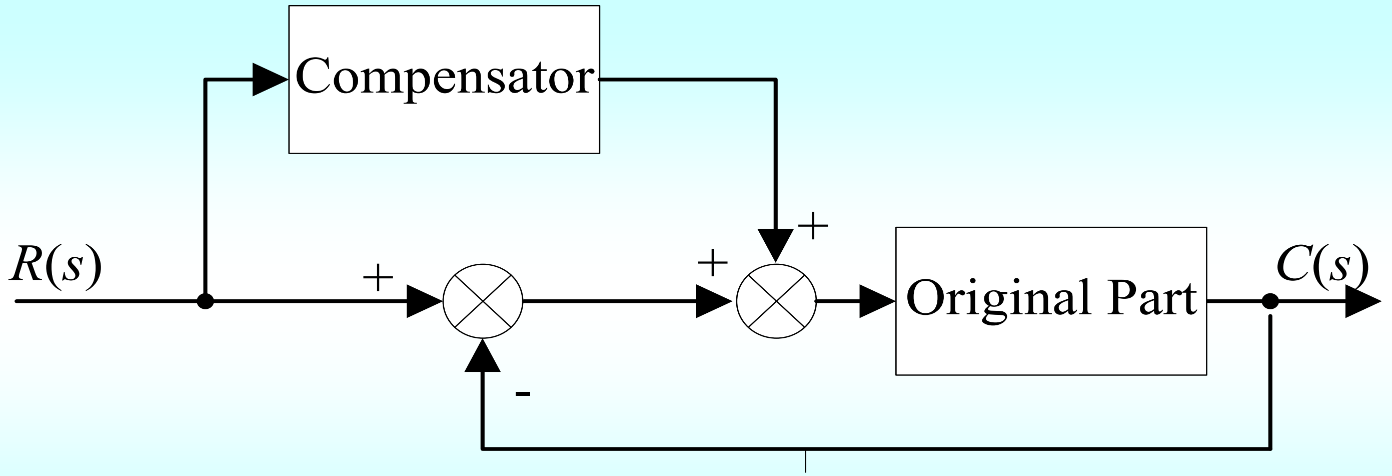
(a) Cascade Compensation



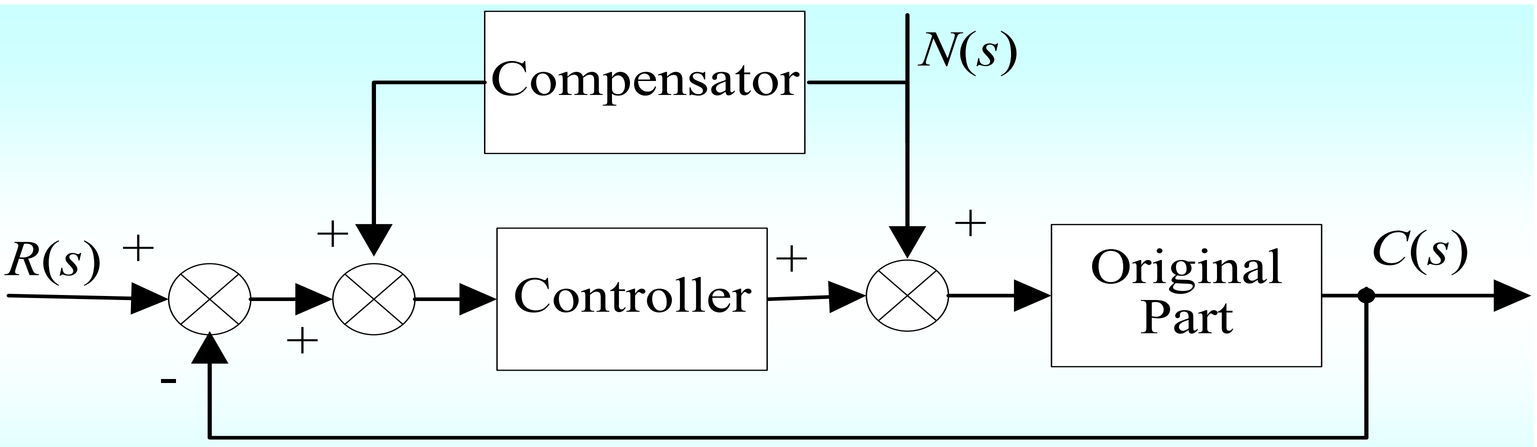
(b) Feedback compensation



(c)Cascade and feedback compensation



(d) Feed-forward compensation



(e) Disturbance compensation

Remark:

- **Cascade compensation and feedback compensation are inside the feedback loop.**
- **Feed-forward compensation and disturbance compensation are outside the feedback loop.**

7.1.3 Methods for Compensator Design

1. Frequency Response Based Method

Main idea： By inserting the compensator, the Bode diagram of the original system is altered to achieve performance requirements.

Original open-loop Bode diagram＋Bode diagram of compensator＋alteration of gain
＝open-loop Bode diagram with compensation

2. Root Locus Based Method

Main idea: Inserting the compensator introduces new open-loop zeros and poles to change the closed-loop root locus to satisfy the requirement.

7.1.2 Cascade Compensation

- **Frequency response based compensation**

Phase lead compensation

Phase lag compensation

Phase lead-lag compensation

- **Fundmantal rule for control design：PID control**

Each requirement relates to a different region of the frequency axis in the Bode diagram.

1. **The steady-state error relates to the magnitude at low frequency.**
2. **The transient response requirement relates to the gain crossover frequency, which usually occurs at higher frequencies.**

Three design rules for cascade compensator:

1. The system is stable with satisfactory steady-state error, but dynamic performance is not good enough.

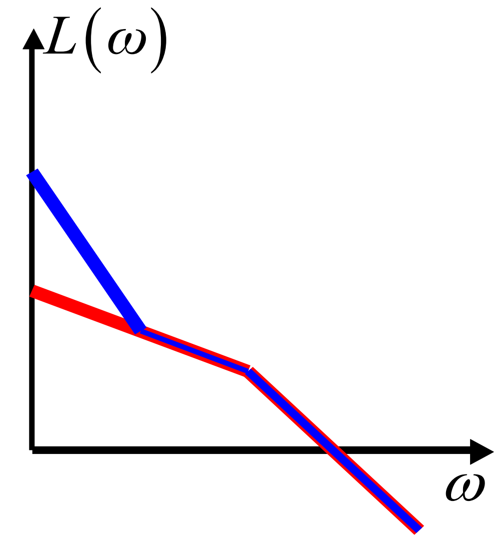
   Compensator is used to change medium and high frequency parts to change crossover frequency and phase margin.

2. The system is stable with satisfactory transient performance, but the steady-state error is large.

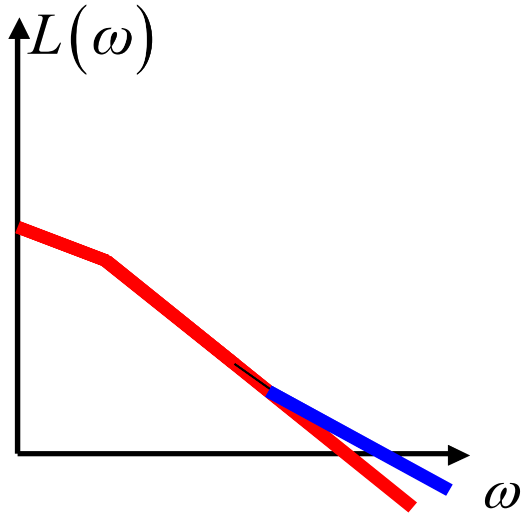
   Compensator is used to increase gain and change lower frequency part, but keep medium and higher frequency parts unchanged.

3. If the steady-state and transient performance are either unsatisfactory, the compensator should be able to increase gain of the lower frequency part and change the medium and higher frequency parts.

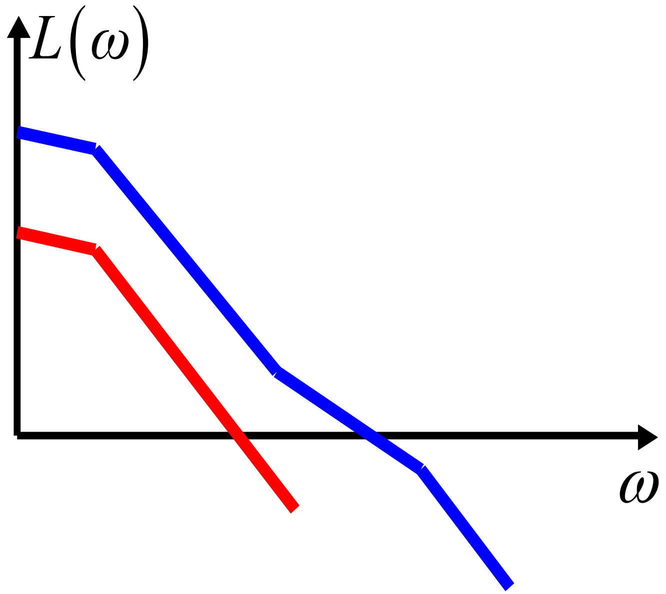
Change of Bode Diagram



a) change of lower frequency part

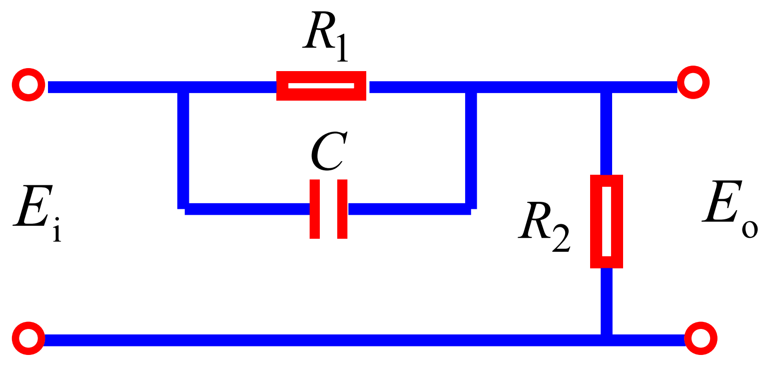


b) change of medium and higher frequency parts



b) change of lower, medium and higher frequency parts

7.2 Phase Lead Compensation



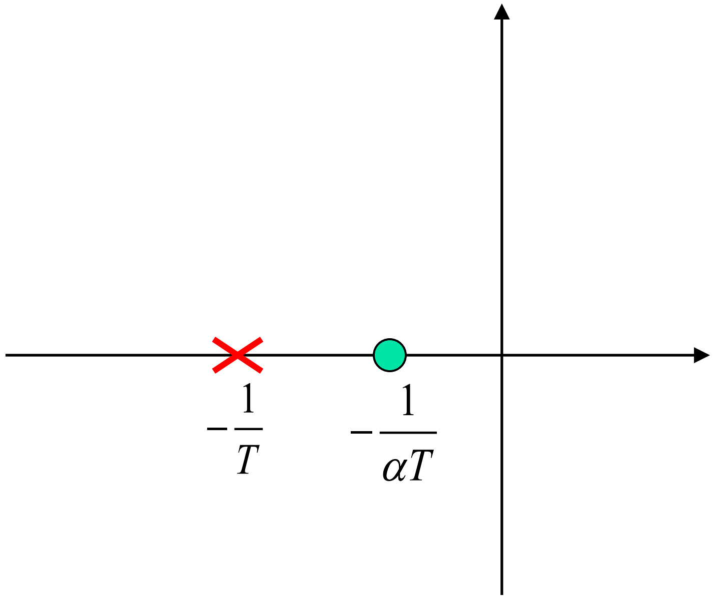
Passive Phase Lead Network

1. Transfer function:

$$G_c(s)=\frac{E_o(s)}{E_i(s)}=\frac{1}{\alpha}\times\frac{1+\alpha Ts}{1+Ts}$$

where

$$\alpha=\frac{R_1+R_2}{R_2}>1, T=\frac{R_1R_2}{R_1+R_2}C$$



Multiplying the transfer function by α

$$L_c(\omega) = 20\lg\sqrt{(\alpha T\omega)^2+1} - 20\log\sqrt{(T\omega)^2+1}$$

$$\varphi_c(\omega) = arctg\,\alpha T\omega - arctg\,T\omega = arctg\frac{(\alpha-1)T\omega}{1+\alpha T^2\omega^2}$$

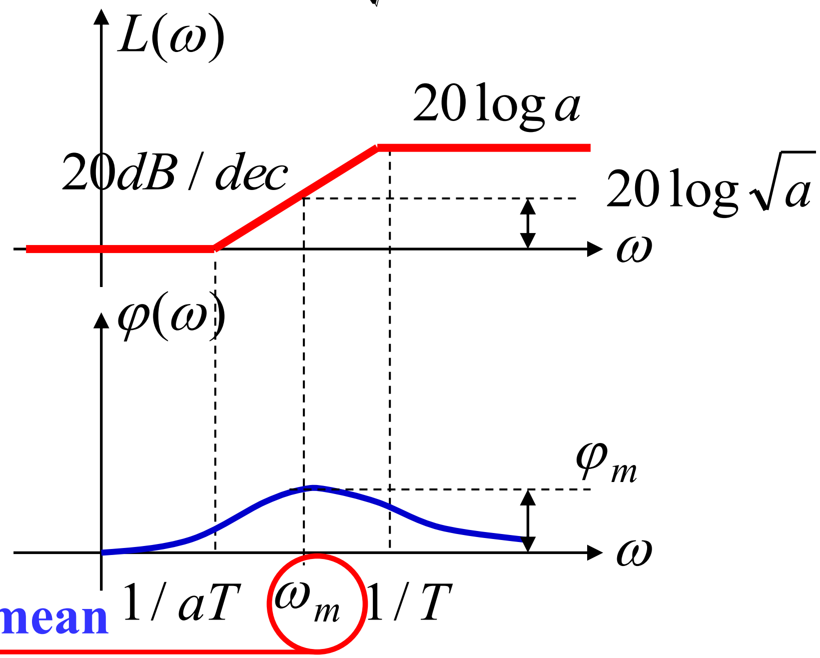
$$\frac{d\phi_c(\omega)}{d\omega} = 0 \quad , \quad \omega_m = \frac{1}{T\sqrt{\alpha}} \quad , \quad \varphi_m = arctg\frac{\alpha-1}{2\sqrt{\alpha}} = \arcsin\frac{\alpha-1}{\alpha+1}$$

$$L_c(\omega_m) = 10\log\alpha$$

Determination of α

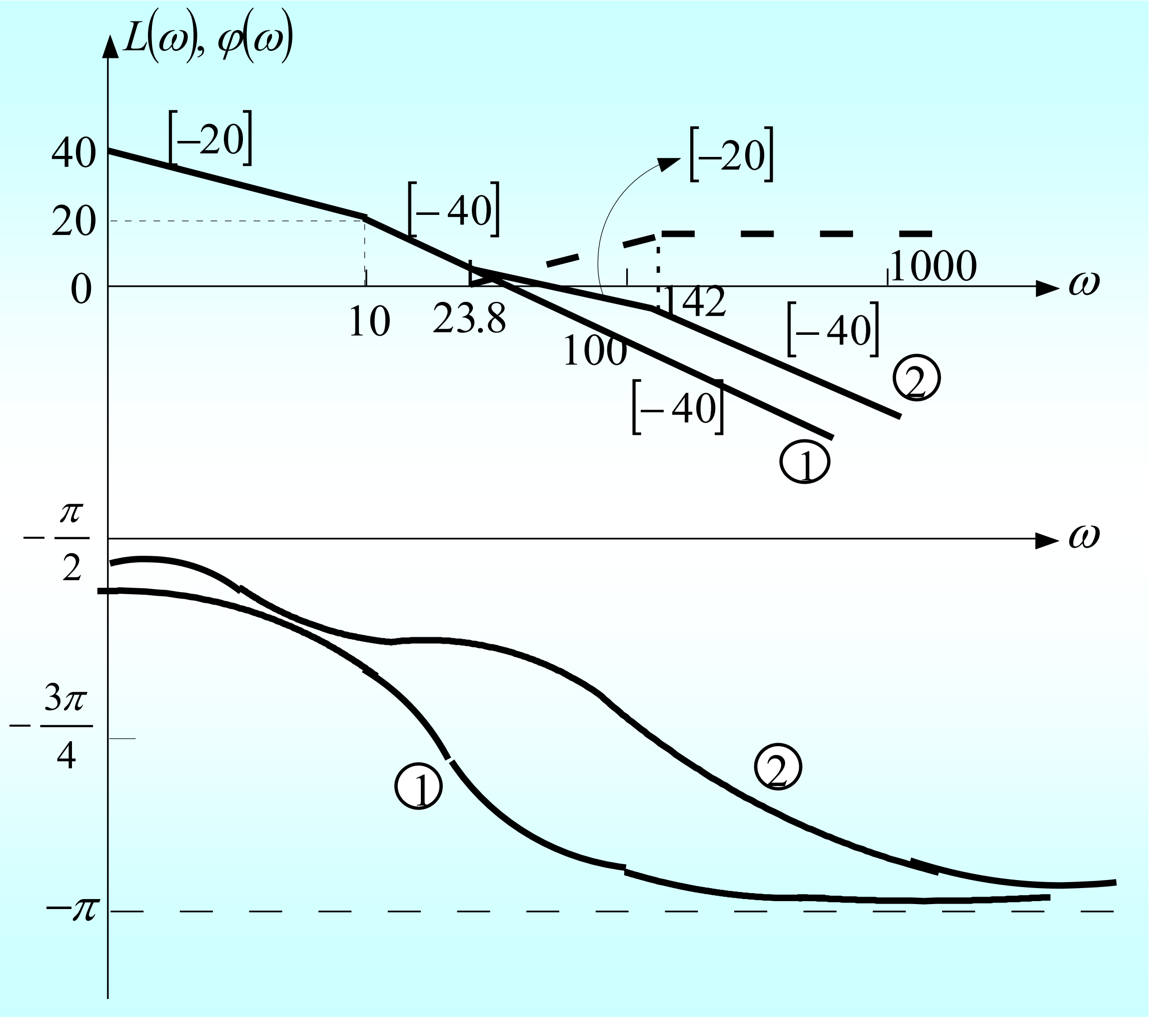
$$\alpha = \frac{1+\sin\varphi_m}{1-\sin\varphi_m}$$

$$0 < \alpha < 20$$



Geometry mean

2、Effect of phase lead compensation

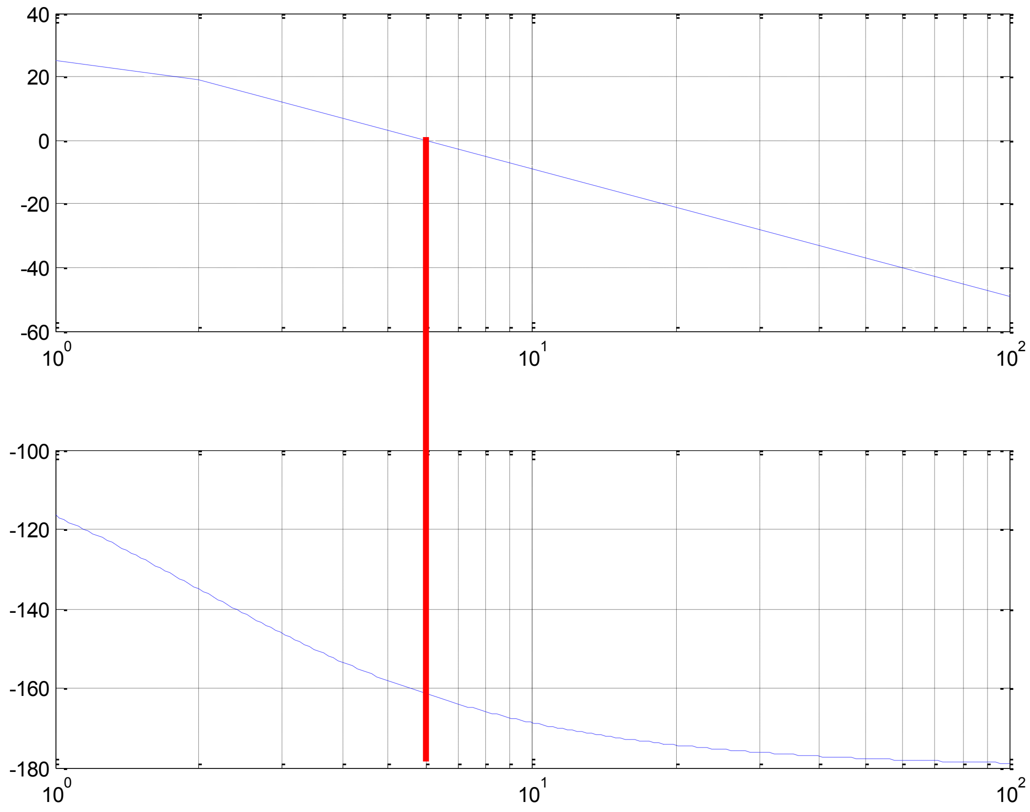


Example 7.1: Given a unity-feedback system with the following open-loop transfer function

$$G(s) = \frac{4K}{s(s+2)}$$

Please design phase lead element to satisfy the following three requirements:

1. **steady speed error constant** $K_v = 20s^{-1}$
2. **phase margin** $\gamma \geq 50°$
3. **gain margin** $GM \geq 10\text{dB}$



Uncompensated system

$$G(s) = \frac{20}{s(0.5s+1)}$$

$$\omega_{c0} = 6.17\text{rad/s}$$
$$\gamma = 17°$$

~~$\varphi_m = 50^\circ - 17^\circ = 33^\circ$~~ **?**

$$\varphi_m = 50^\circ - 17^\circ + 5^\circ = 38^\circ$$

Extra margin:
5°~10°

Remark:

The phase lead compensator not only takes extra phase margin of 33^o at the uncompensated gain crossover frequency.

However, we also have to add the magnitude part of the compensator to the uncompensated magnitude, and the gain crossover frequency moves to a higher value. The ultimate phase margin is less than 33^o .

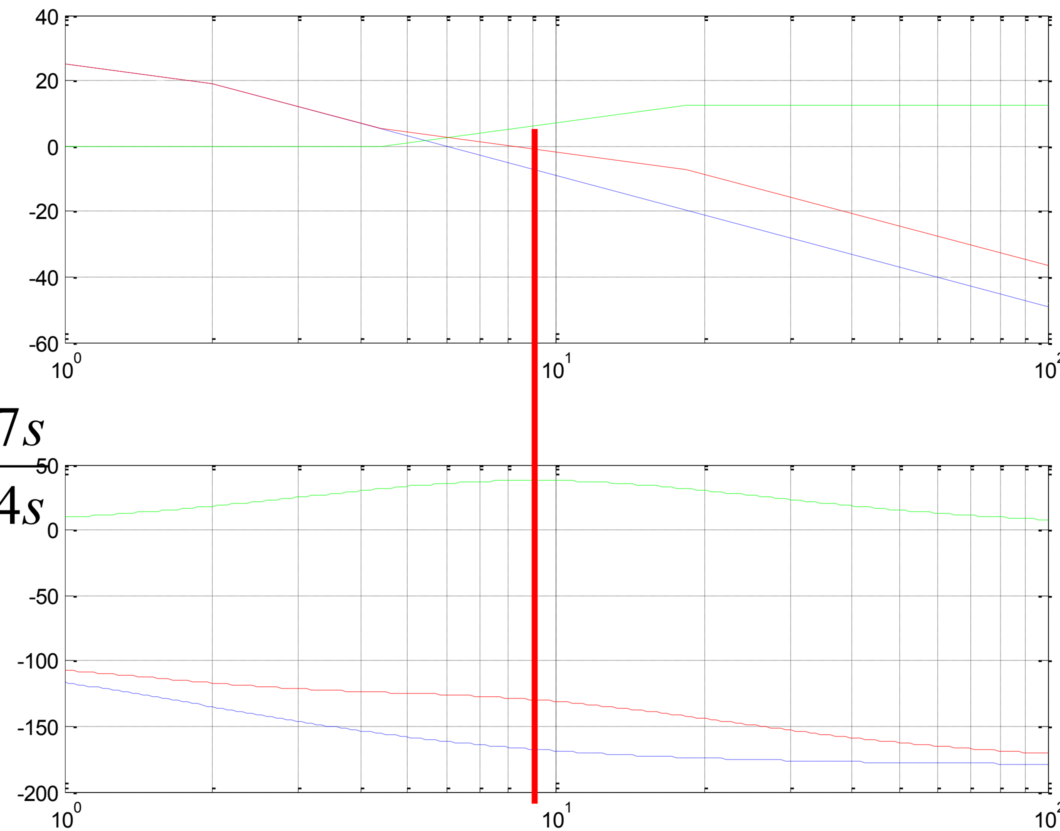
It implies we need to consider extra phase margin for determine α.

$$\alpha = \frac{1+\sin\varphi_m}{1-\sin\varphi_m} = \frac{1+\sin 38^\circ}{1-\sin 38^\circ} = 4.2$$

$$10\lg\alpha = 6.2\text{dB} \qquad \omega_m = 9\text{rad/s} = \frac{1}{\sqrt{\alpha}T} = \omega_{c1}$$

$$\omega_1 = \frac{1}{\alpha T} = \frac{\omega_m}{\sqrt{\alpha}} = 4.4\text{rad/s}$$

$$\omega_2 = \frac{1}{T} = \omega_m\sqrt{\alpha} = 18.4\text{rad/s}$$



$$G_c(s) = \frac{s+4.4}{s+18.4} = 0.238\cdot\frac{1+0.227s}{1+0.054s}$$

Complete Compensator

$$\alpha G_c(s) = \frac{1+0.227s}{1+0.054s}$$

Comments on phase lead compensation

1、The slop around the gain crossover frequency is increased. It improves the relative stability.

2、The closed-loop resonance peak is reduced. Also, the overshoot is reduced.

3、Increase the open-loop phase margin.

4、The open-loop (and usually the closed-loop) bandwidths are increased. It is beneficial for fast response. But it may cause problems if noise exists at the higher and unattenuated frequencies.

5、Take no effect on the steady-state performance.

Rules to design phase lead compensation

(1) Determine K to satisfy steady-state error constraint

(2) Determine the uncompensated phase margin γ_0

(3) estimate the phase margin φ_m in order to satisfy the transient response performance constraint

(4) Determine α

(5) Calculate ω_m

(6) Determine T

(7) Confirmation

- **Constraints for application of lead compensation:**
- **Constraint 1: The system is stable.**

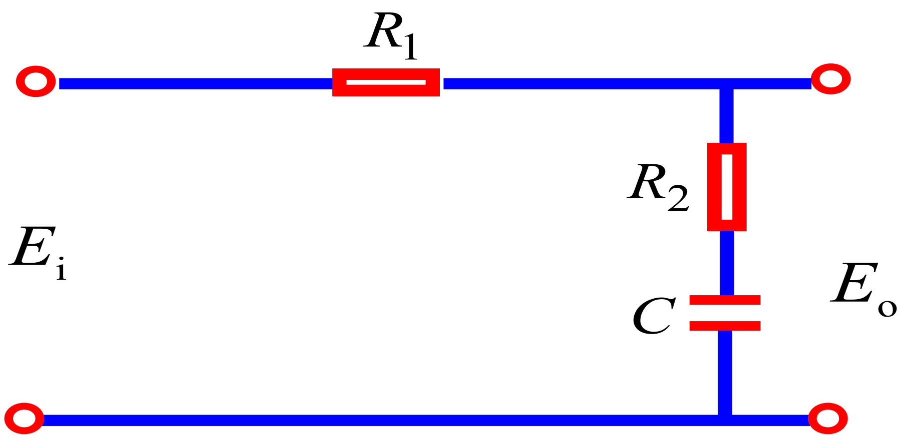
  If it is unstable, the phase need to compensate is too big. The noise takes severe effects on the system.
- **Constraint 2: The phase cannot reduce very fast around the gain crossover frequency.**

  The phase lead compensation can only provide less than 60^o extra phase margin.

6.3 Phase Lag Compensation

Transfer function:

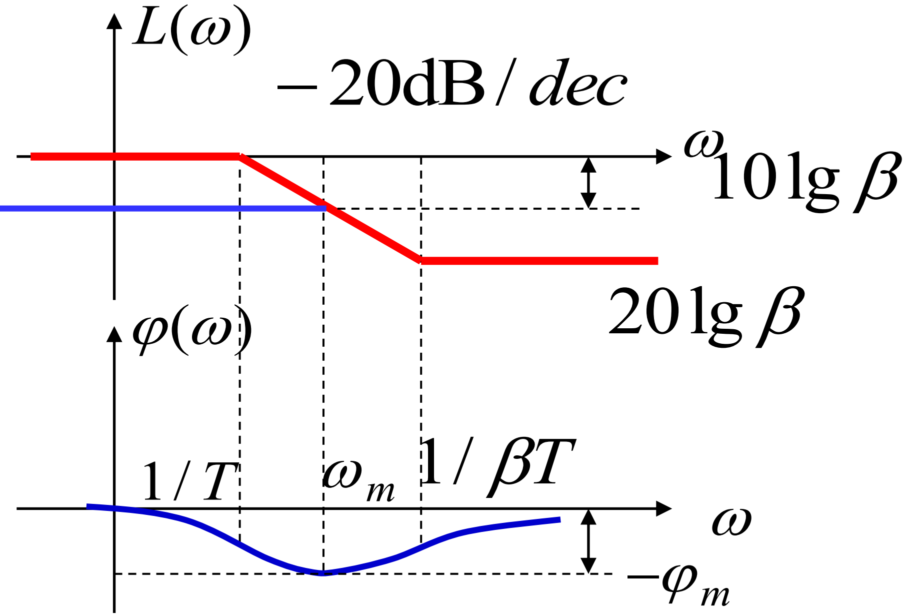
$$G_c(s) = \frac{R_2 + \dfrac{1}{CS}}{R_1 + (R_2 + \dfrac{1}{CS})} = \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1} = \frac{\beta Ts + 1}{Ts + 1}$$



Passive phase lag network

$$T = (R_1 + R_2)C, \quad \beta = \frac{R_2}{R_1 + R_2} < 1$$

$$\omega_m = \frac{1}{T\sqrt{\beta}}$$



Maximum phase lag

$$\phi_m = \arcsin\frac{1-\beta}{1+\beta}$$

The compensator has no filtering effect on the low frequency signal, but filters high frequency noise. The smaller β is，the lower the noise frequency where the noise can pass.

Comments on phase lag compensation:

1、Phase lag compensator is a low-pass filter. It changes the low-frequency part to reduce gain crossover frequency. The phase is of no consequence around the gain crossover frequency.

2、Be able to amplify the magnitude of low-frequency part, and thus reduce the steady-sate error.

3、The slope around gain crossover freuency is -20dB/dec.

Resonance peak is reduced, and the system is more stable.

4、Reduce the gain crossover frequency, and then reduce the bandwidth. The rising time is increased. The system response slows down.

Rules for phase lag compensator design:

1、Determine *K* to satisfy the steady-state error;

2、Draw bode diagram for umcompensated system

3、Find the frequency ω_{c1} from the uncompensated open-loop Bode diagram where the phase margin satisfy the performance requirement;

4、Calculate the magnitude at the frequency ω_{c1}.

Determine β by

$$\beta = 10^{-L(\omega_{c1})/20}$$

5、Determine the two break frequencies.

$$\omega_z = \frac{1}{\beta T} < \frac{1}{10}\omega_{c1} \qquad \omega_p = \frac{1}{T}$$

6. Comfirmation

Avoid introduce the maximum phase lag around the gain crossover frequency ω_{c1} . So the break frequency $\frac{1}{\beta T}$ is much less than ω_{c1}

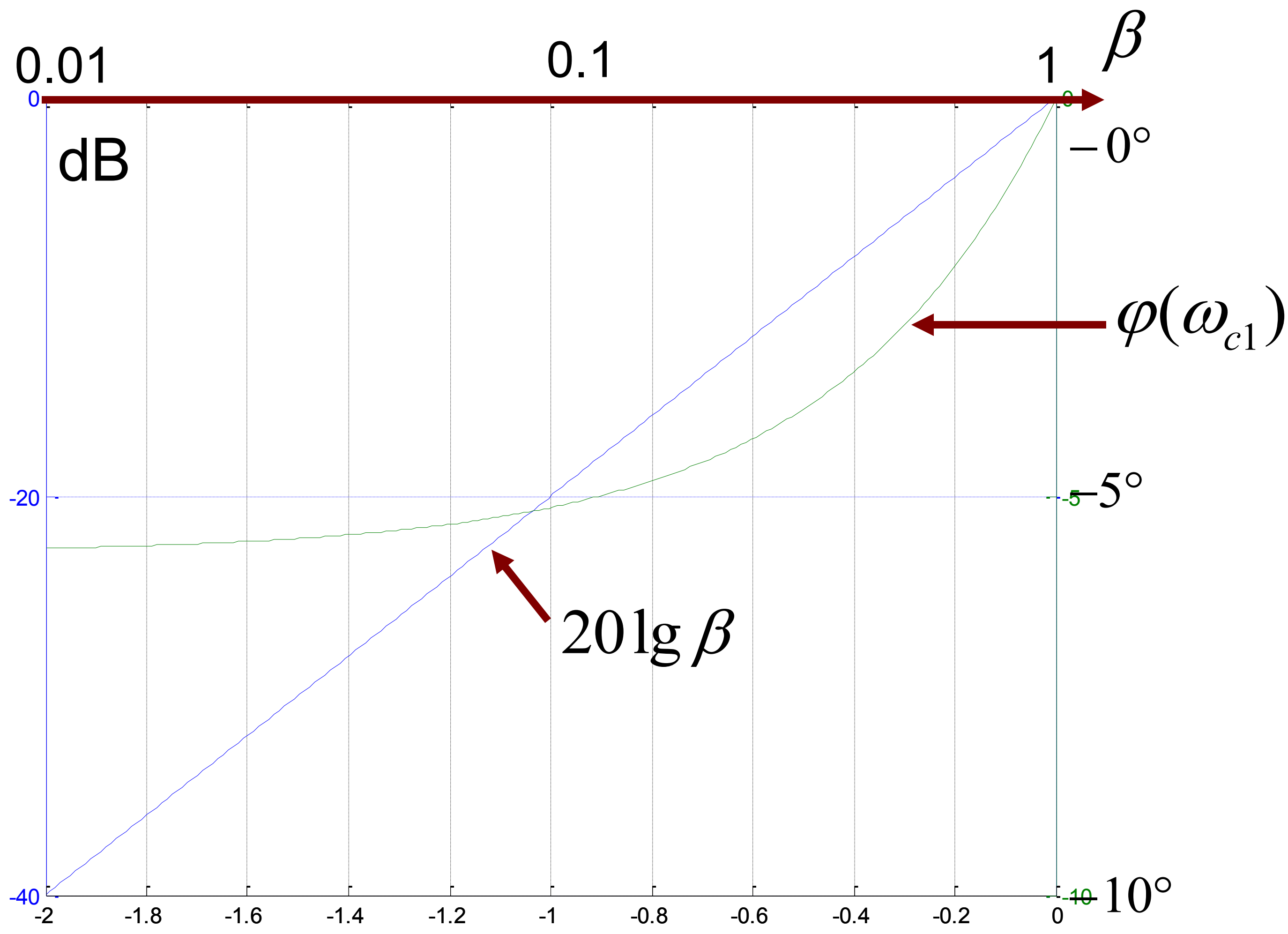
$$\frac{1}{\beta T} = \frac{1}{10}\omega_{c1}$$

The phase lag at *ω_{c1} is*

$$\varphi_c(\omega_{c1}) = arctg\,\beta T\omega_{c1} - arctg T\omega_{c1} = \frac{(\beta-1)T\omega_{c1}}{1+\beta(T\omega_{c1})^2}$$

Substitude $\omega_{c1}T = \frac{10}{\beta}$

$$\varphi(\omega_{c1}) = arctg\frac{(\beta-1)\frac{10}{\beta}}{1+\beta(\frac{10}{\beta})^2} = arctg\frac{10(\beta-1)}{100+\beta} \approx arctg[0.1(\beta-1)]$$



$\varphi\left(\omega_{c1}\right)$ and $20\lg\beta$ versus β

Example 6.2: Given a unity-feedback controller with the following open-loop transfer function

$$G(s)=\frac{K}{s(0.1s+1)(0.2s+1)}$$

Please design a cascade compensator to satisfy the following requirements:

$$K_v=30 \qquad \gamma \geq 40^o \qquad GM \geq 10(dB)$$

$$\omega_{c1} \geq 2.3(rad/s)$$

Notice: If the compensated system cannot satisfy the requirements, we need to further alter crossover frequency and phase of the lag compensator. $|\varphi(\omega_{c1})|$

Solution:

(1) $K_v = \lim_{s \to 0} sG(s) = K = 30$

(2) Draw the uncompensated Bode diagram

$\gamma_0 = -38.3^o$

The system is unstable. The needed extra phase compensation is 83.3^o . The phase lead compensation can not provide so big phase margin.

Two ways:

(a)Apply two cascade phase lead compensation

(b)Apply phase lag compensation

(3) For ω=2.5 rad/s, γ=45^o **Set ω_{c1}=2.5**

(4) $L(\omega_{c1})=22\text{dB}$

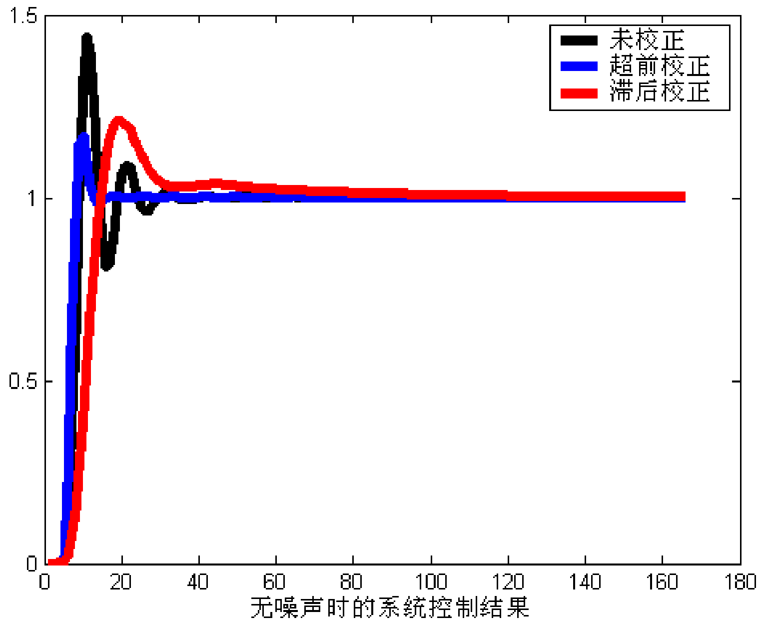
$$\beta=10^{-L(\omega_{c1})/20}=0.08$$

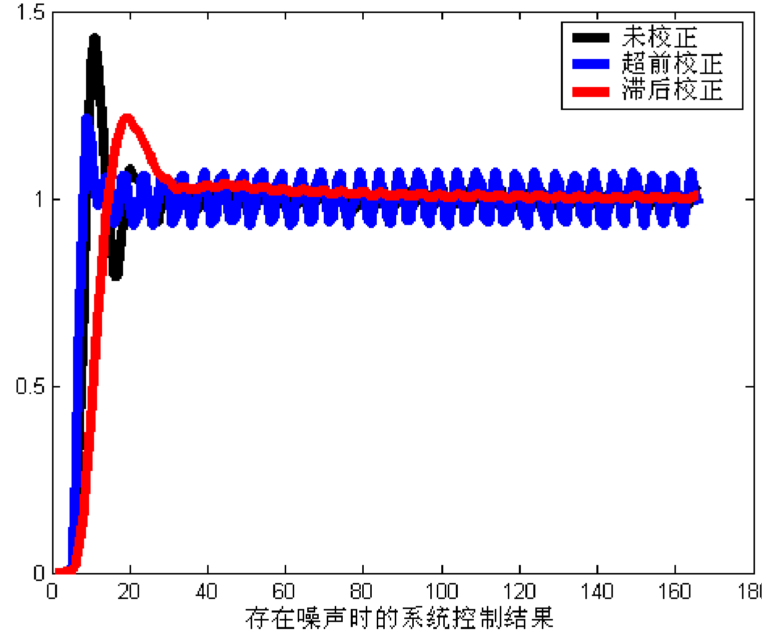
(5) $$\frac{1}{\beta T}=\frac{1}{10}\omega_{c1}=0.25$$

$$\omega_p=\frac{1}{T}=0.25\times 0.08=0.02\text{rad/s}$$

(6) $$G_c(s)=\frac{4s+1}{50s+1}$$

Confirmation: $\gamma(\omega_{c1})=44^o, \omega_g=6.7rad/s, GM\approx 10\text{dB}$





➢**Applicable for the following systems:**

（1）The transient performance is satisfactory, but the steady-state performance is desired to be improved.

（2）High requirement for noise attenuation.

➢**Drawback: The system response is slow down.**

Comparison of phase lead and lag compensation

	Phase lead compensation	Phase lag compensation
Main Idea	Improve transient performance by using phase lead characteristics	Improve the steady-state performance by using magnitude attenuation at the high-frequency part
Effect	(1) Around ω_c, the absolute value of slope is reduced. Phase margin γ and gain margin GM are increased. (2) Increase the bandwidth (3) With bigger γ, overshoot is reduced. (4) Take no effect on the steady-state performance.	(1) Keep relative stability unchanged, but reduce the steady-state error. (2) Reduce ω_c and then closed-loop bandwidth (3) For specific open-loop gain, γ, GM and resonant peak M_r are all improved due to magnitude attenuation around ω_c
Weak ness	(1) Broad bandwidth reduces the filtering for noise. (2) For passive network implementation, need an extra amplifier.	Narrow Bandwidth increase the response time.
Applic ation	(1) Extra phase lead compensation is less than 55^0. (2) Require broad bandwidth and fast response (3) No matter the noise at high-frequency part.	(1) The phase lag of the uncompensated system is fast around ω_c. (2) Bandwidth and transient response are satisfactory. (3) Require attenuation of noise (4) The phase margin can be satisfied at the low frequency.