DESIGN OF CLOSED LOOP SYSTEMS USING COMPENSATION TECHNIQUES IN TIME DOMAIN

Lecture Outline

- Introduction to Lead Compensation
- Electronic Lead Compensator
- Electrical Lead Compensator
- Mechanical Lead Compensator

Lead Compensation

- Lead Compensation essentially yields an appreciable improvement in transient response and a small change in steady state accuracy.

- There are many ways to realize lead compensators and lag compensators, such as electronic networks using operational amplifiers, electrical *RC* networks, and mechanical spring-dashpot systems.

Lead Compensation

- Generally Lead compensators are represented by following transfer function

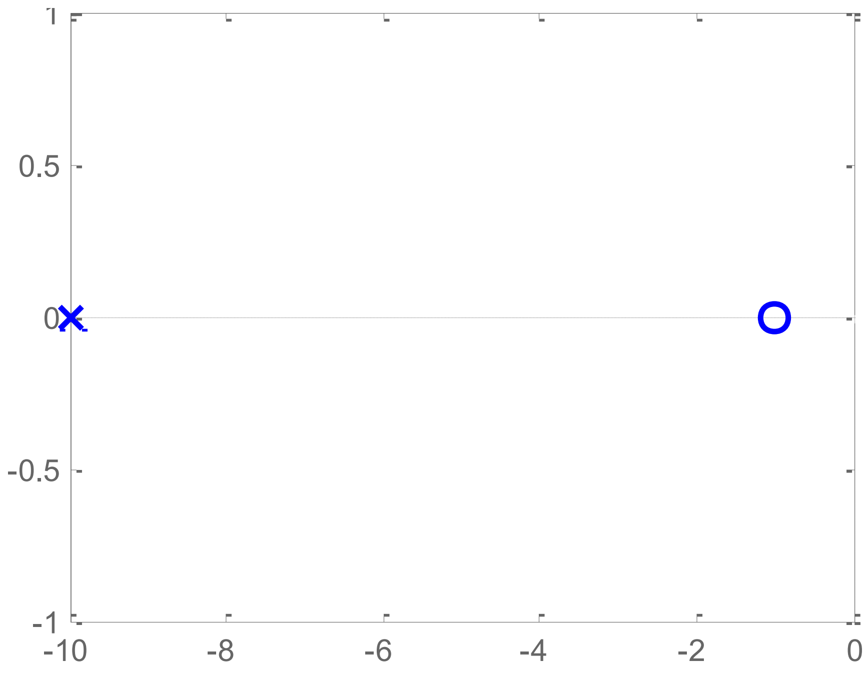
$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1}, \qquad (0 < \alpha < 1)$$

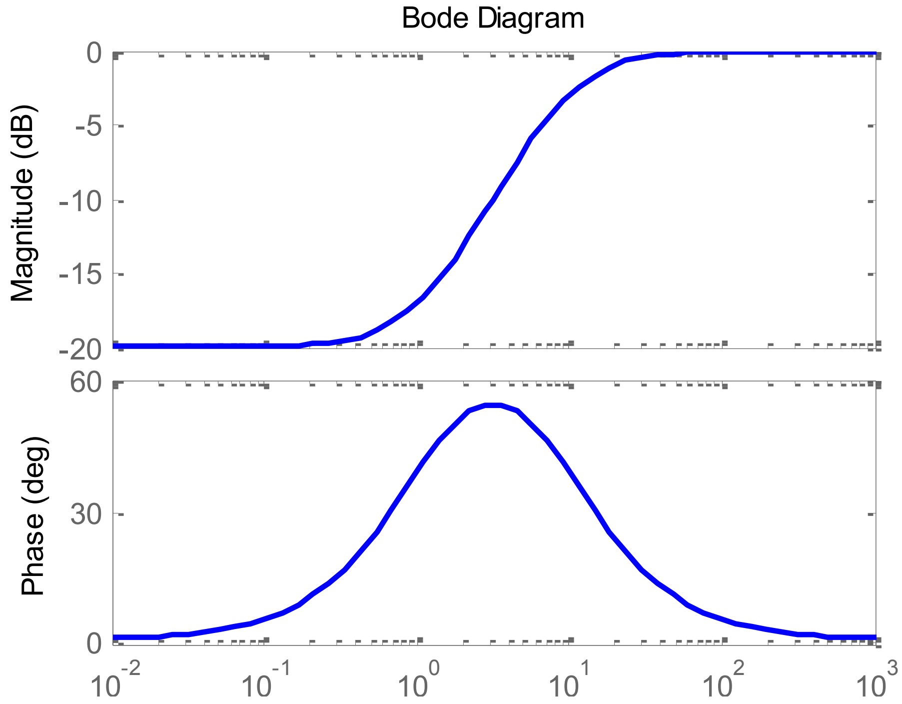
- or

$$G_c(s) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

Lead Compensation

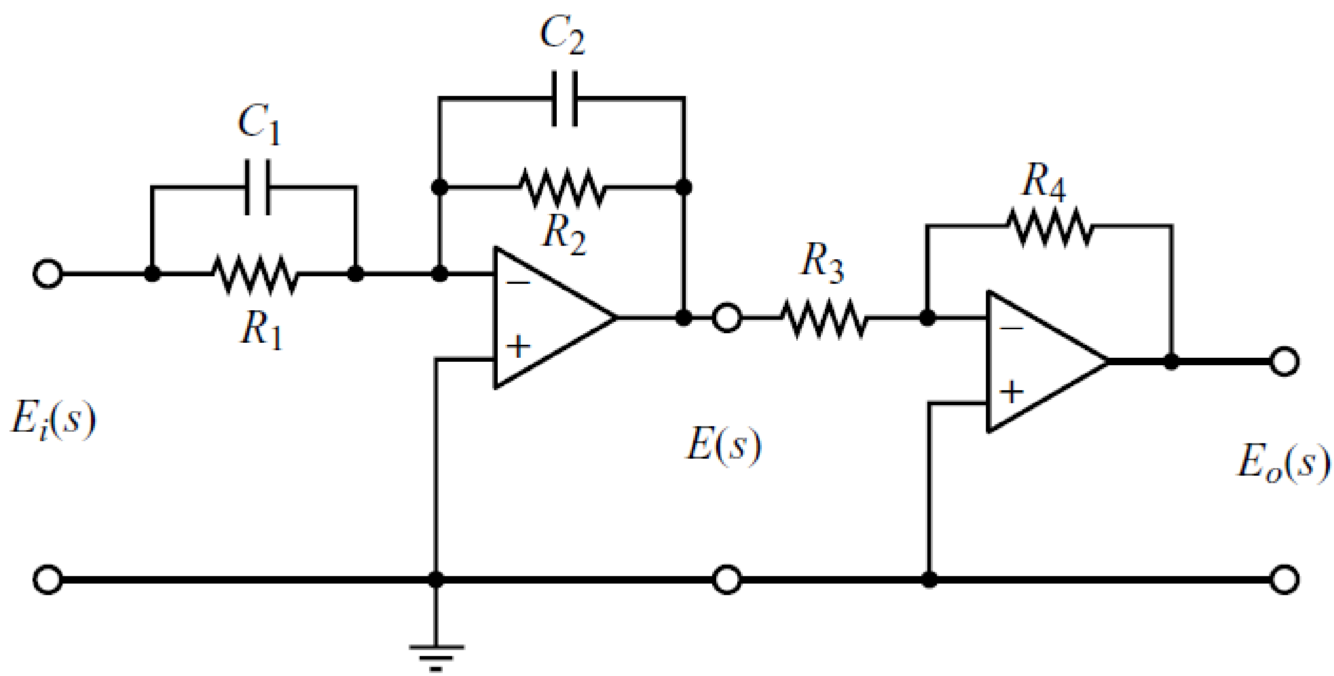
$$G_c(s) = 3\,\frac{s+1}{s+10}, \qquad (\alpha = 0.1)$$





Electronic Lead Compensator

- Following figure shows an electronic lead compensator using operational amplifiers.



$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$$

Electronic Lead Compensator

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$$

- This can be represented as

$$\frac{E_o(s)}{E_i(s)} = \frac{R_4 C_1}{R_3 C_2} \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}}$$

- Where,

$$T = R_1 C_1 \qquad aT = R_2 C_2 \qquad K_c = \frac{R_4 C_1}{R_3 C_2}$$

- Then,

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

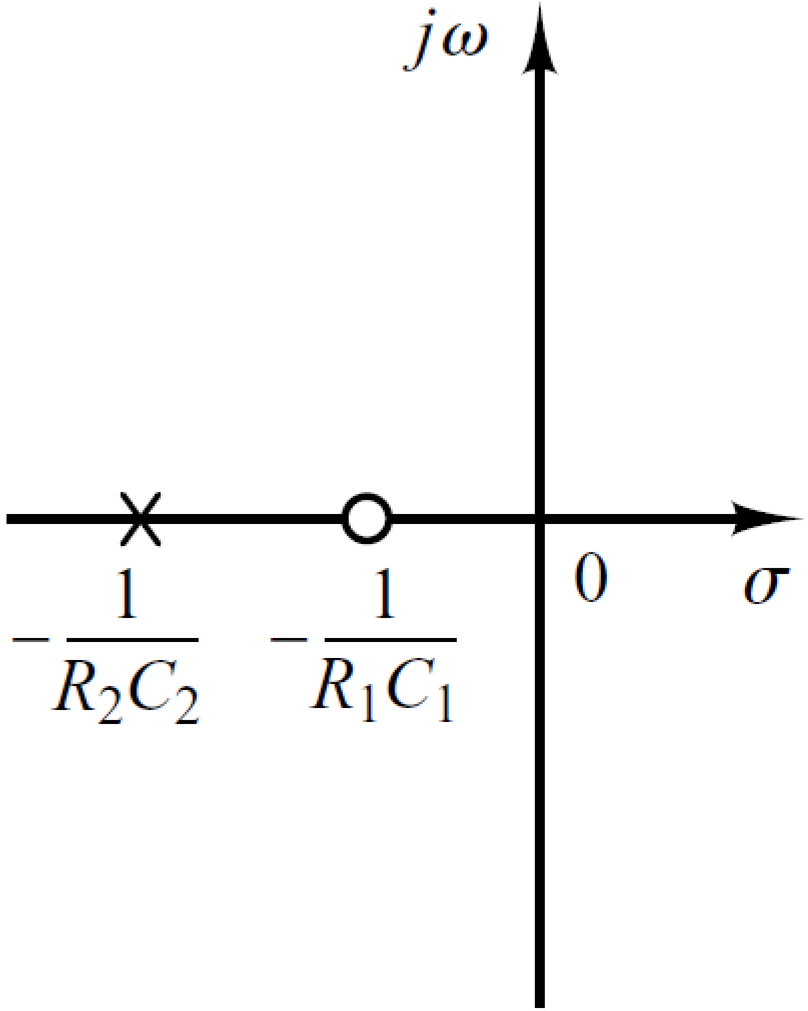
- Notice that

$$R_1 C_1 > R_2 C_2$$

Electronic Lead Compensator

- Pole-zero Configuration of Lead Compensator

$$R_1C_1 > R_2C_2$$



Lead Compensation Techniques Based on the Root-Locus Approach.

- The root-locus approach to design is very powerful when the specifications are given in terms of time-domain quantities, such as

  - damping ratio
  - undamped natural frequency
  - desired dominant closed-loop poles
  - maximum overshoot
  - rise time
  - settling time.

Lead Compensation Techniques Based on the Root-Locus Approach.

- The procedures for designing a lead compensator by the root-locus method may be stated as follows:
  - **Step-1:** Analyze the given system via root locus.

Step-2

- From the performance specifications, determine the desired location for the dominant closed-loop poles.

Step-3

- From the root-locus plot of the uncompensated system (original system), ascertain whether or not the gain adjustment alone can yield the desired closed loop poles.

- If not, calculate the angle deficiency.

- This angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

Step-4

- Assume the Lead Compensator to be:

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

- Where α and T are determined from the angle deficiency.
- K_c is determined from the requirement of the open-loop gain.

Step-5

- If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle.

- If no other requirements are imposed on the system, try to make the value of α as large as possible.

- A larger value of α generally results in a larger value of K_v, which is desirable.

- Larger value of α will produce a larger value of K_v and in most cases, the larger the K_v is, the better the system performance.

Step-6

- Determine the value of K_c of the lead compensator from the magnitude condition.

Final Design check

- Once a compensator has been designed, check to see whether all performance specifications have been met.

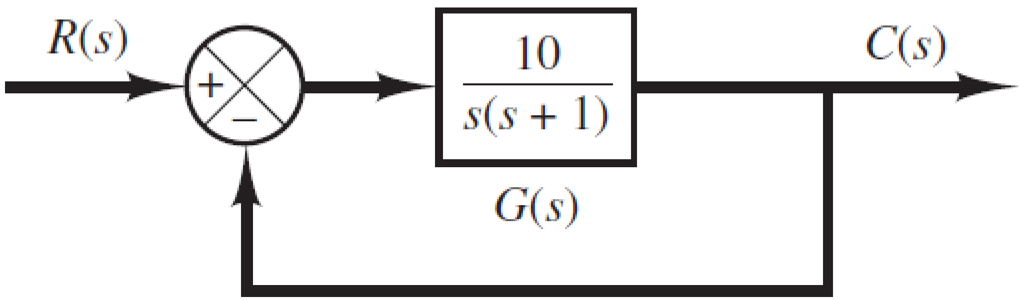
- If the compensated system does not meet the performance specifications, then repeat the design procedure by adjusting the compensator pole and zero until all such specifications are met.

Final Design check

- If the selected dominant closed-loop poles are not really dominant, or if the selected dominant closed-loop poles do not yield the desired result, it will be necessary to modify the location of the pair of such selected dominant closed-loop poles.

Example-1

- Consider the position control system shown in following figure.



- It is desired to design an Electronic lead compensator $G_c(s)$ so that the dominant closed poles have the damping ratio 0.5 and undamped natural frequency 3 rad/sec.

Step-1 (Example-1)

- Draw the root Locus plot of the given system.

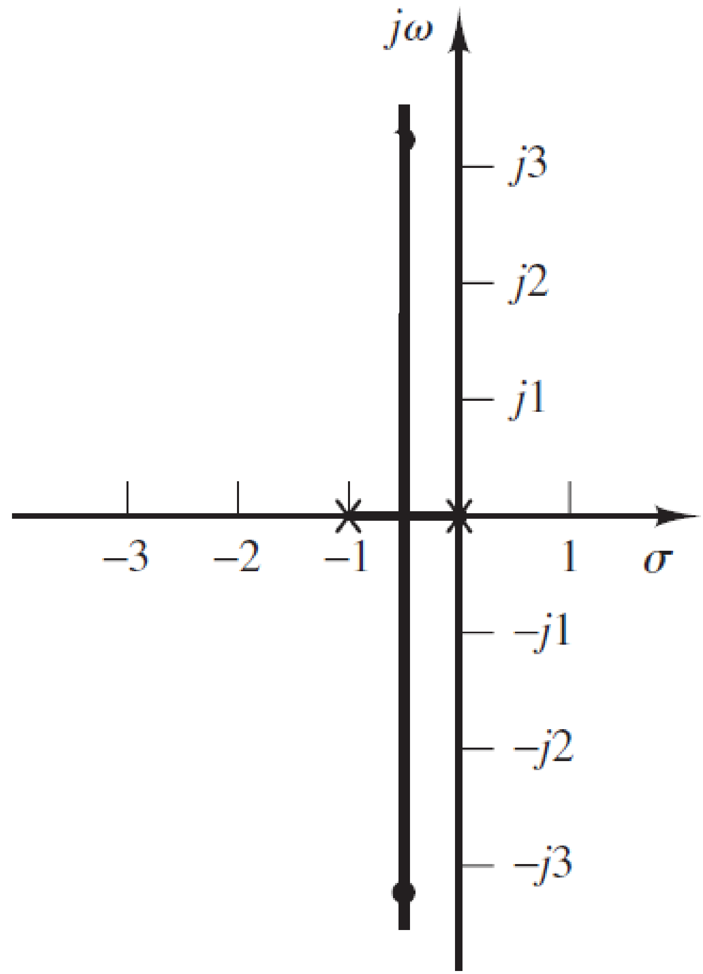
$$G(s)H(s) = \frac{10}{s(s+1)}$$

- The closed loop transfer function of the given system is:

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + s + 10}$$

- The closed loop poles are

$$s = -0.5 \pm j3.1225$$

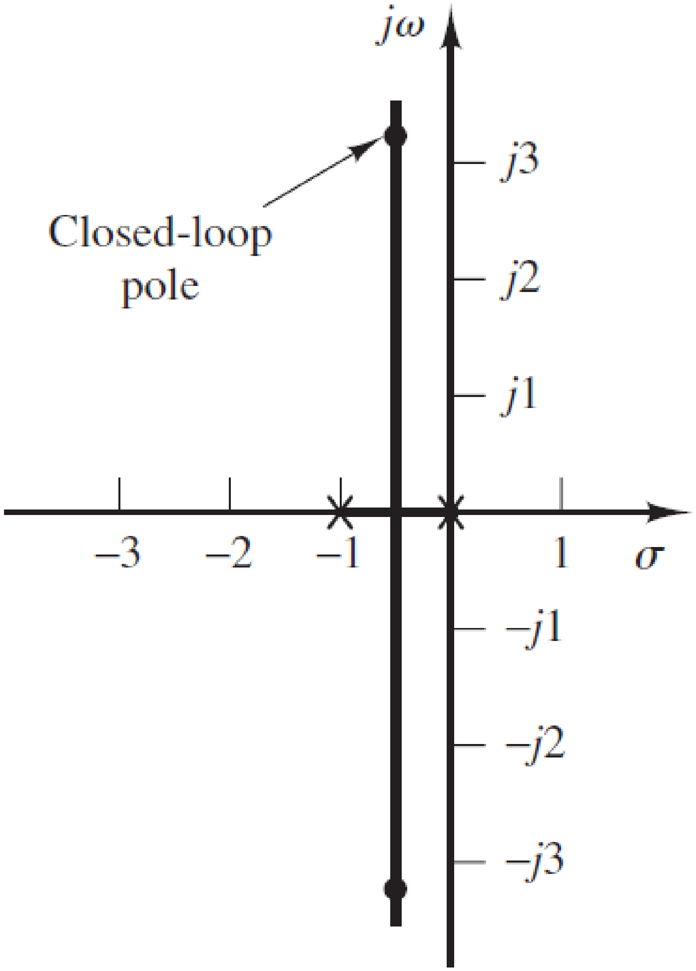


Step-1 (Example-1)

- Determine the characteristics of given system using root loci.

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + s + 10}$$

- The damping ratio of the closed-loop poles is 0.158.
- The undamped natural frequency of the closed-loop poles is 3.1623 rad/sec.
- Because the damping ratio is small, this system will have a large overshoot in the step response and is not desirable.



Step-2 (Example-1)

- From the performance specifications, determine the desired location for the dominant closed-loop poles.

- Desired performance Specifications are:
  - It is desired to have damping ratio 0.5 and undamped natural frequency 3 rad/sec.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{9}{s^2 + 3s + 9}$$

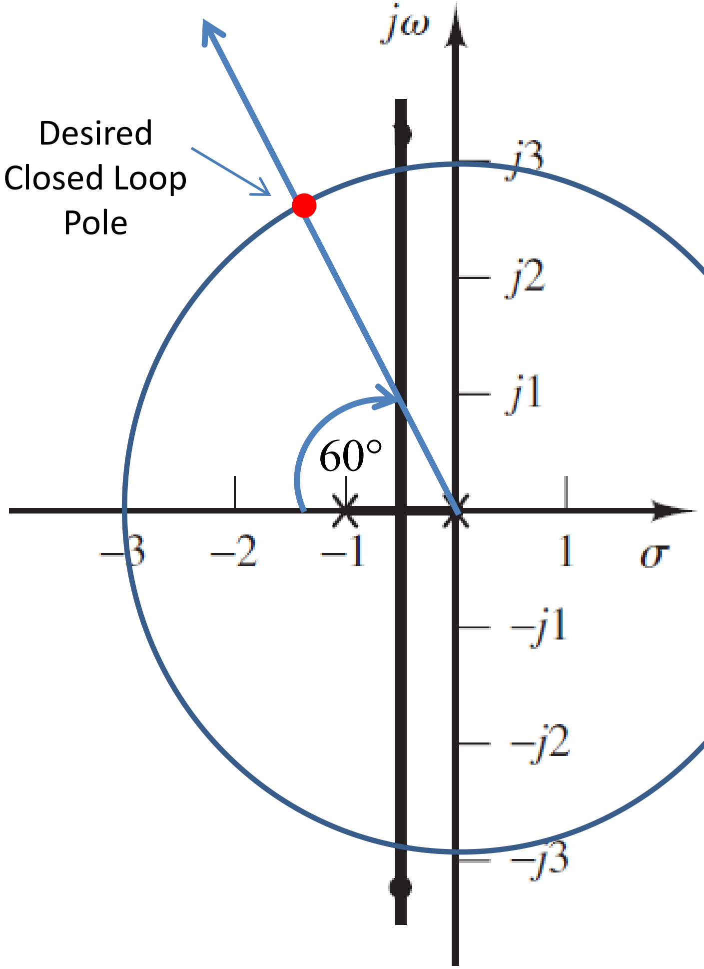
$$s = -1.5 \pm j2.5981$$

Step-2 (Example-1)

- Alternatively desired location of closed loop poles can also be determined graphically
  - Desired ω_n= 3 rad/sec
  - Desired damping ratio= 0.5

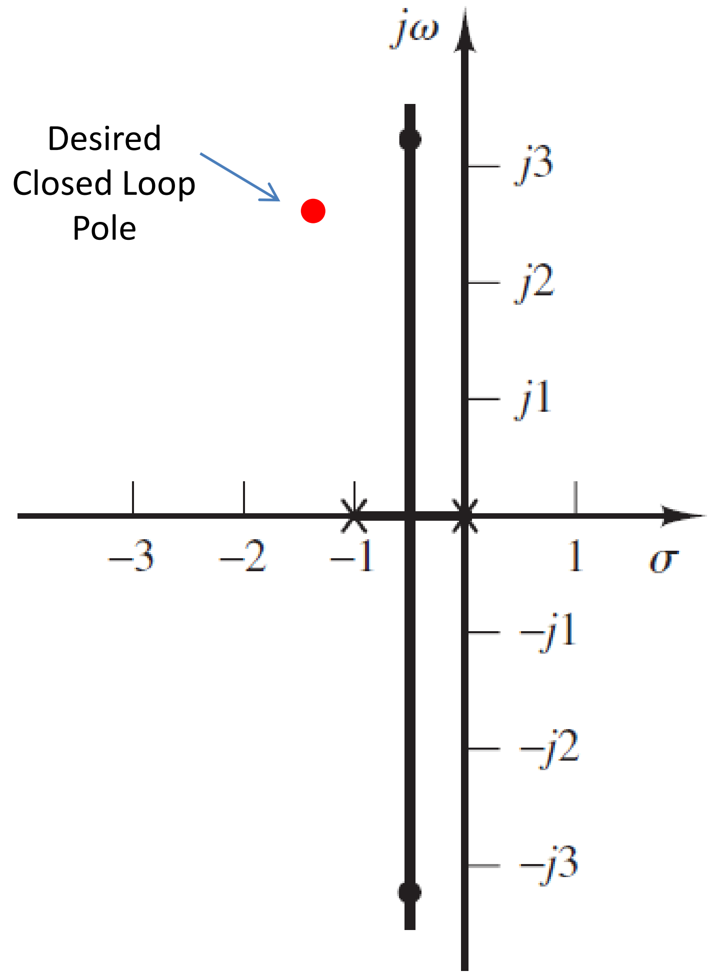
$$\theta = \cos^{-1}\zeta$$

$$\theta = \cos^{-1}(0.5) = 60°$$



Step-3 (Exampl-1)

- From the root-locus plot of the uncompensated system ascertain whether or not the gain adjustment alone can yield the desired closed loop poles.



Step-3 (Exampl-1)

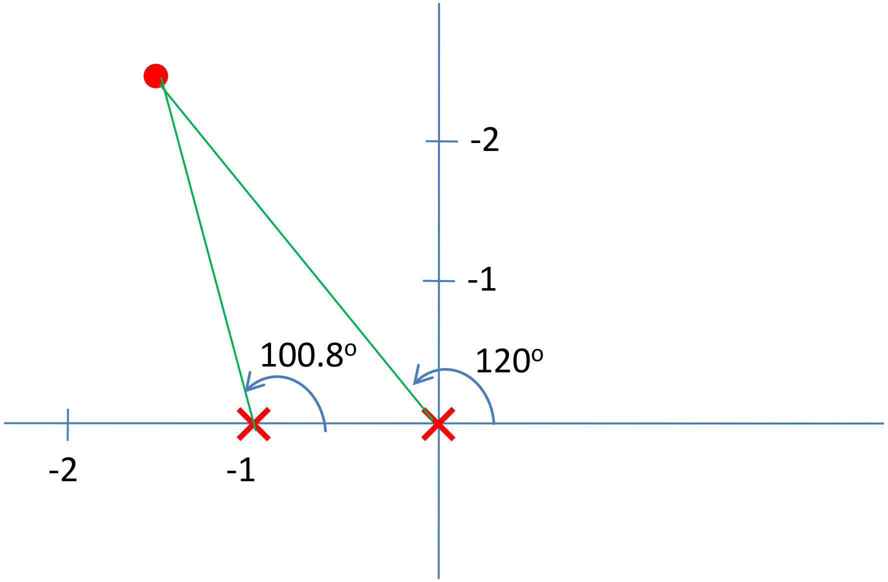
- If not, calculate the angle deficiency.
- To calculate the angle of deficiency apply Angle Condition at desired closed loop pole.

Desired Closed Loop Pole

$$s = -1.5 \pm j2.5981$$

$$\theta_d = 180° - 120° - 100.8°$$

$$\theta_d = -40.89°$$



Step-3 (Exampl-1)

- Alternatively angle of deficiency can be calculated as.

$$\theta_d = 180° + \angle \frac{10}{s(s+1)} \bigg|_{s=-1.5+j2.5981}$$

Where $s = -1.5 \pm j2.5981$ are desired closed loop poles

$$\theta_d = 180° + \angle 10 - \angle s \big|_{s=-1.5+j2.5981} - \angle (s+1) \big|_{s=-1.5+j2.5981}$$

$$\theta_d = 180° - 120° - 100.8°$$

$$\theta_d = -40.89°$$

Step-4 (Exampl-1)

- This angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

$$G_c(s) = K_c \alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

- Note that the solution to such a problem is not unique. There are infinitely many solutions.

Step-5 (Exampl-1)

Solution-1

- **Solution-1**
  - If we choose the zero of the lead compensator at s = -1 so that it will cancel the plant pole at s =-1, then the compensator pole must be located at s =-3.