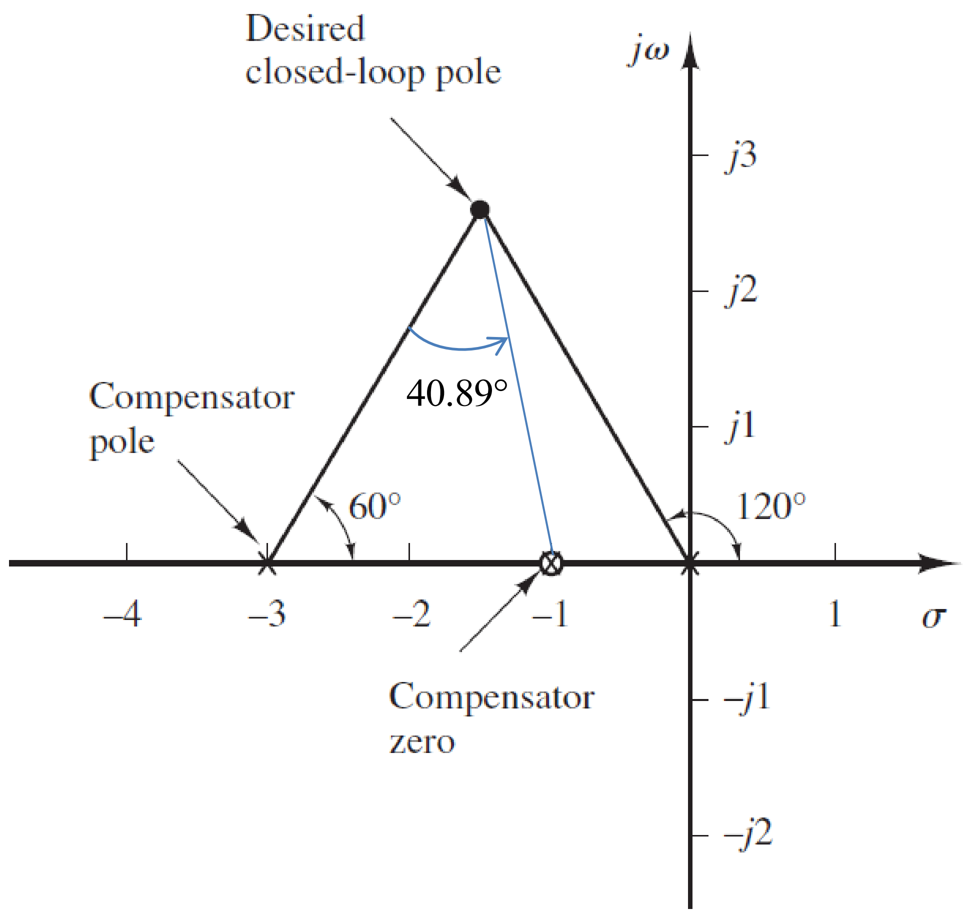
Step-5 (Example-1)

Solution-1

- If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle.



Step-5 (Example-1)

Solution-1

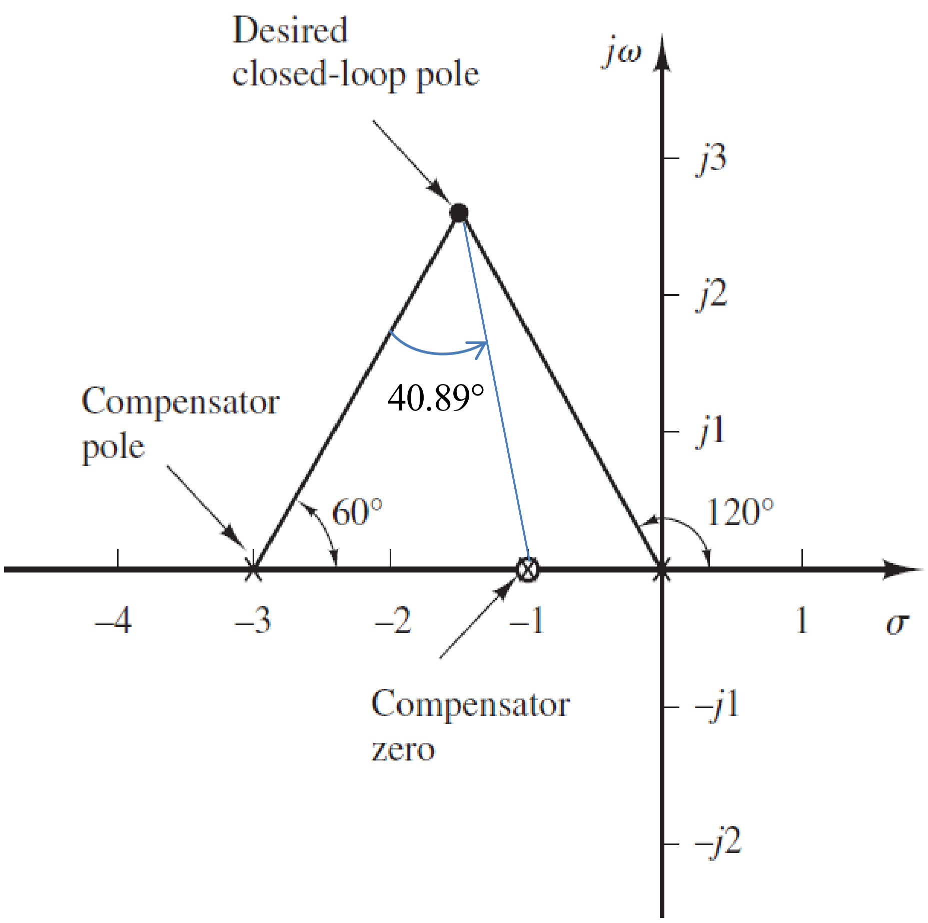
- The pole and zero of compensator are determined as

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = K_c \frac{s+1}{s+3}$$

- The Value of α can be determined as

$$\frac{1}{T} = 1 \xrightarrow{\text{yields}} T = 1$$

$$\frac{1}{\alpha T} = 3 \xrightarrow{\text{yields}} \alpha = 0.333$$



Step-6 (Example-1)

Solution-1

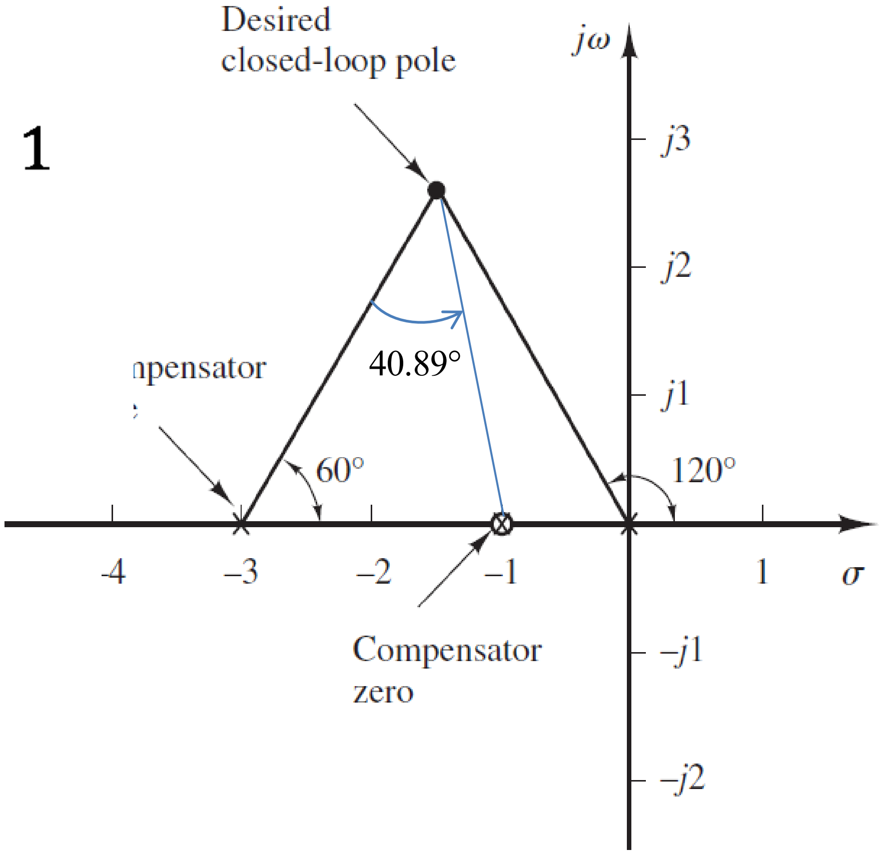
- The Value of K_c can be determined using magnitude condition.

$$\left| K_c \frac{(s+1)}{s+3} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$

$$\left| K_c \frac{10}{s(s+3)} \right|_{s=-1.5+j2.5981} = 1$$

$$K_c = \left| \frac{s(s+3)}{10} \right|_{s=-1.5+j2.5981} = 0.9$$

$$G_c(s) = 0.9 \frac{s+1}{s+3}$$



Solution-1

Final Design Check

- The open loop transfer function of the designed system then becomes

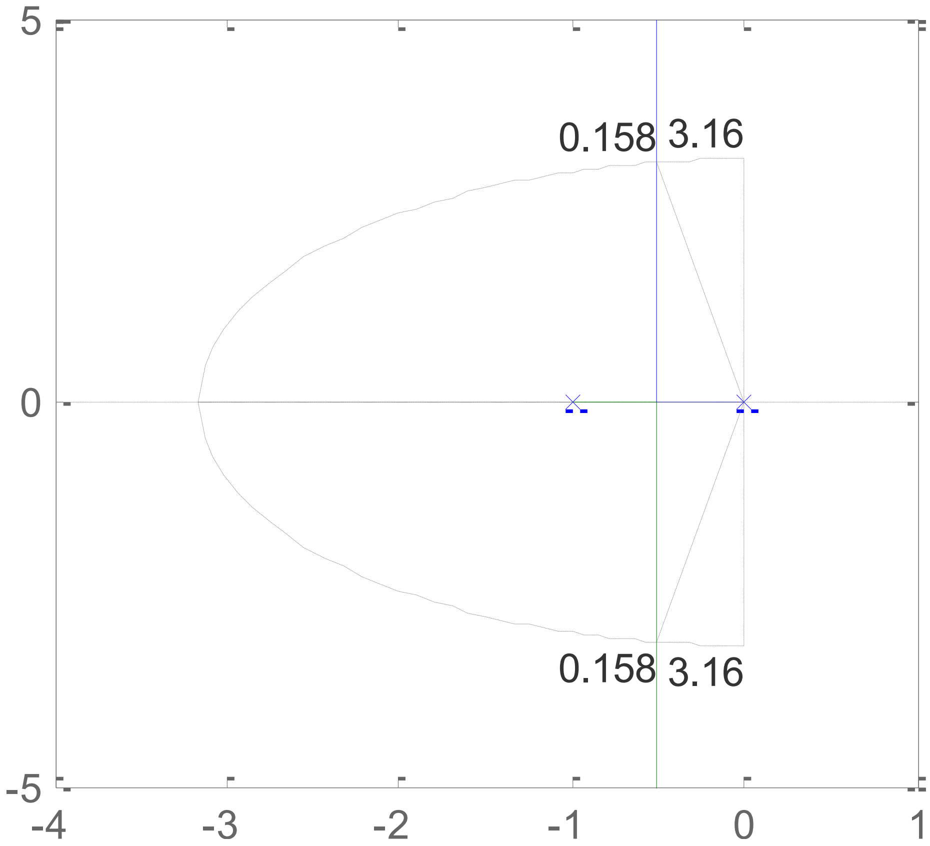
$$G_c(s)G(s) = \frac{9}{s(s+3)}$$

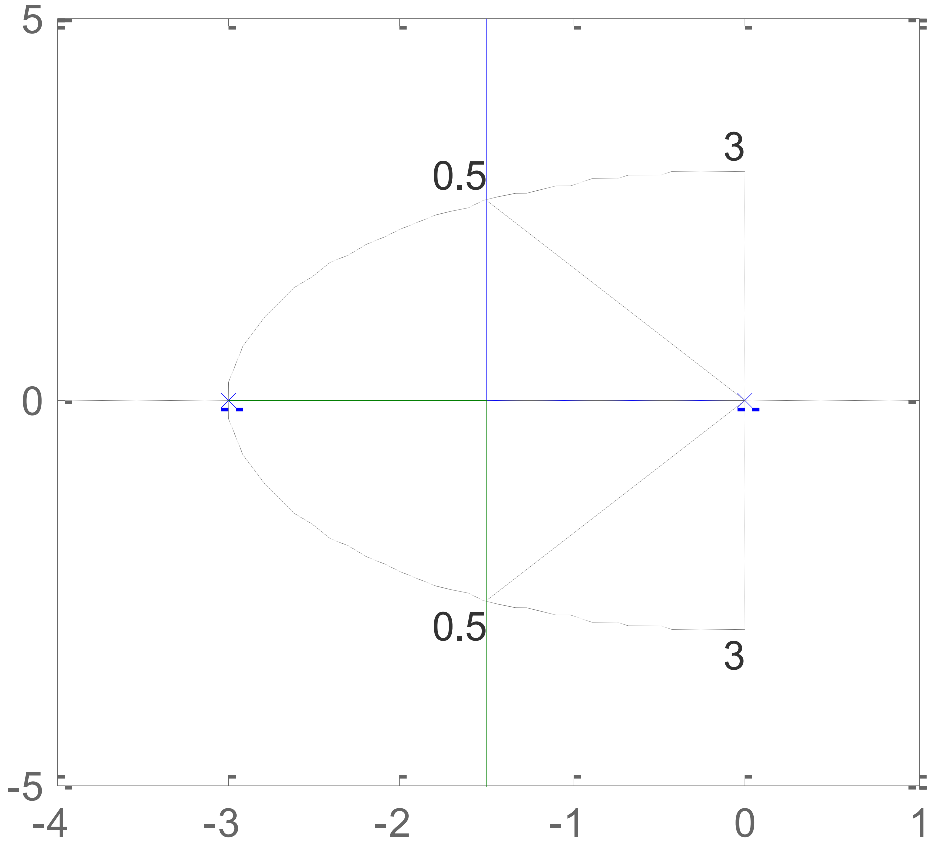
- The closed loop transfer function of compensated system becomes.

$$\frac{C(s)}{R(s)} = \frac{9}{s^2 + 3s + 9}$$

Final Design Check

Solution-1





$$G(s) = \frac{10}{s(s+1)}$$

$$G_c(s)G(s) = \frac{9}{s(s+3)}$$

Solution-1

Final Design Check

- The static velocity error constant for original system is obtained as follows.

$$K_v = \lim_{s \to 0} sG(s)$$

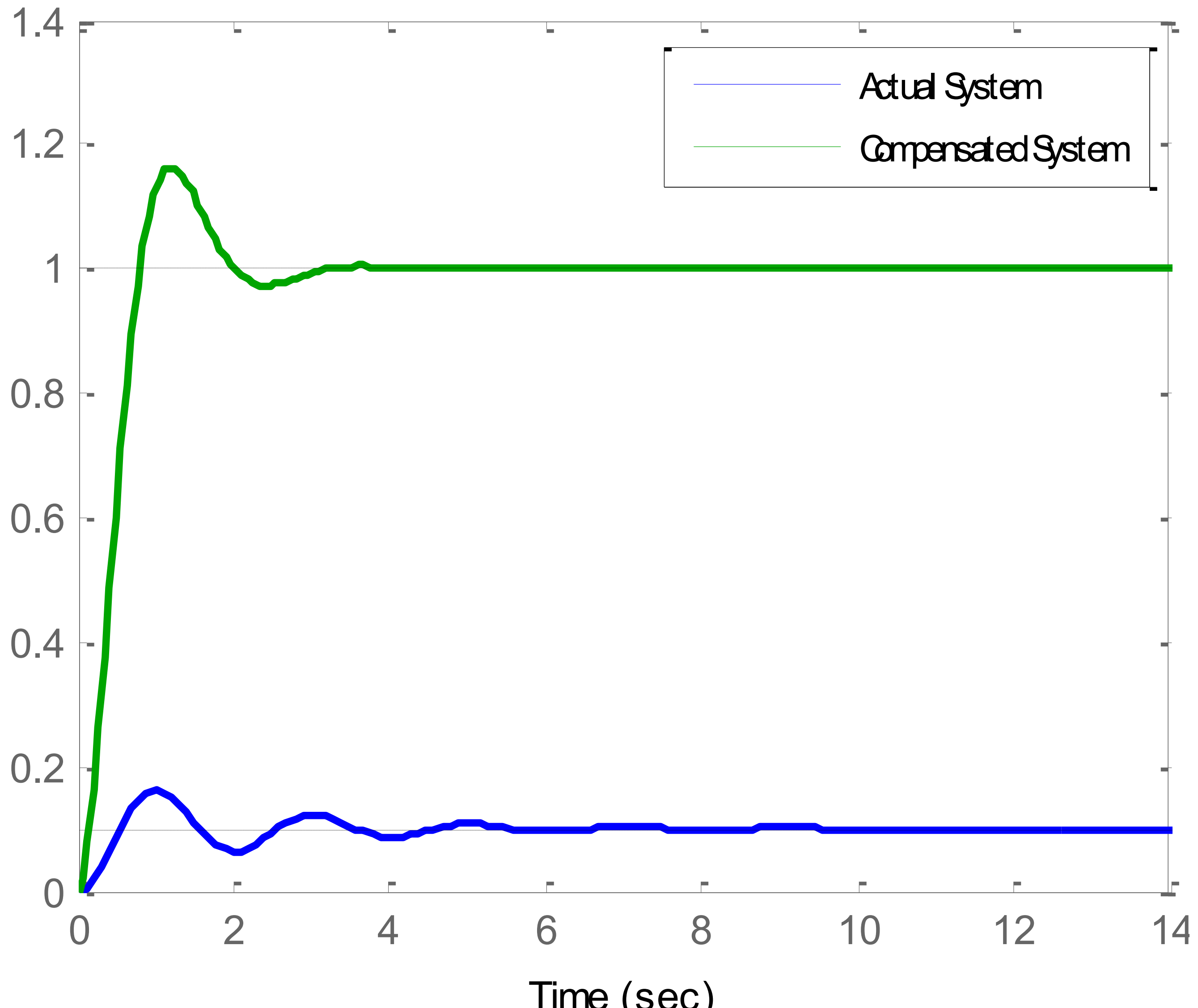
$$K_v = \lim_{s \to 0} s\left[\frac{10}{s(s+1)}\right] = 10$$

- The steady state error is then calculated as

$$e_{ss} = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

Final Design Check

Solution-1



Solution-1

Final Design Check

- The static velocity error constant for the compensated system can be calculated as

$$K_v = \lim_{s \to 0} sG_c(s)G(s)$$

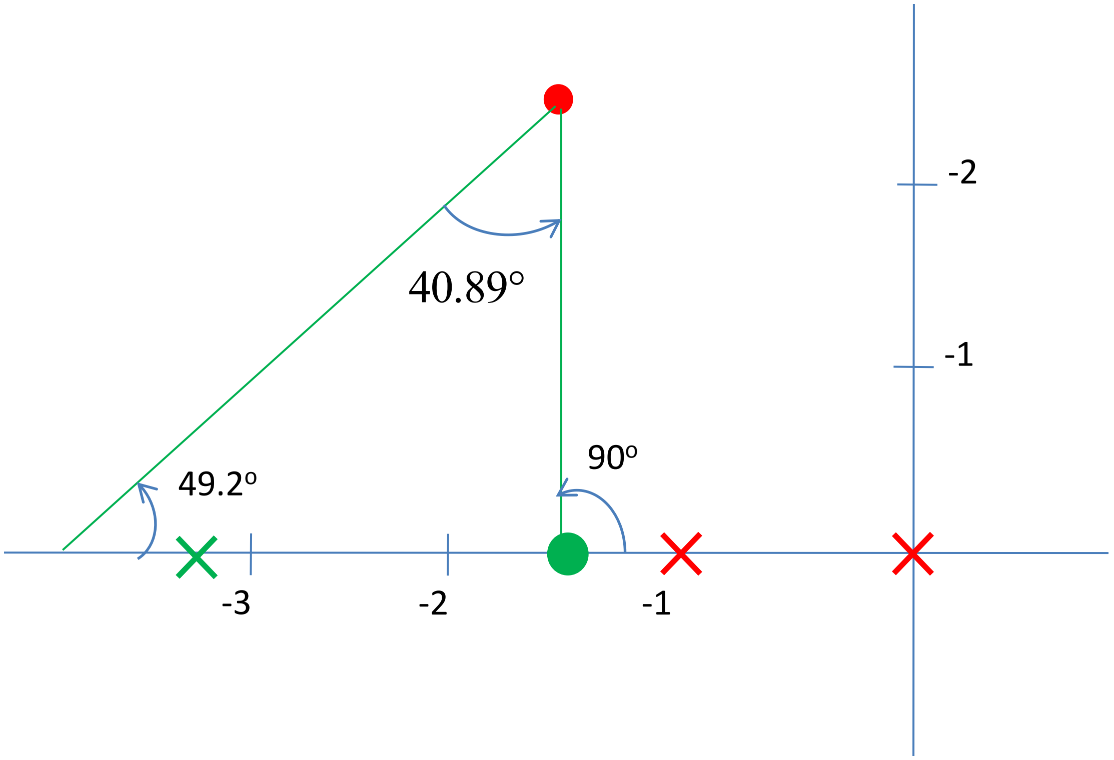
$$K_v = \lim_{s \to 0} s\left[\frac{9}{s(s+3)}\right] = 3$$

- The steady state error is then calculated as

$$e_{ss} = \frac{1}{K_v} = \frac{1}{3} = 0.333$$

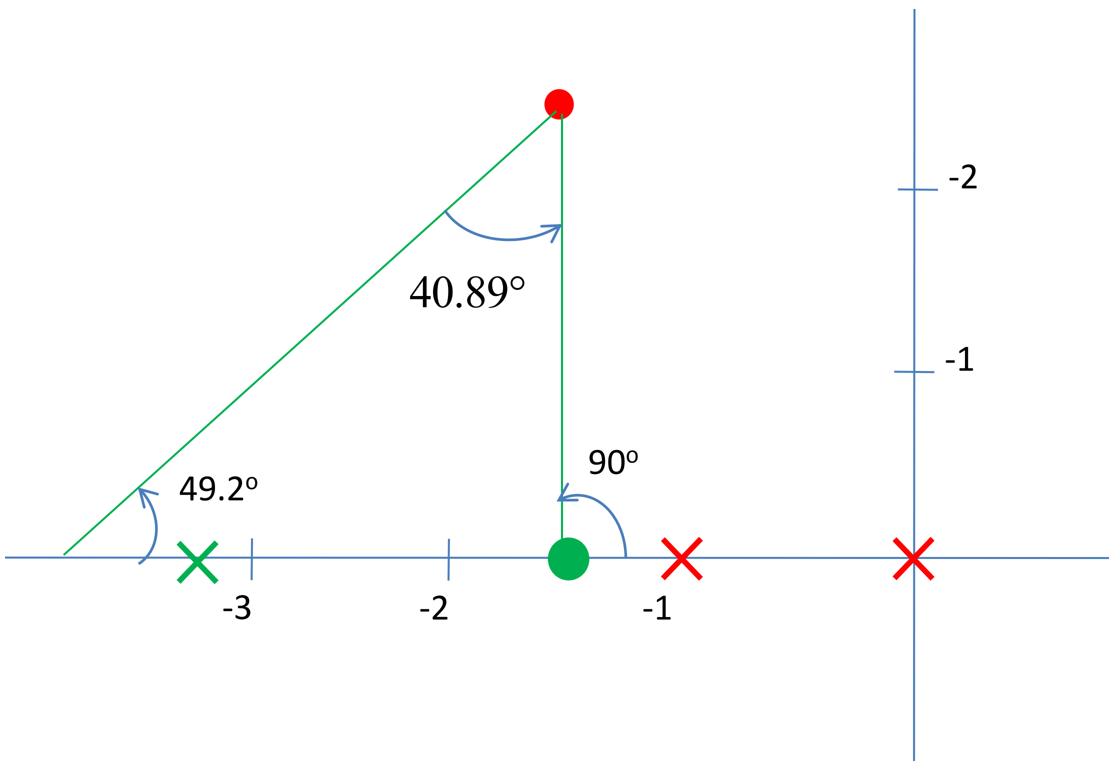
Step-5 (Exampl-1)

- **Solution-2**



Step-5 (Exampl-1)

- **Solution-2**



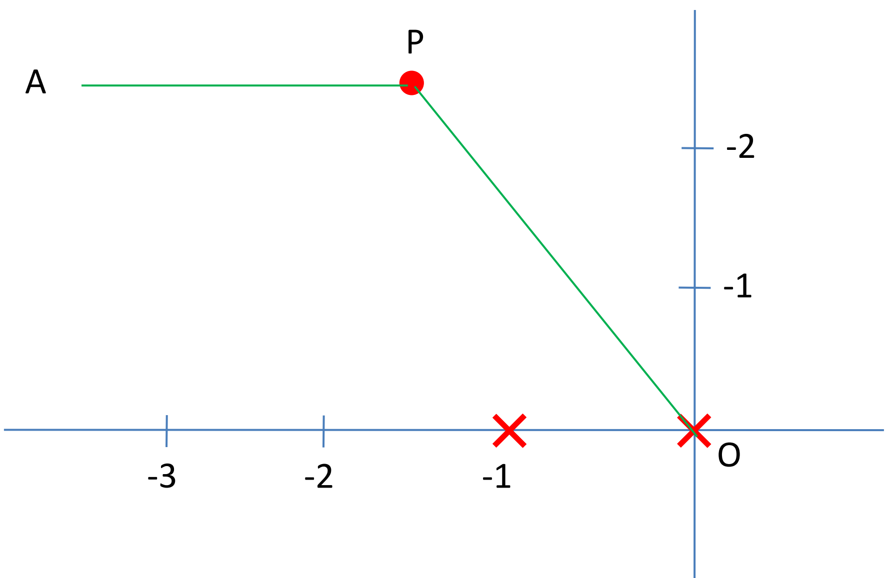
$$G_c(s) = 1.03\,\frac{s + 1.5}{s + 3.6}$$

Step-5 (Example-1)

Solution-3

- If no other requirements are imposed on the system, try to make the value of α as large as possible. A larger value of α generally results in a larger value of K_v, which is desirable.

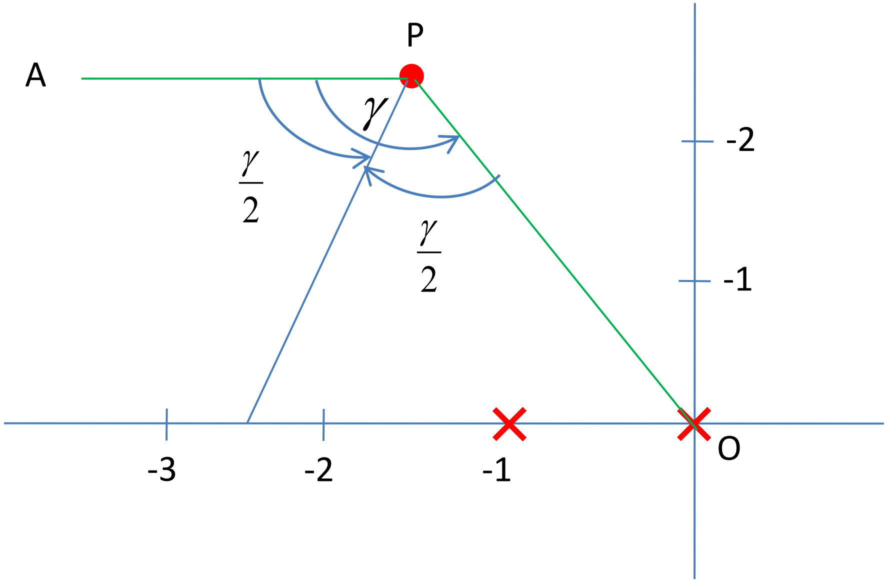
- Procedure to obtain a largest possible value for α.
  - First, draw a horizontal line passing through point *P*, the desired location for one of the dominant closed-loop poles. This is shown as line PA in following figure.
  - Draw also a line connecting point P and the origin O.



Step-5 (Example-1)

Solution-3

- Bisect the angle between the lines PA and PO, as shown in following figure.

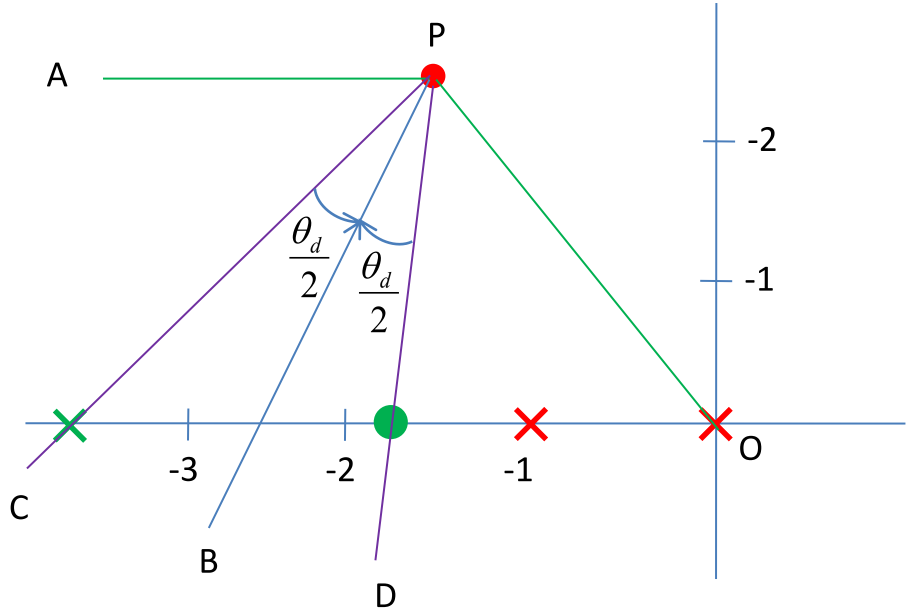


Step-5 (Example-1)

Solution-3

- Draw two lines **PC** and **PD** that make angles $\pm\frac{\theta_d}{2}$ with the the bisector **PB**.

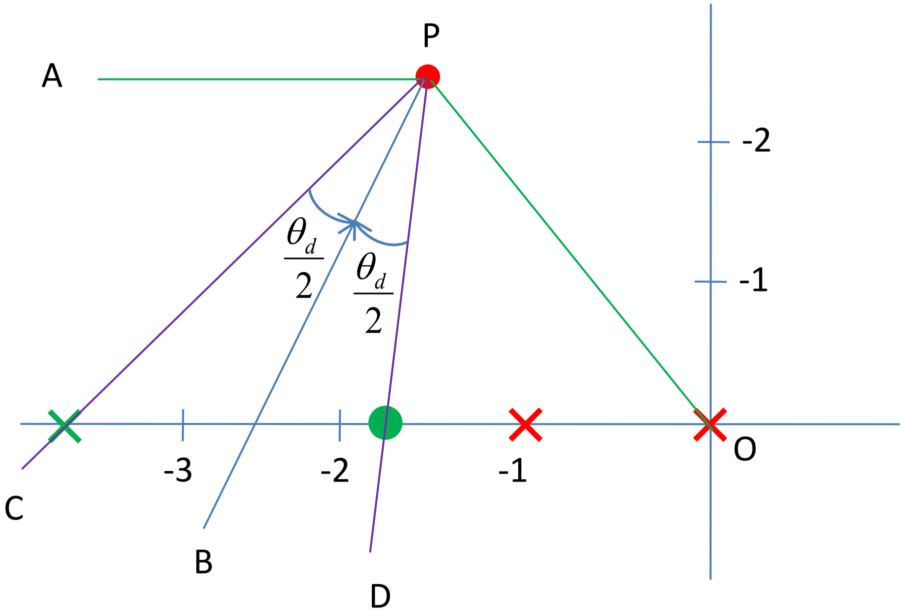
- The intersections of **PC** and **PD** with the negative real axis give the necessary locations for the pole and zero of the lead network.

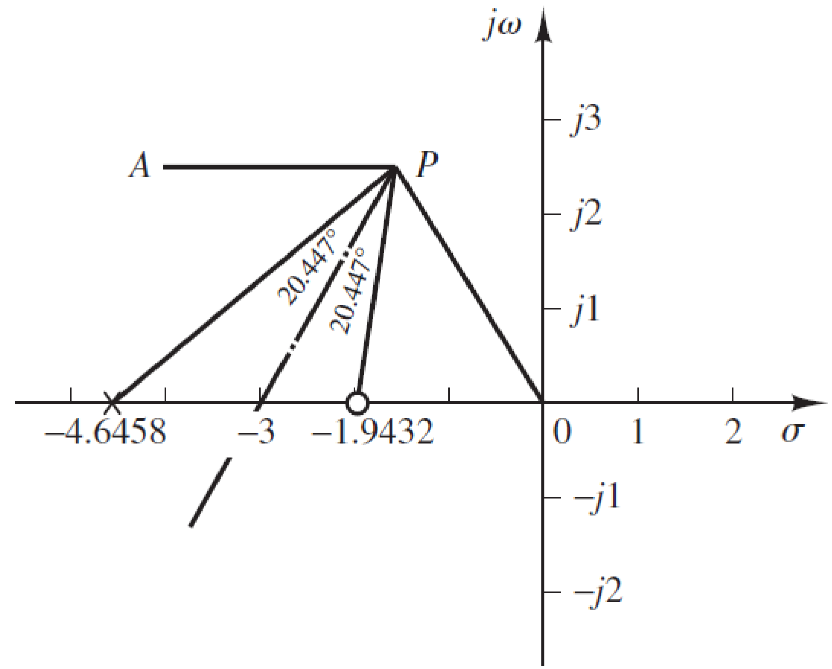


Step-5 (Example-1)

Solution-3

- The lead compensator has zero at s=–1.9432 and pole at s=–4.6458.





- Thus, $G_c(s)$ can be given as

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = K_c \frac{s+1.9432}{s+4.6458}$$

Step-5 (Example-1)

Solution-3

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = K_c \frac{s+1.9432}{s+4.6458}$$

- For this compensator value of α is

$$\frac{1}{T} = 1.9432 \xrightarrow{\text{yields}} T = 0.514$$

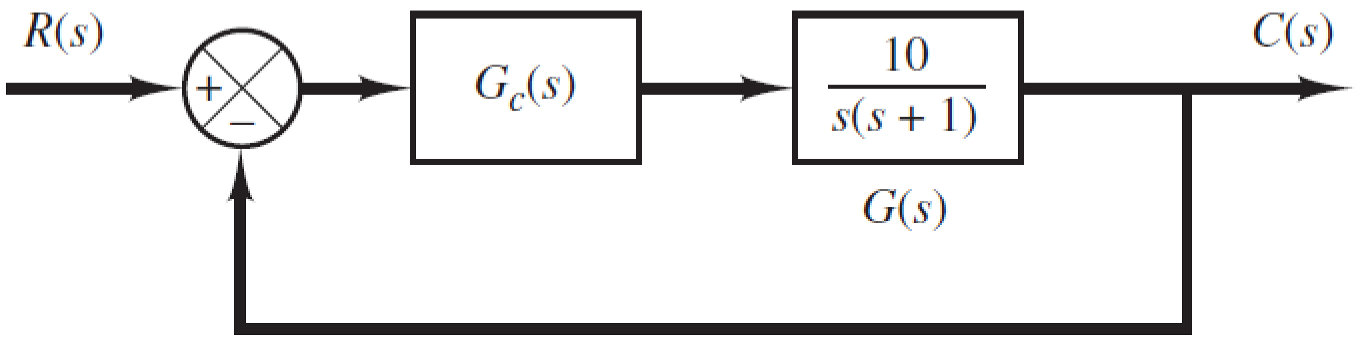
- Also

$$\frac{1}{\alpha T} = 4.6458 \xrightarrow{\text{yields}} \alpha = 0.418$$

Step-6 (Example-1)

Solution-3

- Determine the value of K_c of the lead compensator from the magnitude condition.



$$G(s)G_c(s)H(s) = \frac{10K_c(s+1.9432)}{s(s+1)(s+4.6458)}$$

$$\left|\frac{10K_c(s+1.9432)}{s(s+1)(s+4.6458)}\right|_{s=-1.5+j2.5981} = 1$$

Step-6 (Example-1)

Solution-3

- The K_c is calculated as

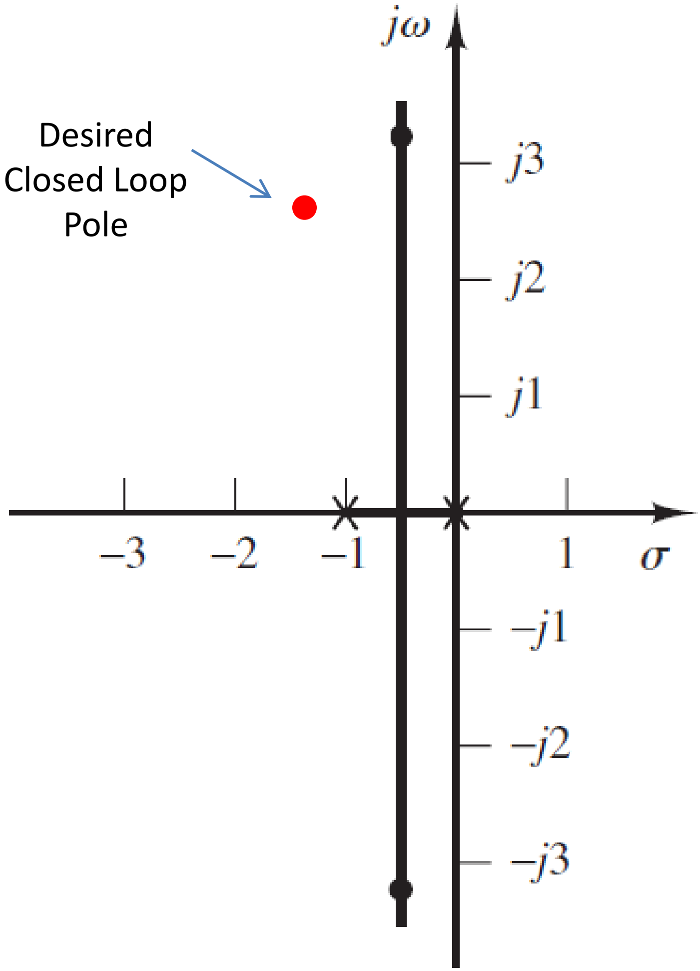
$$K_c = 1.2287$$

- Hence, the lead compensator $G_c(s)$ just designed is given by

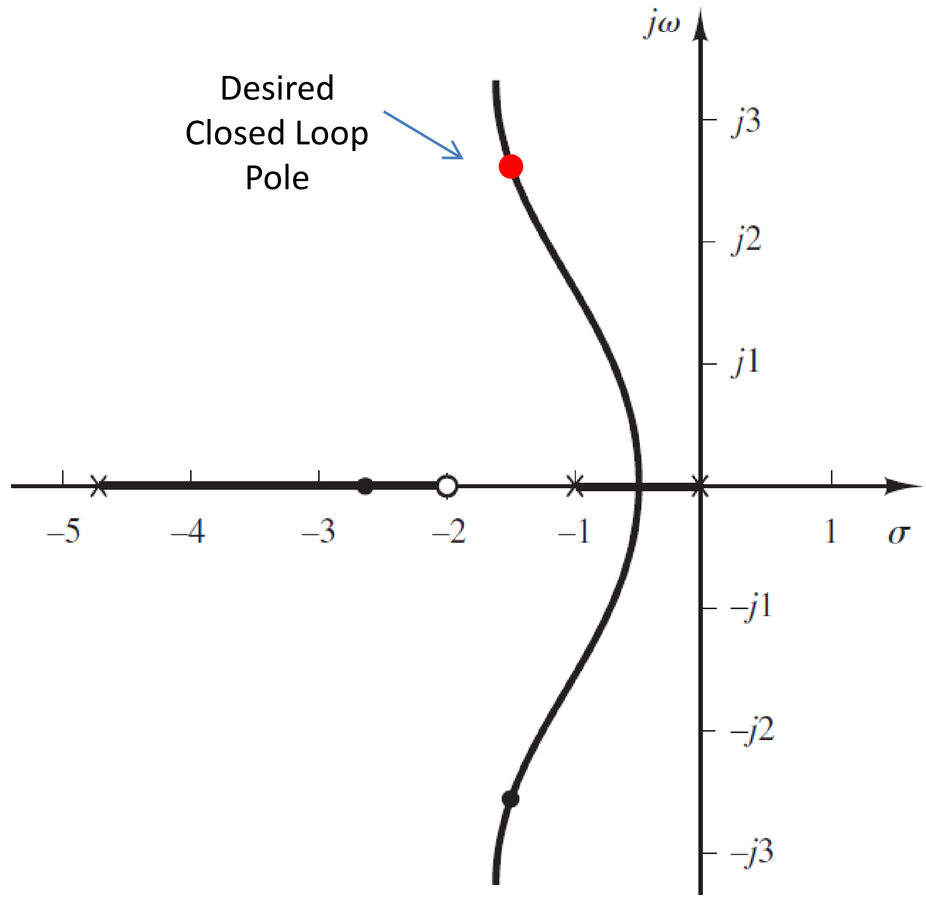
$$G_c(s) = 1.2287\frac{s + 1.9432}{s + 4.6458}$$

Final Design Check

Solution-3



Uncompensated System



Compensated System

Final Design Check

Solution-3

- It is worthwhile to check the static velocity error constant K_v for the system just designed.

$$K_v = \lim_{s\to 0} sG_c(s)G(s)$$

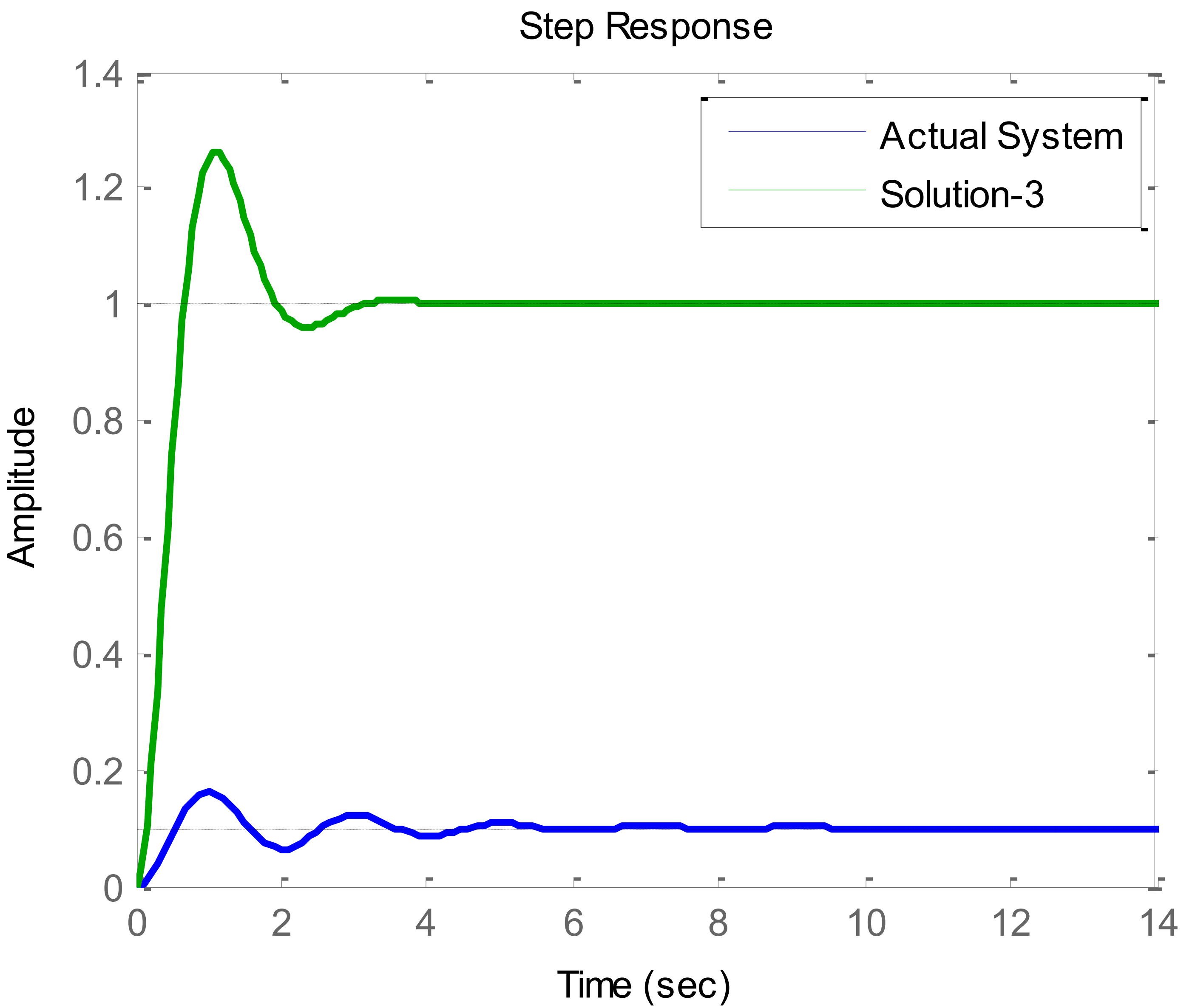
$$K_v = \lim_{s\to 0} s\left[1.2287\frac{s+1.9432}{s+4.6458}\frac{10}{s(s+1)}\right] = 5.139$$

- Steady state error is

$$e_{ss} = \frac{1}{K_v} = \frac{1}{5.139} = 0.194$$

Final Design Check

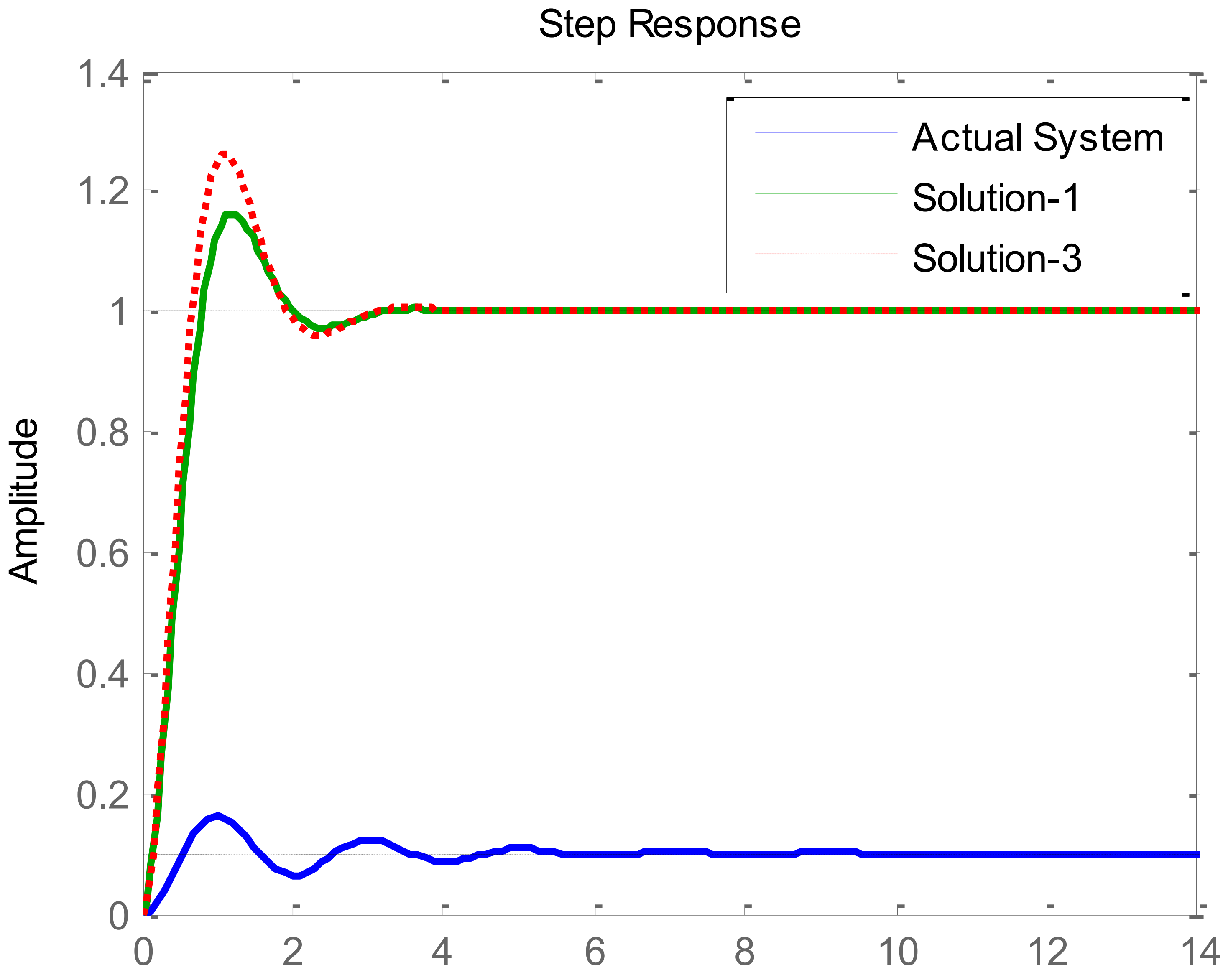
Solution-3



Final Design Check

Solution-1

Solution-3



Mechanical Lead Compensator

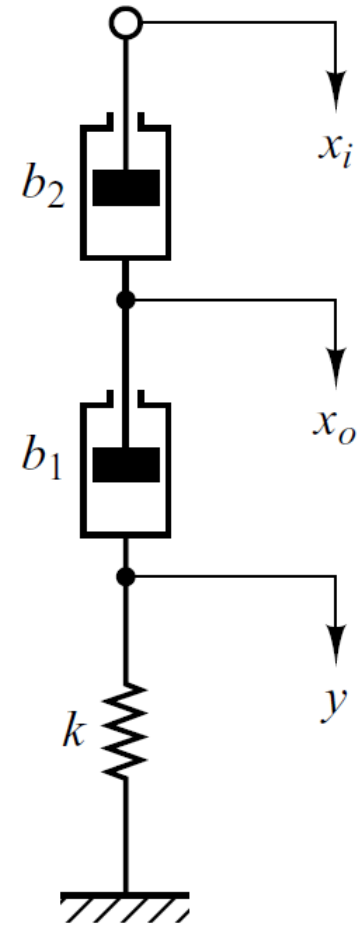
- Figure shows the mechanical lead compensator.
- Equations are obtained as

$$b_2(\dot{x}_i - \dot{x}_o) = b_1(\dot{x}_o - \dot{y})$$

$$b_1(\dot{x}_o - \dot{y}) = ky$$

- Taking Laplace transform of these equatior assuming zero initial conditions and eliminatir Y(s), we obtain

$$\frac{X_o(s)}{X_i(s)} = \frac{b_2}{b_1 + b_2} \frac{\frac{b_1}{k}s + 1}{\frac{b_2}{b_1 + b_2}\frac{b_1}{k}s + 1}$$



Mechanical Lead Compensator

$$\frac{X_o(s)}{X_i(s)} = \frac{b_2}{b_1 + b_2} \frac{\frac{b_1}{k} s + 1}{\frac{b_2}{b_1 + b_2} \frac{b_1}{k} s + 1}$$

- By defining

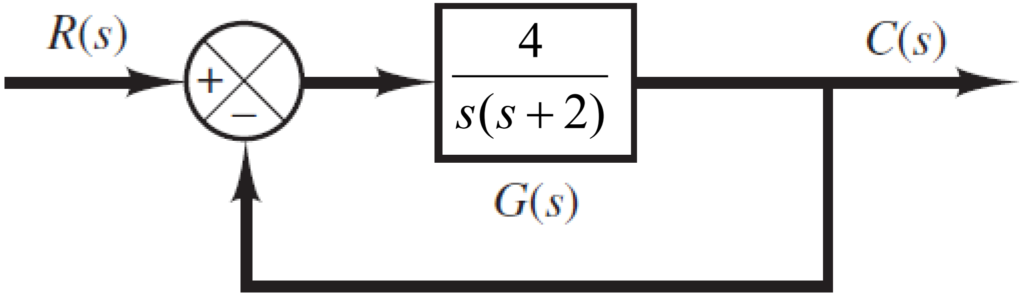
$$\frac{b_1}{k} = T, \qquad \frac{b_2}{b_1 + b_2} = \alpha < 1$$

- We obtain

$$\frac{X_o(s)}{X_i(s)} = \alpha \frac{Ts + 1}{\alpha Ts + 1} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

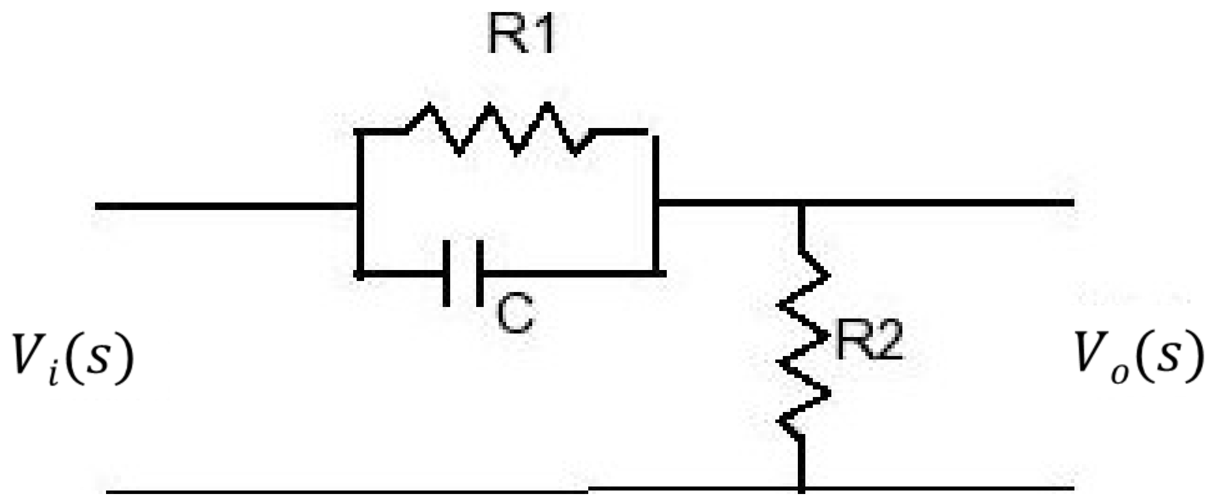
Exampl-2

- Design a mechanical lead compensator for following system.



- The damping ratio of closed loop poles is 0.5 and natural undamped frequency 2 rad/sec. It is desired to modify the closed loop poles so that natural undamped frequency becomes 4 rad/sec without changing the damping ratio.

Electrical Lead Compensator

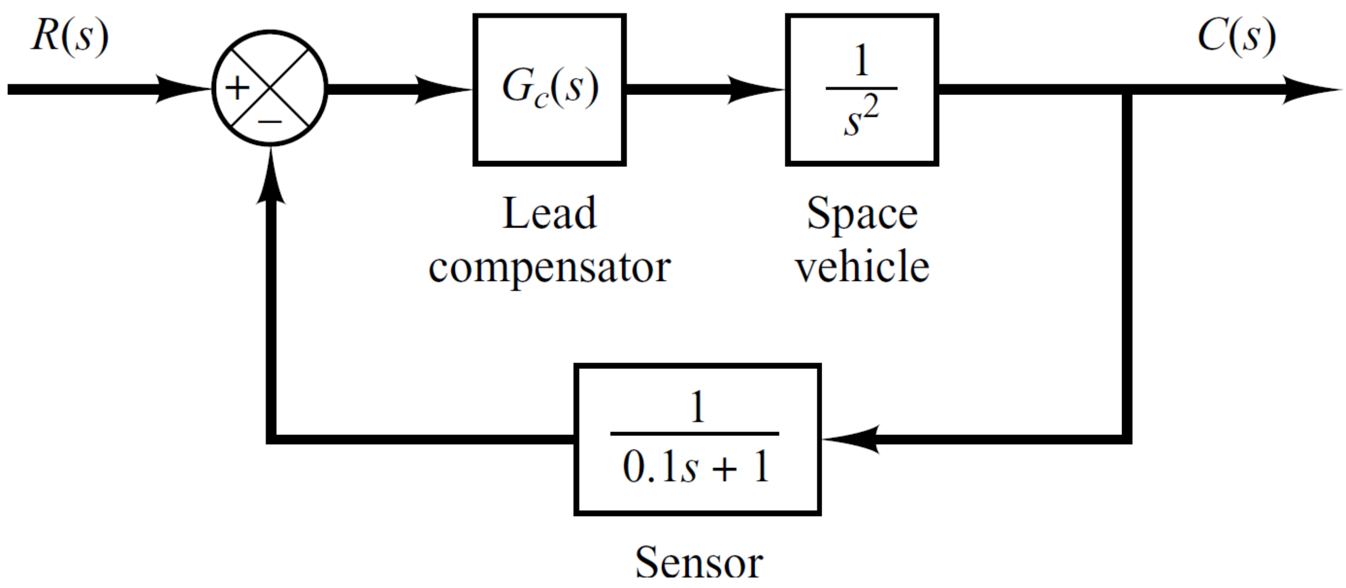


$$\frac{V_o(s)}{V_i(s)}_c = \frac{R_2}{R_1 + R_2} \frac{R_1 C s + 1}{\frac{R_1 R_2}{R_1 + R_2} C s + 1}$$

$T = R_1 \mathrm{C}$ $\quad aT = \frac{R_1 R_2 C}{R_1 + R_2}$ $\quad a = \frac{R_2}{R_1 + R_2}$ $\quad K_c = 1$

Example-3

- Consider the model of space vehicle control system depicted in following figure.



- Design an Electrical lead compensator such that the damping ratio and natural undamped frequency of dominant closed loop poles are 0.5 and 2 rad/sec.