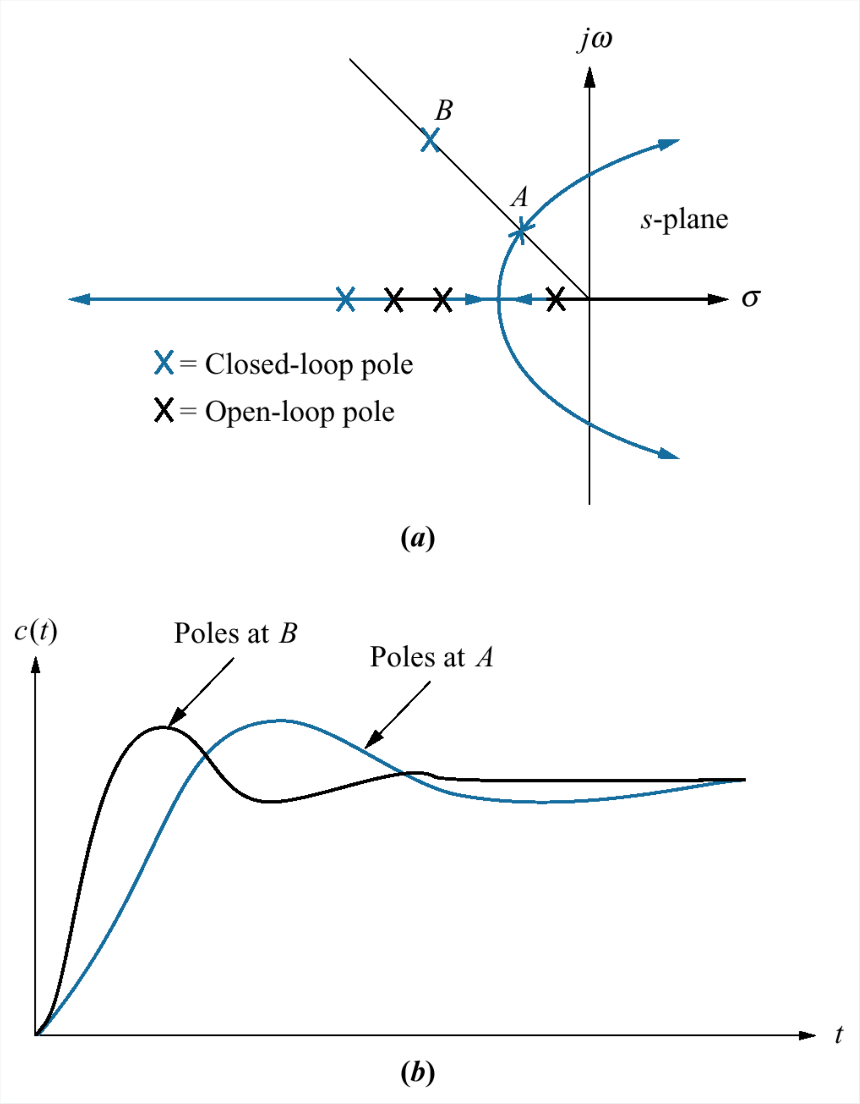
Chapter 9

Design via Root Locus

Improving transient response

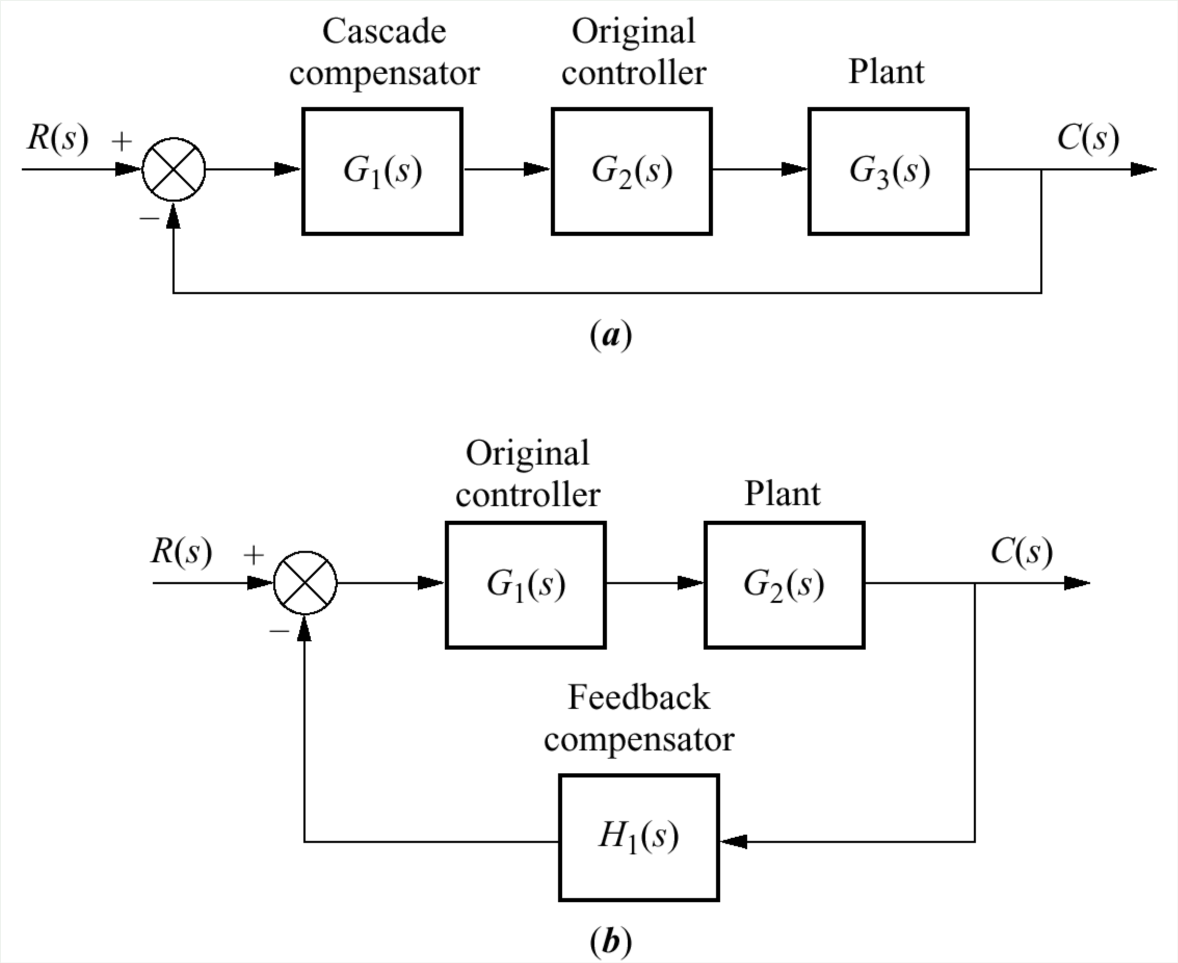
Figure 9.1
a. Sample root locus, showing possible design point via gain adjustment (*A*) and desired design point that cannot be met via simple gain adjustment (*B*);
b. responses from poles at *A* and *B*



Improving steady-state error

Compensation techniques:
a. cascade;
b. feedback

Ideal compensators are implemented with active networks.



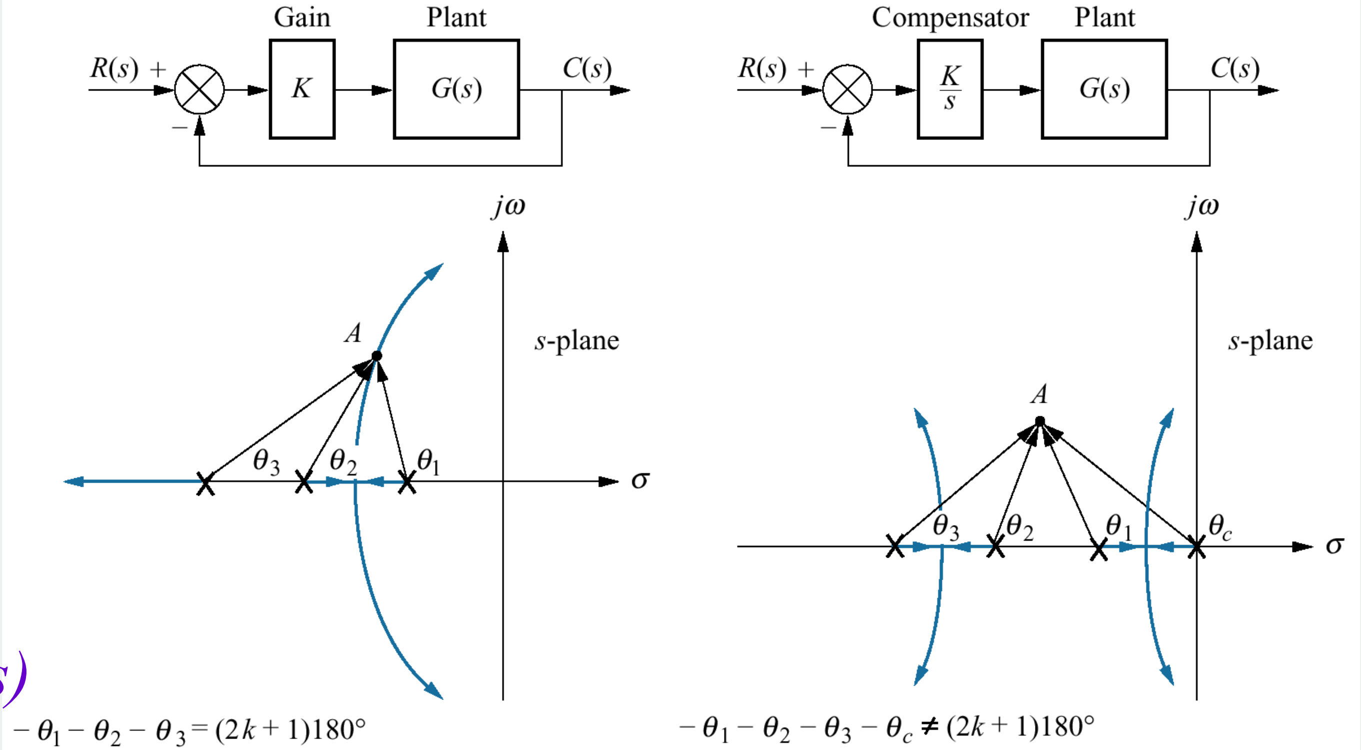
Improving steady-state error via cascade compensation

Pole at A is:

a. on the root locus without compensator;

b. not on the root locus with compensator pole added;

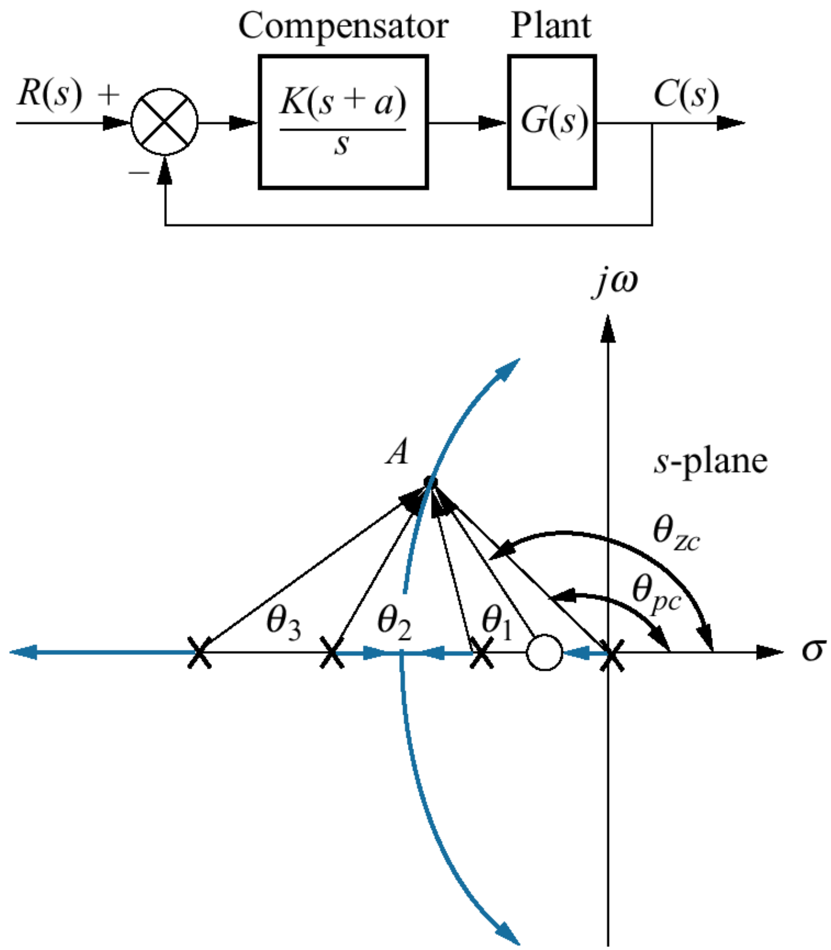
(figure continues)



(a) ***(b)***

Ideal Integral compensation (PI)

c. approximately on the root locus with compensator pole and zero added

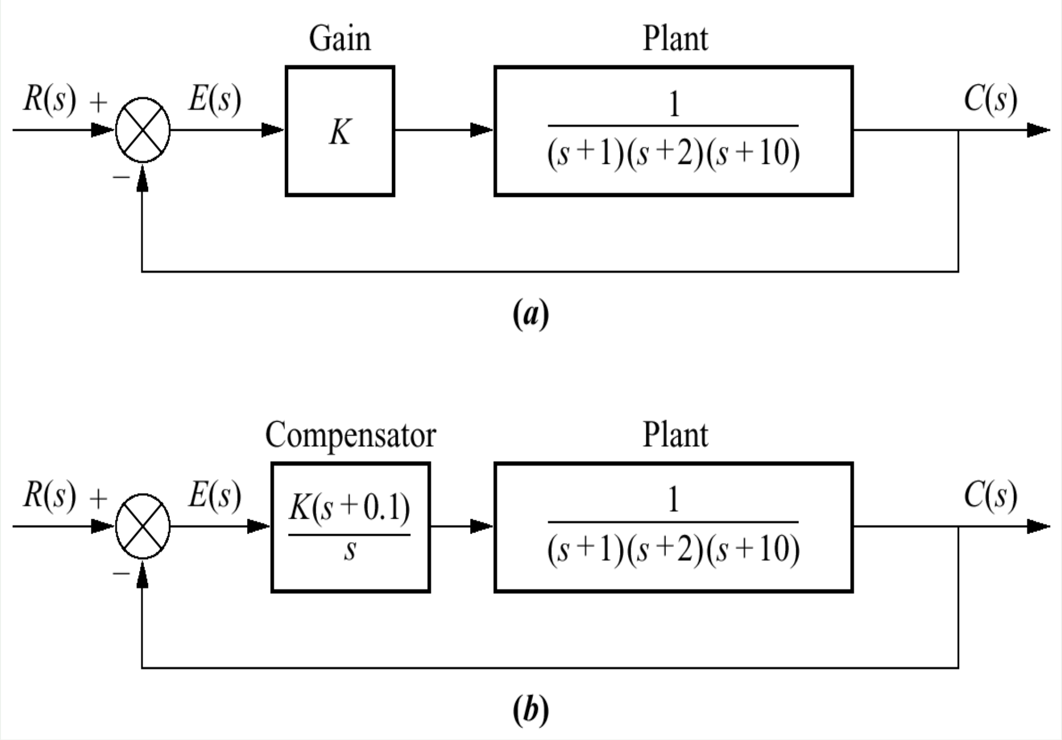


$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k+1)180°$$

(c)

Closed-loop system for Example 9.1

a. before compensation;
b. after ideal integral compensation



Problem: The given system operating with damping ratio of 0.174. Add an ideal integral compensator to reduce the ss error.

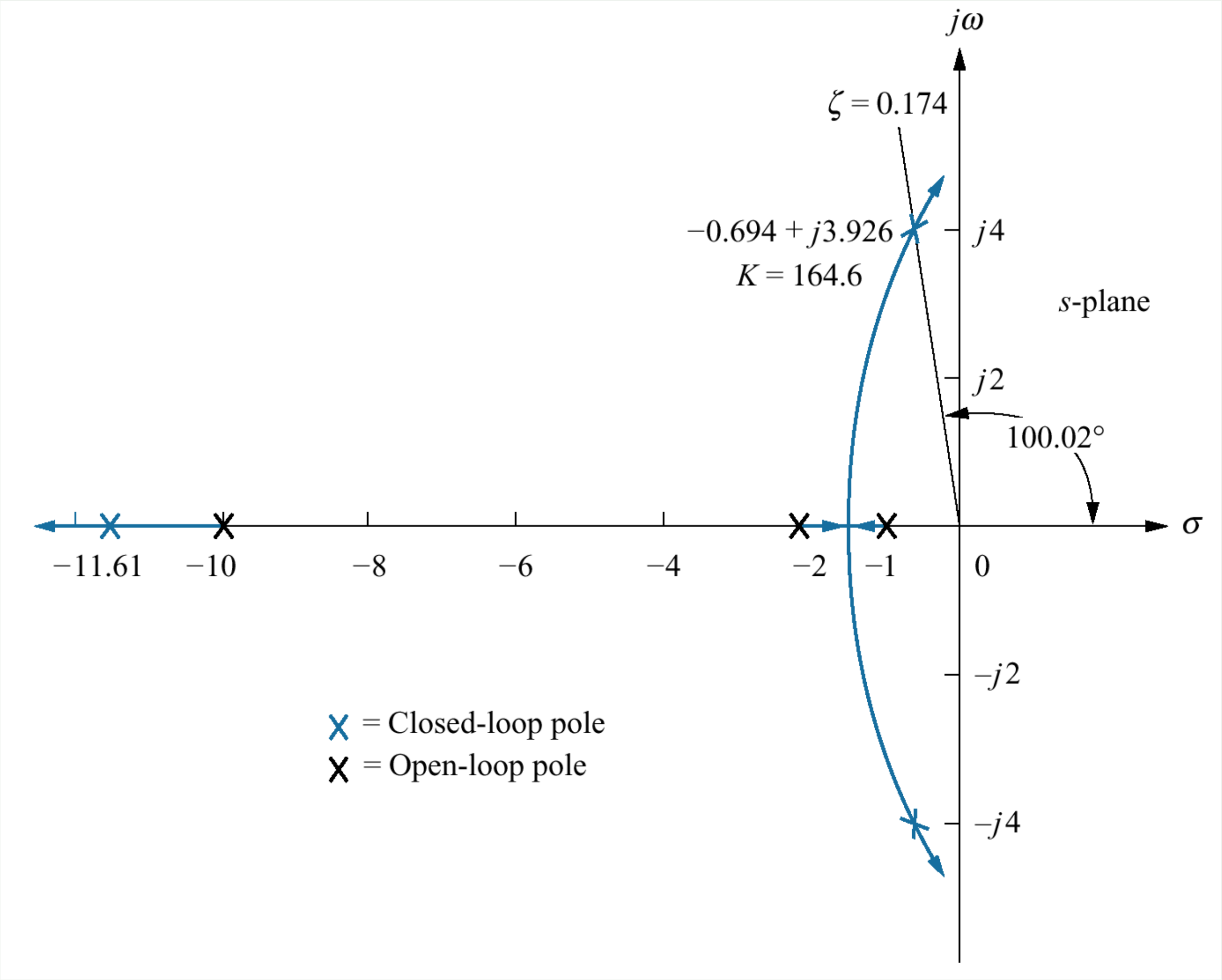
Solution:

We compensate the system by choosing a pole at the origin and a zero at -0.1

Root locus for uncompensated system of Figure 9.4(a)

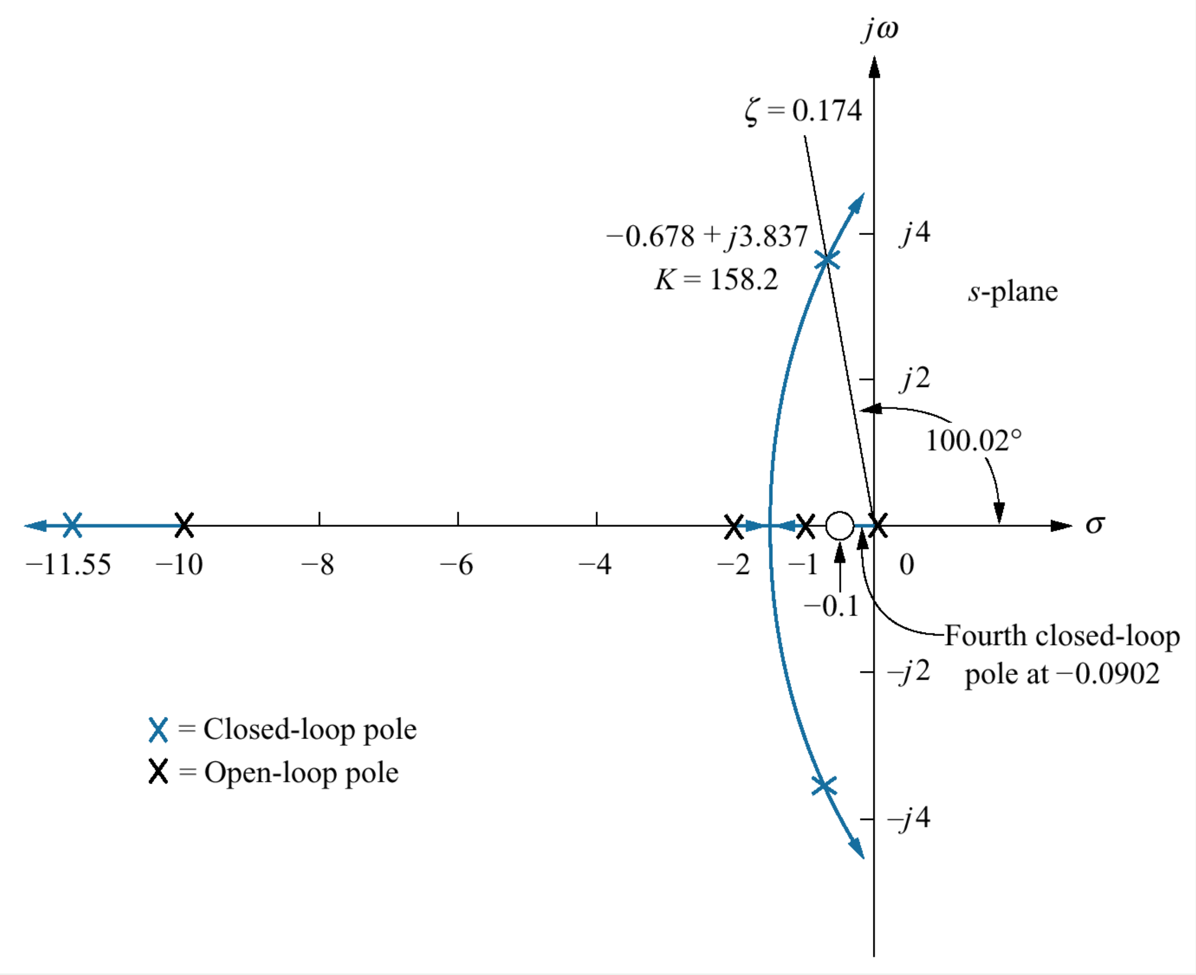
The gain K = 164.6 yields K_p= 8.23 and

$$e(\infty) = \frac{1}{1+K_p} = 0.108$$

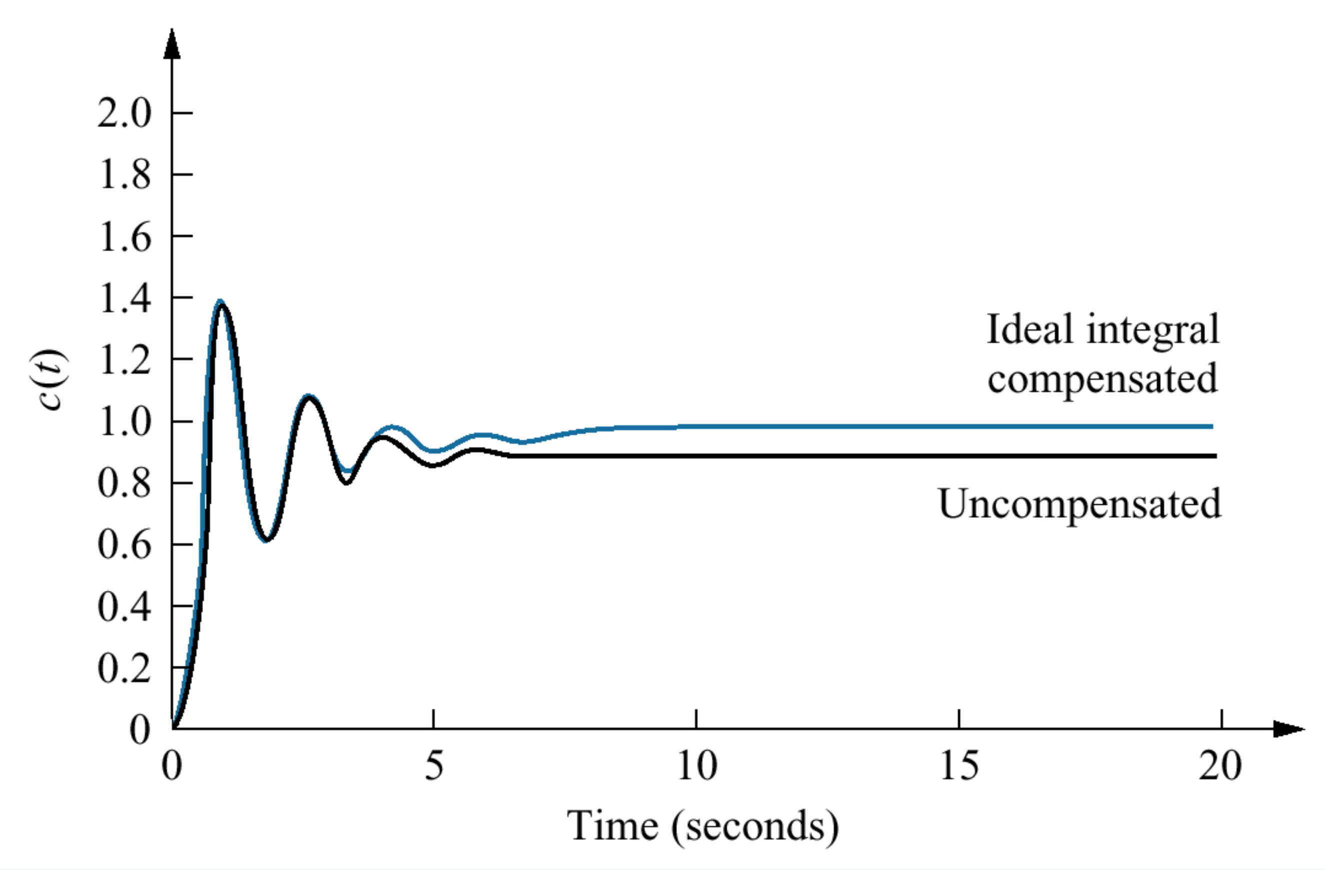


Root locus for compensated system of Figure 9.4(b)

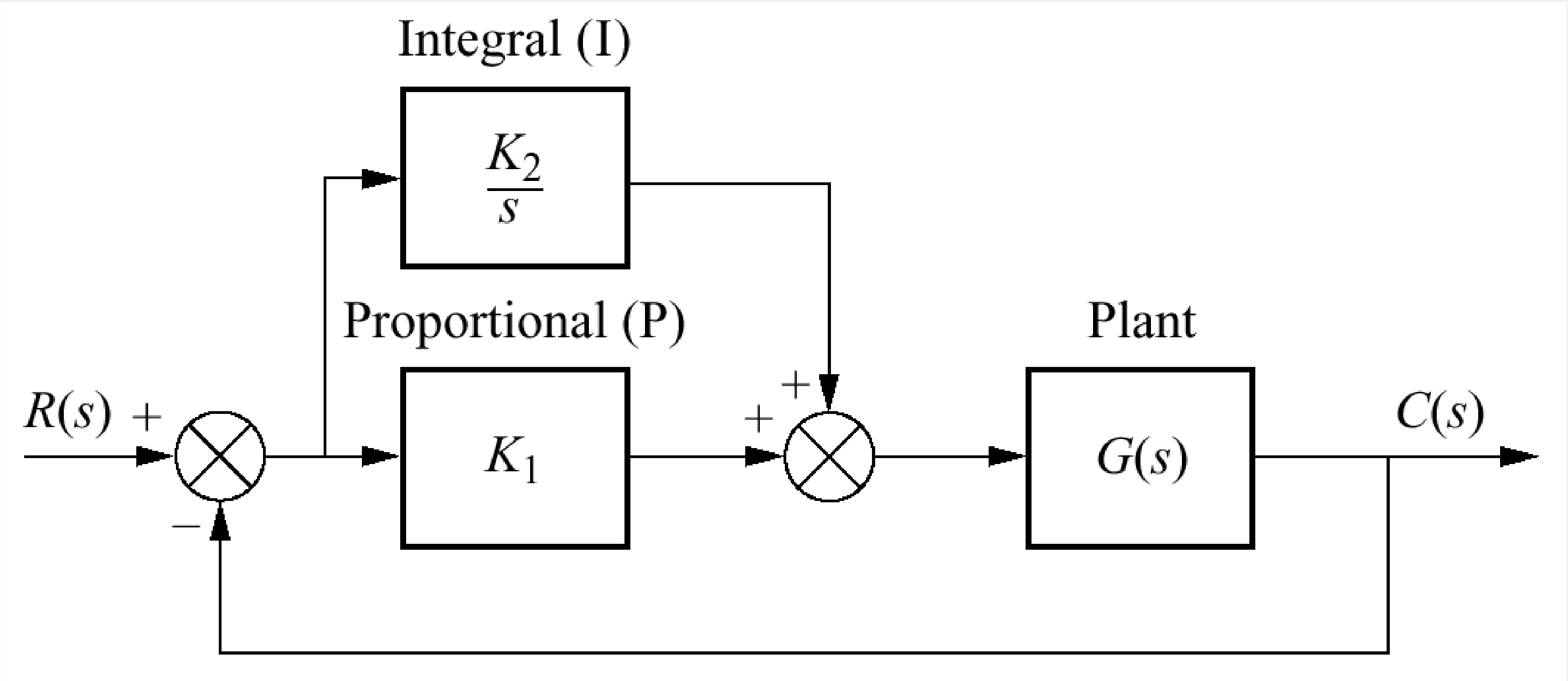
Almost same transient response and gain, but with zero ss error since we have a type one system.



Ideal integral compensated system response and the uncompensated system response of Example 9.1



PI controller



A method to implement an Ideal integral compensator is shown.

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{s}$$

Lag Compensator

a. Type 1 uncompensated system;
b. Type 1 compensated system;
c. compensator pole-zero plot

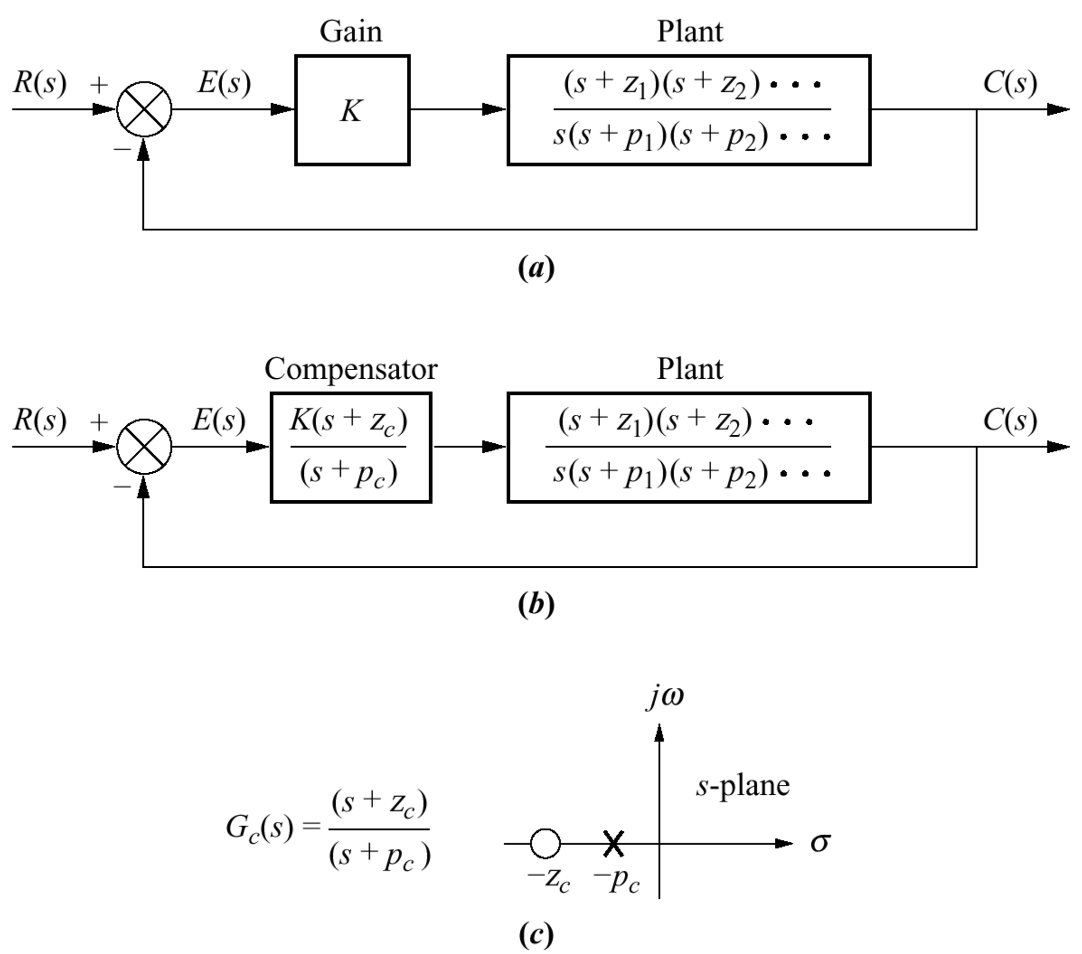
Using passive networks, the compensation pole and zero is moved to the left, close to the origin.

The static error constant for uncompensated system is

$$K_{vo} = \frac{K z_1 z_2 \ldots.}{p_1 p_2 \ldots\ldots.}$$

Assuming the compensator is used as in b & c the static error is

$$K_{vN} = \frac{(K z_1 z_2 \ldots.)(z_c)}{(p_1 p_2 \ldots\ldots.)(p_c)}$$

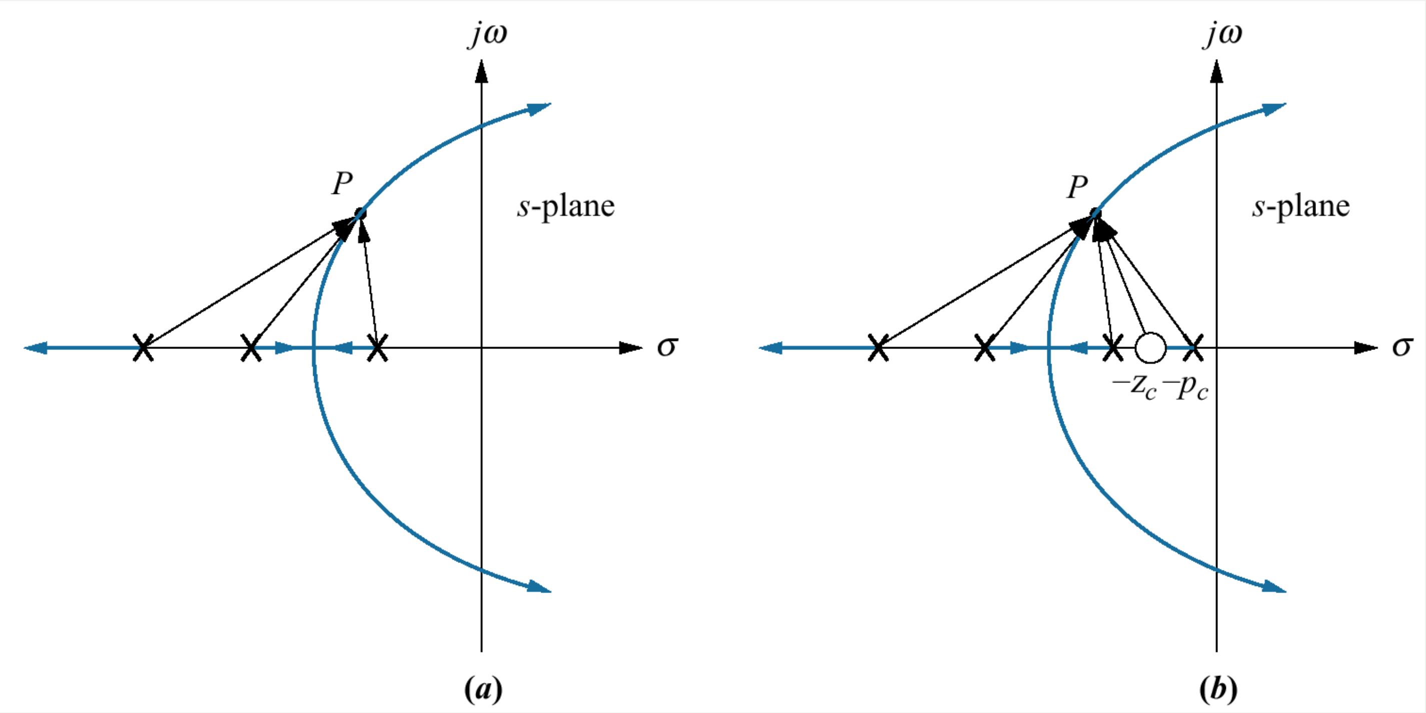


Effect on transient response

Root locus:

a. before lag compensation;

b. after lag compensation

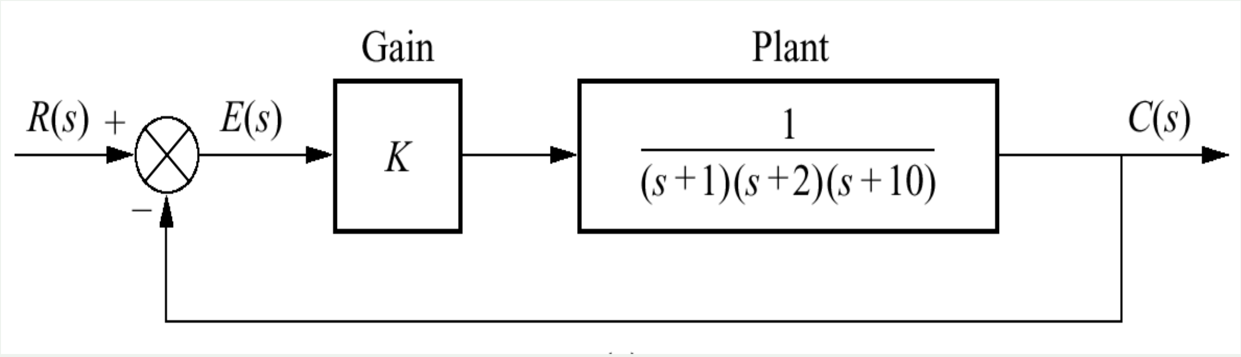


Almost no change on the transient response and same gain K. While the ss error is effected since

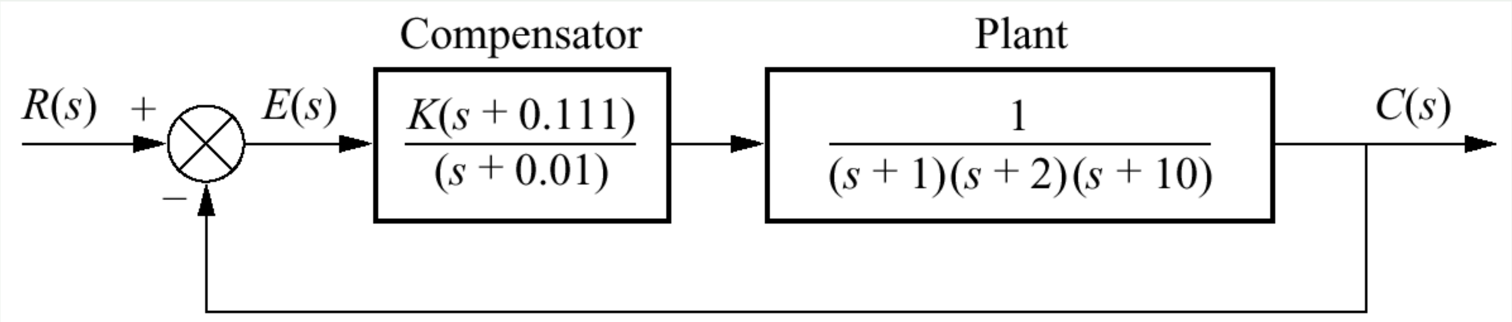
$$K_{vN} = K_{vo}\frac{z_c}{p_c} > K_{vo}$$

Lag compensator design Example 9.2

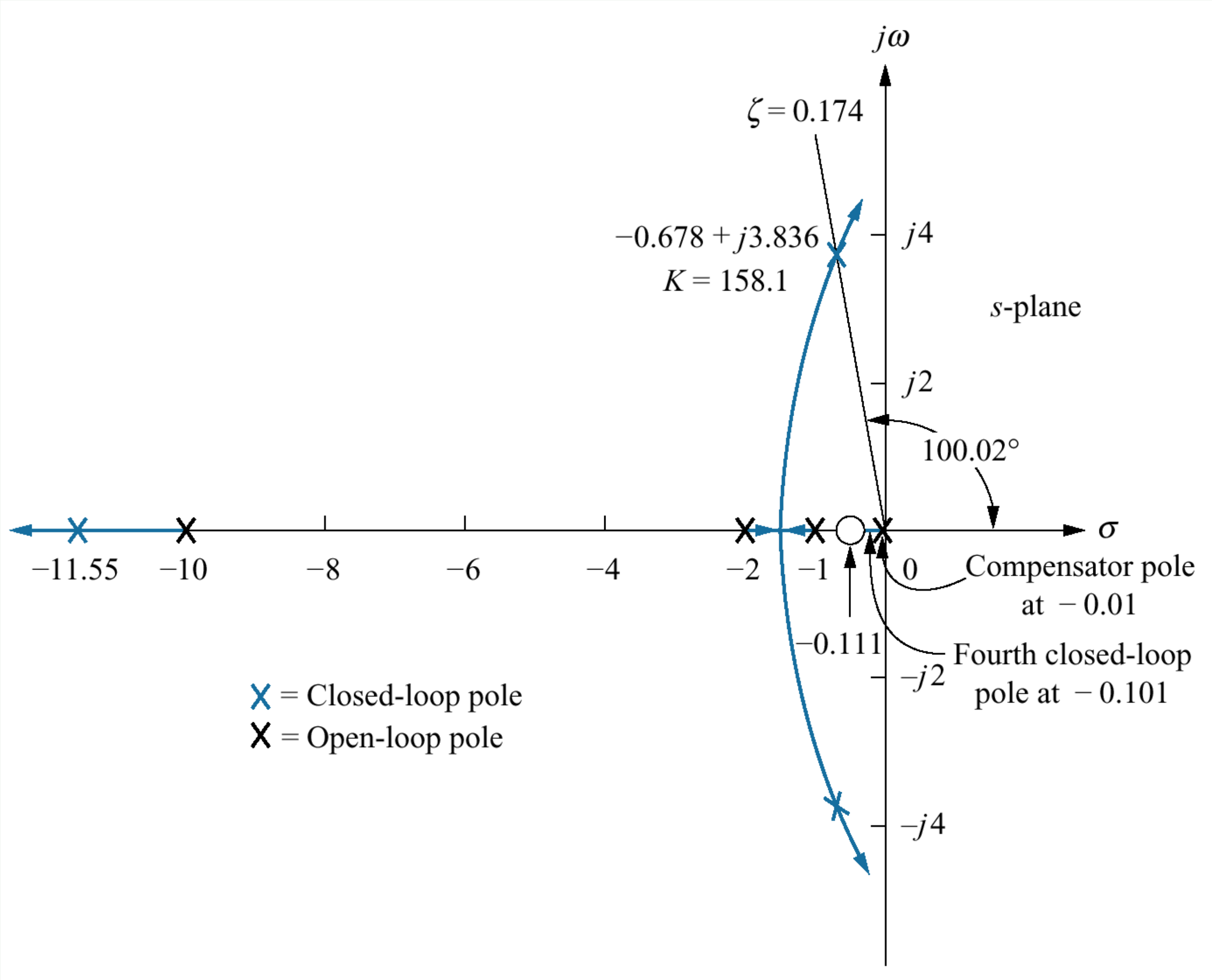
Problem: Compensate the shown system to improve the ss error by a factor of 10 if the system is operating with a damping ratio of 0.174



Solution: the uncompensated system error from previous example is 0.108 with $K_p = 8.23$. a ten fold improvement means ss error = 0.0108 so $Kp = 91.59$. so the ratio $\frac{z_c}{p_c} = \frac{K_{PN}}{K_{Po}} = \frac{91.59}{8.23} = 11.13$ arbitrarily selecting $P_c = 0.01$ and $Z_c = 11.13P_c \approx 0.111$



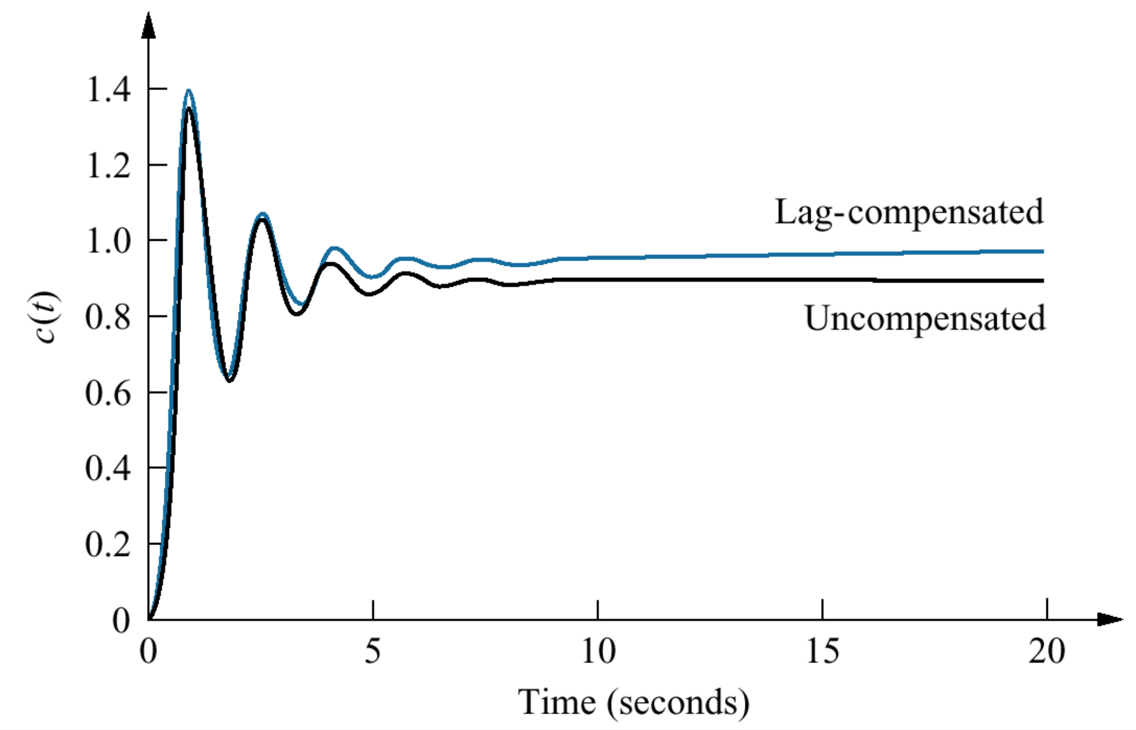
Root locus for compensated system



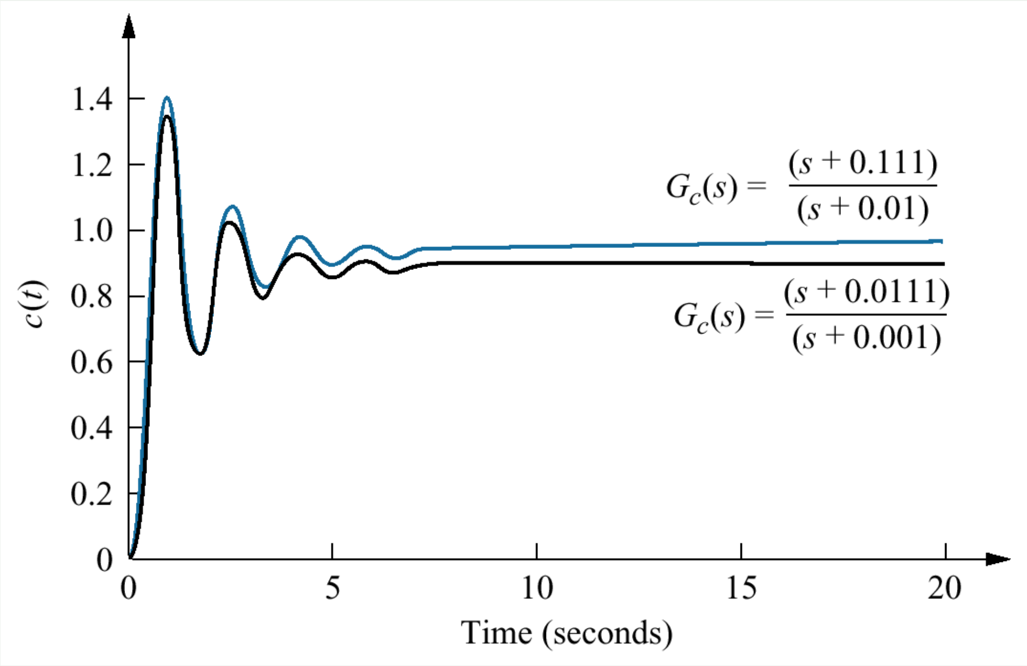
Predicted characteristics of uncompensated and lag-compensated systems for Example 9.2

Parameter	**Uncompensated**	**Lag-compensated**
Plant and compensator	$\dfrac{K}{(s+1)(s+2)(s+10)}$	$\dfrac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
K_p	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111

Step responses of uncompensated and lag-compensated systems for Example 9.2



Step responses of the system for Example 9.2 using different lag compensators



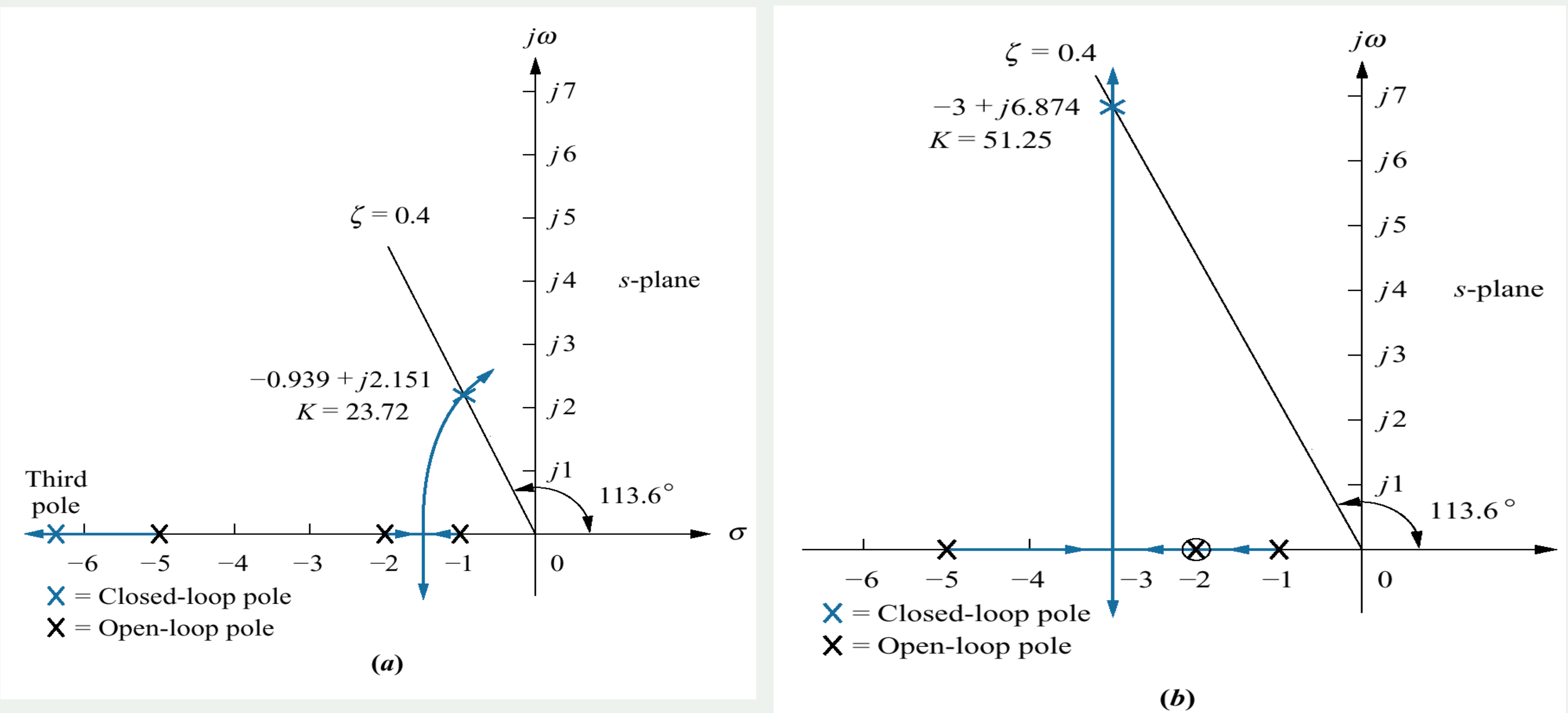
Improving Transient response via Cascade Compensation

Ideal Derivative compensator is called PD controller
When using passive network it's called lead compensator

Using ideal derivative compensation: $$G_c(s) = s + z_c$$

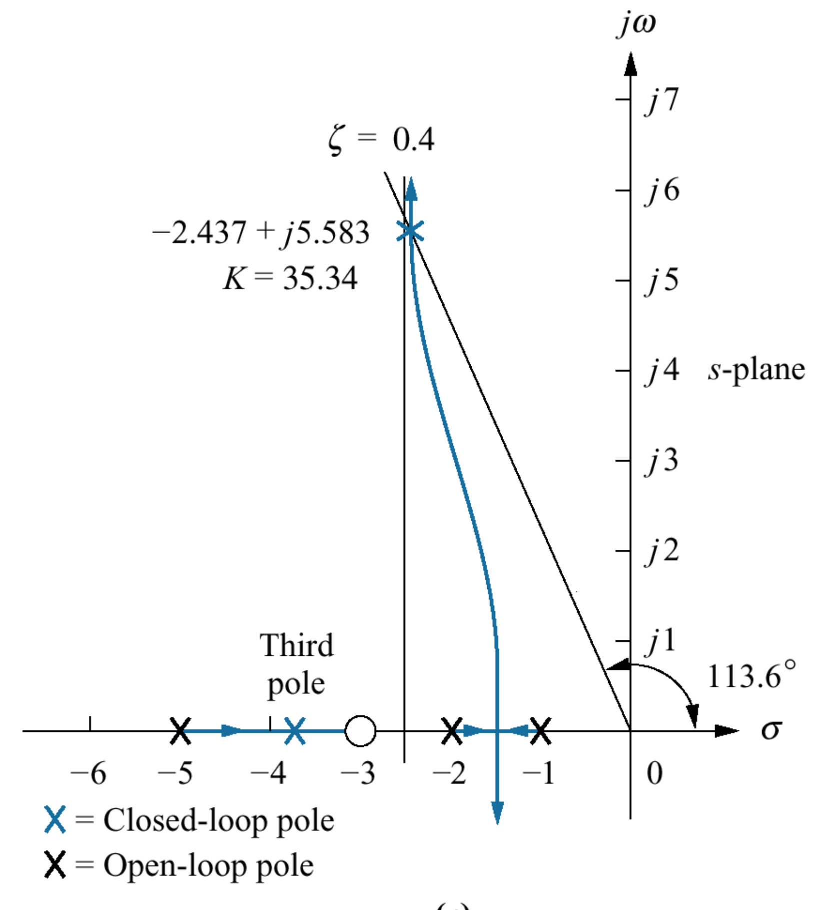
a. uncompensated;

b. compensator zero at –2;

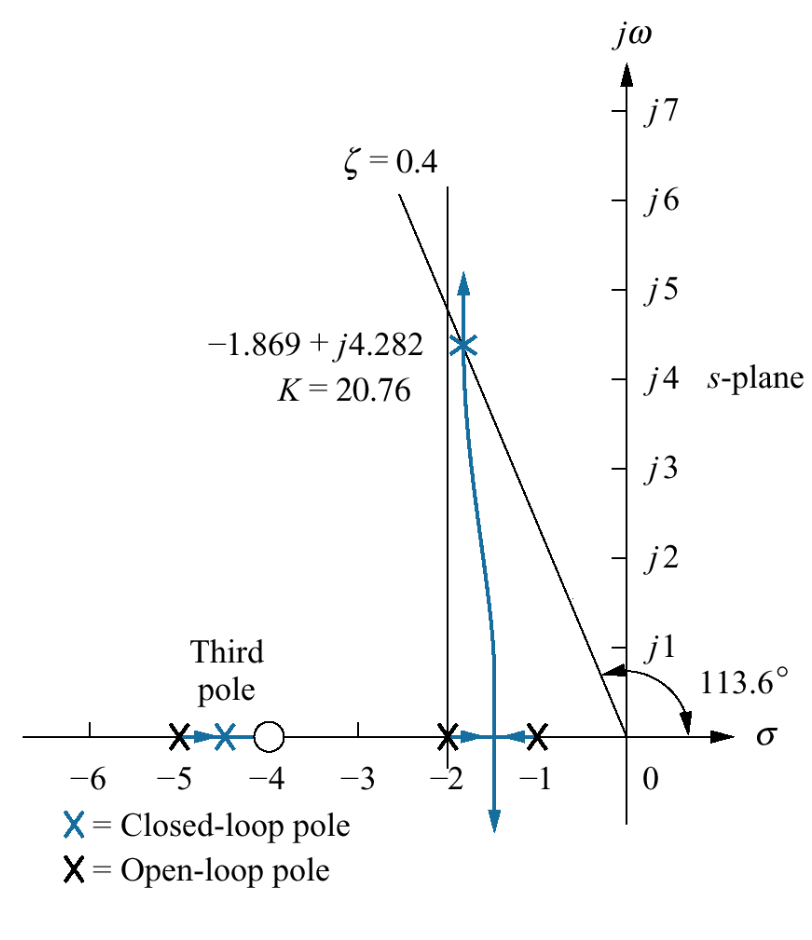


Improving Transient response via Cascade Compensation

c. Compensator zero at –3;
d. Compensator zero at – 4



(c)



(d)

Uncompensated system and ideal derivative compensation solutions from Table 9.2

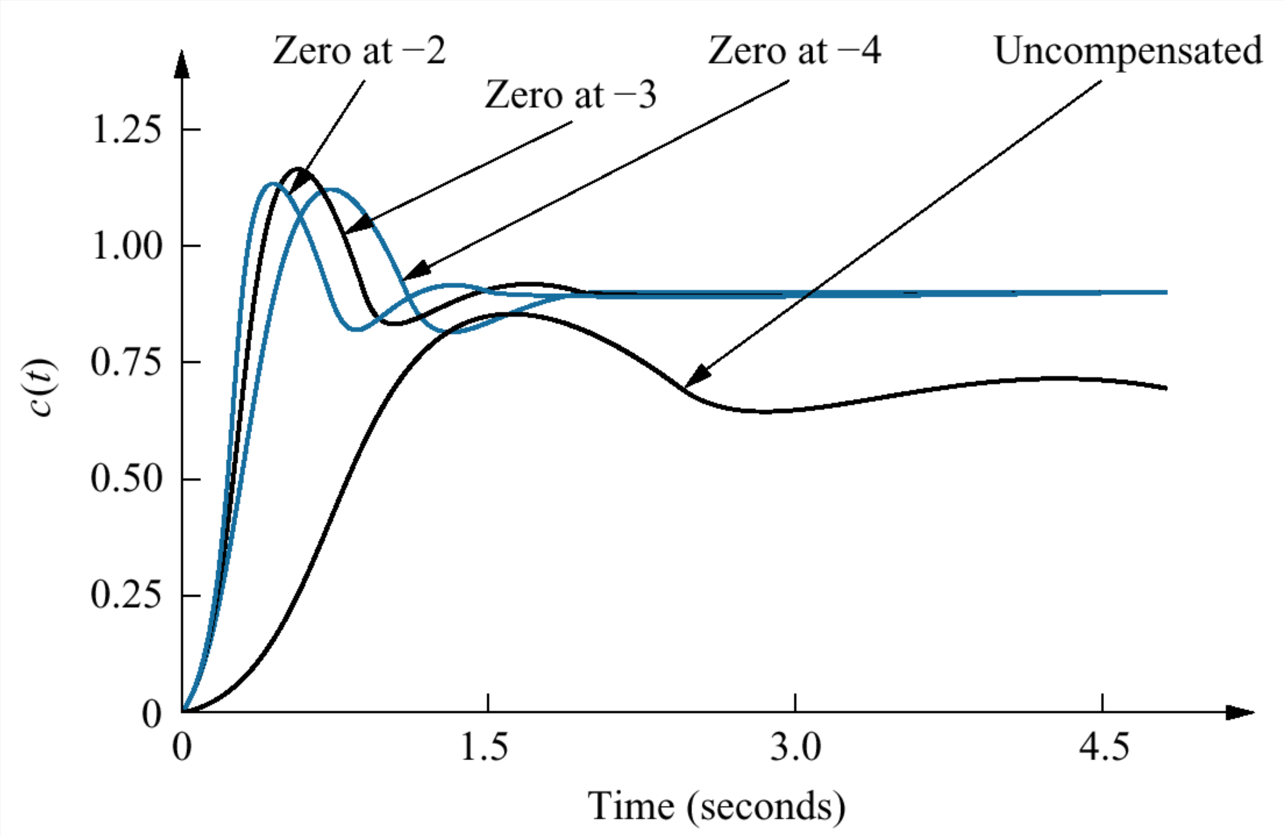
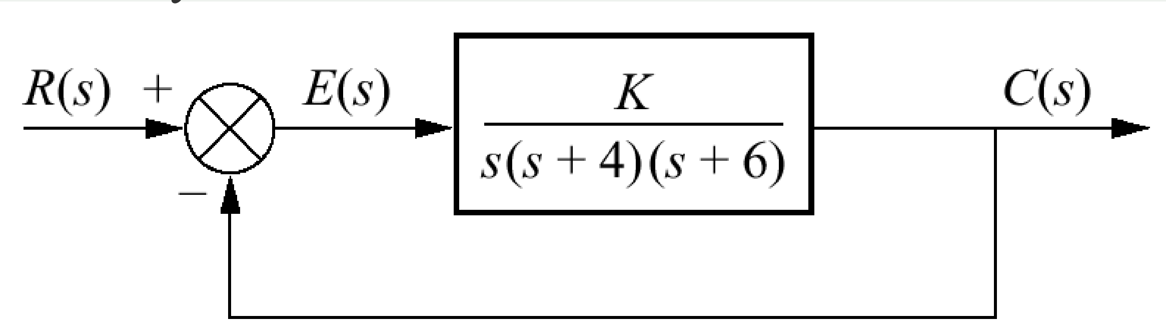


Table 9.2 Predicted characteristics for the systems of previous slides

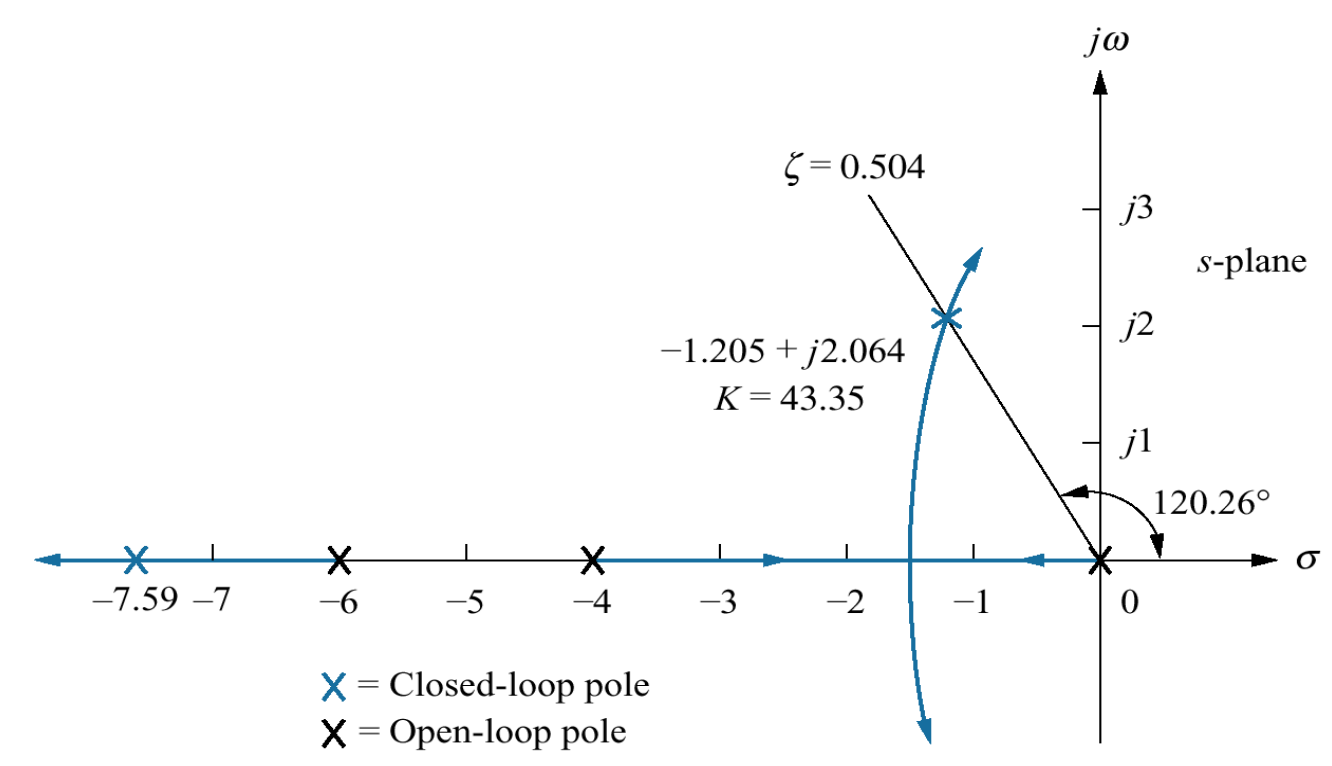
	Uncompensated	**Compensation b**	**Compensation c**	**Compensation d**
Plant and compensator	$\dfrac{K}{(s+1)(s+2)(s+5)}$	$\dfrac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\dfrac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\dfrac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
$\%OS$	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	-6.123	None	-3.127	-4.262
Zero	None	None	-3	-4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK

Feedback control system for Example 9.3

Problem: Given the system in the figure, design an ideal derivative compensator to yield a 16% overshoot with a threefold reduction in settling time.



Root locus for uncompensated system of Example 9.3



Compensated dominant pole superimposed over the uncompensated root locus for Example 9.3

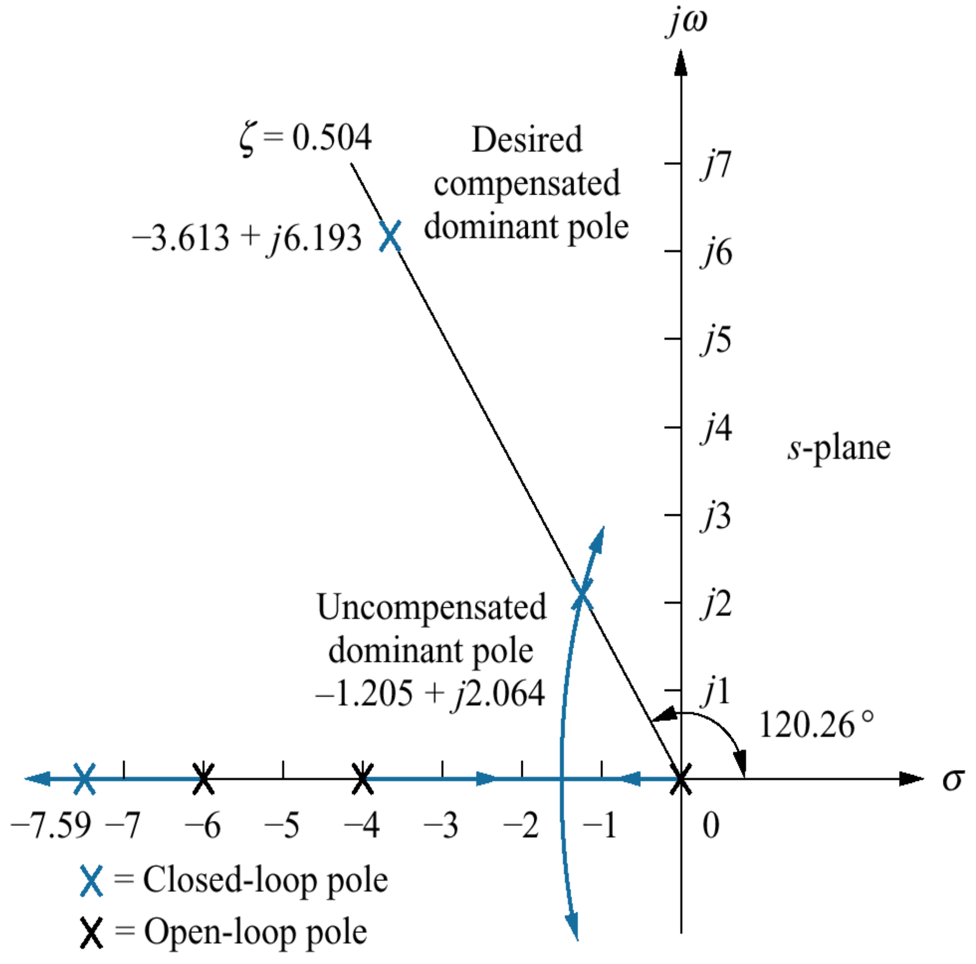
The settling time for the uncompensated system shown in next slide is $T_s = \frac{4}{\xi\omega_n} = \frac{4}{1.205} = 3.320$

In order to have a threefold reduction in the settling time, the settling time of the compensated system will be one third of 3.32 that is 1.107, so the real part of the compensated system's dominant second order pole is $\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$

And the imaginary part is

$$\omega_d = 3.613\tan(180^\circ - 120.26^\circ) = 6.193$$

The figure shows the designed dominant 2nd order poles.



Evaluating the location of the compensating zero for Example 9.3

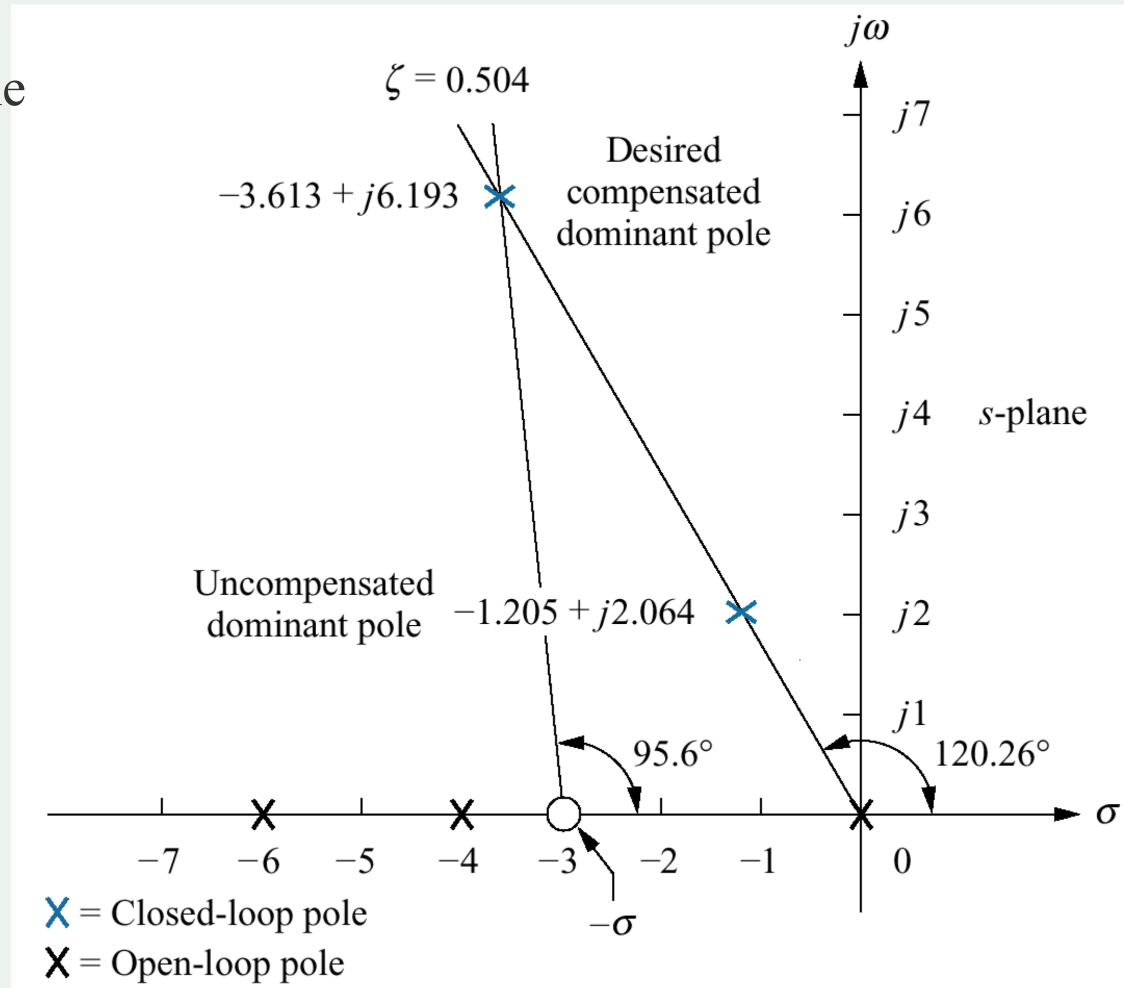
The sum of angles from all poles to the desired compensated pole -3.613+j6.193 is -275.6

The angle of the zero to be on the root locus is 275.6-180=95.6

The location of the compensator zero is calculated as

$$\frac{6.193}{3.613-\sigma} = \tan(180^\circ - 95.6^\circ)$$

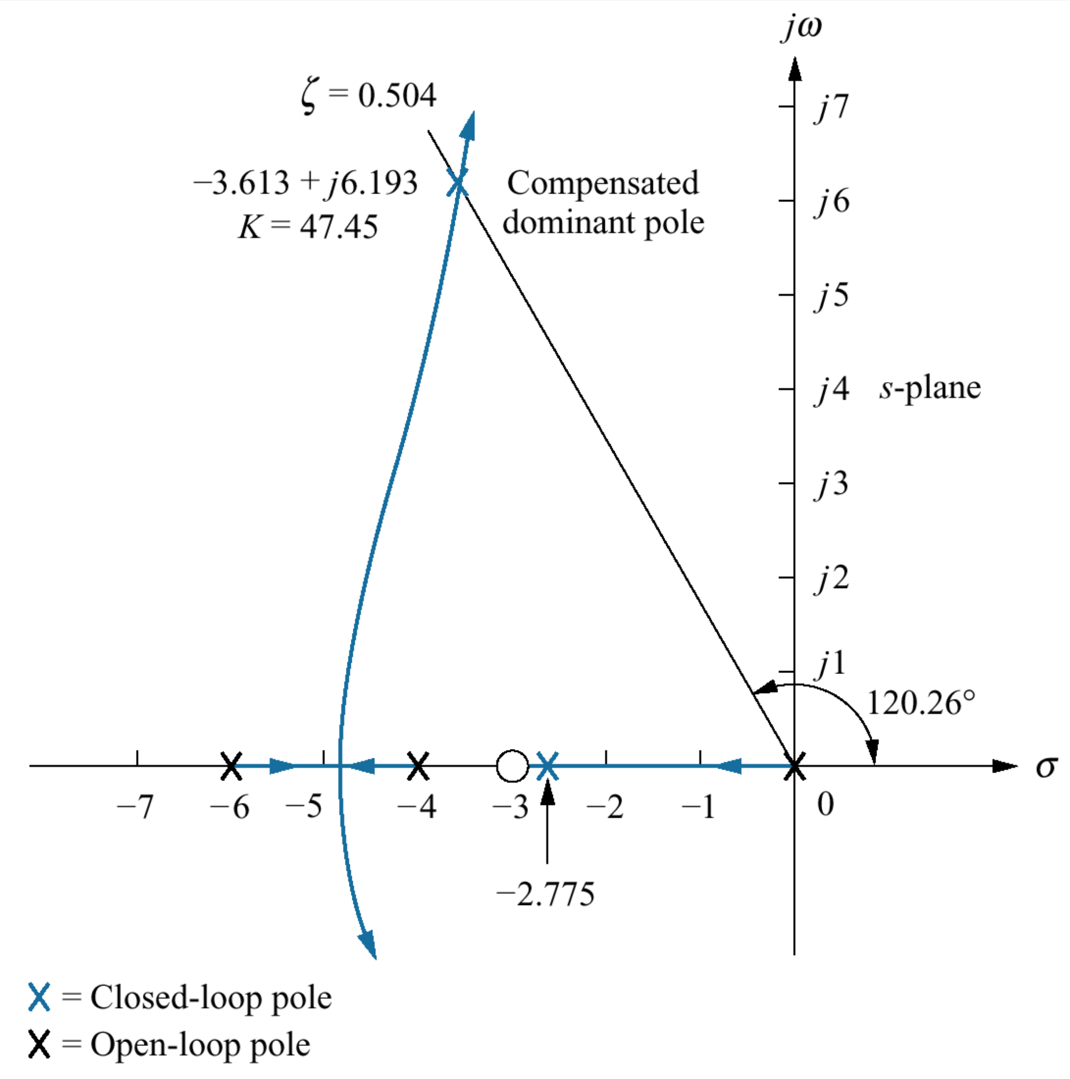
Thus $\sigma = 3.006$



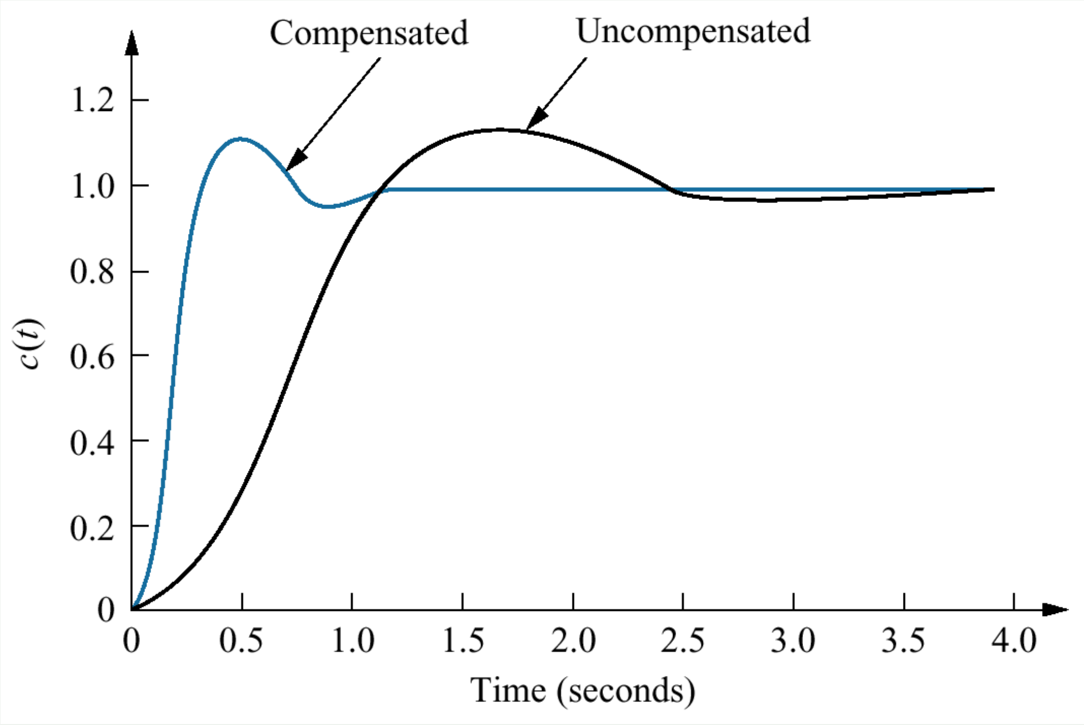
Uncompensated and compensated system characteristics for Example 9.3

	Uncompensated	**Simulation**	**Compensated**	**Simulation**
Plant and compensator	$\dfrac{K}{s(s+4)(s+6)}$		$\dfrac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
$\%OS$	16	14.8	16	11.8
T_s	3.320	3.6	1.107	1.2
T_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	−7.591		−2.775	
Zero	None		−3.006	
Comments	Second-order approx. OK		Pole-zero not canceling	

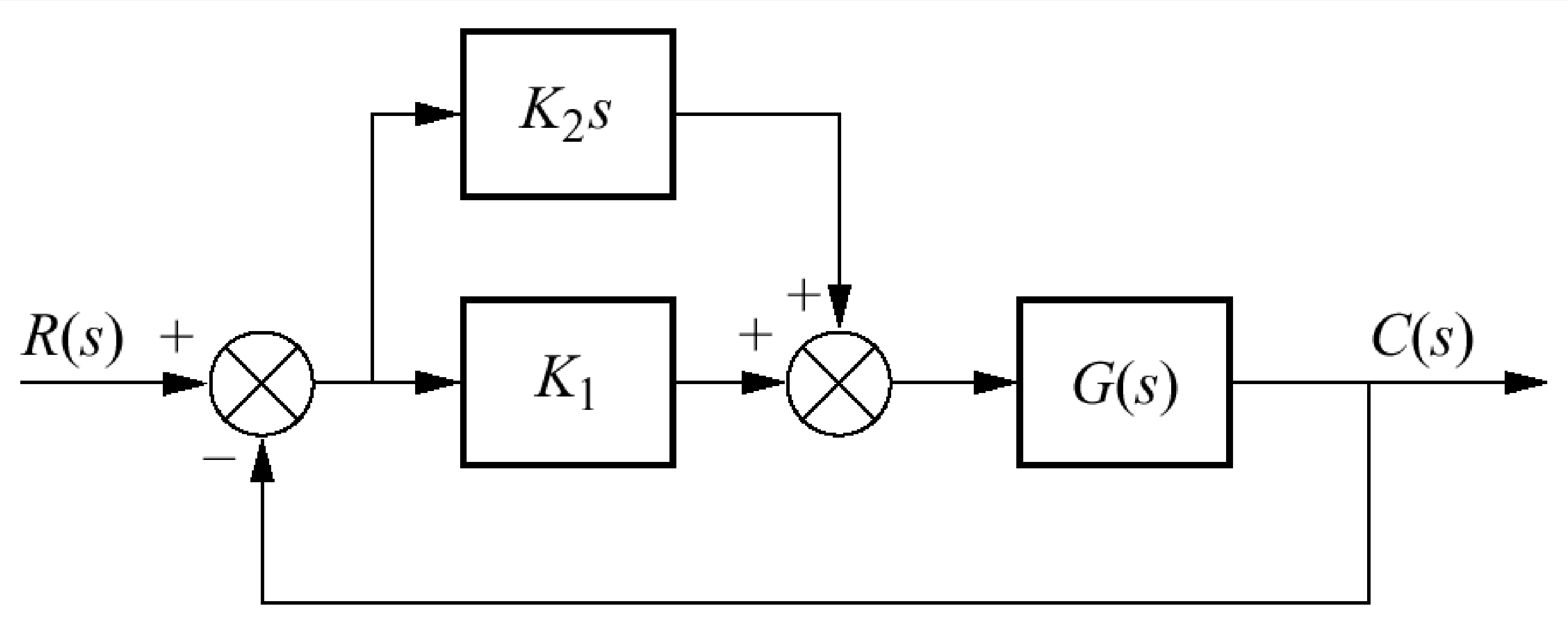
Root locus for the compensated system of Example 9.3



Uncompensated and compensated system step responses of Example 9.3



PD controller implementation



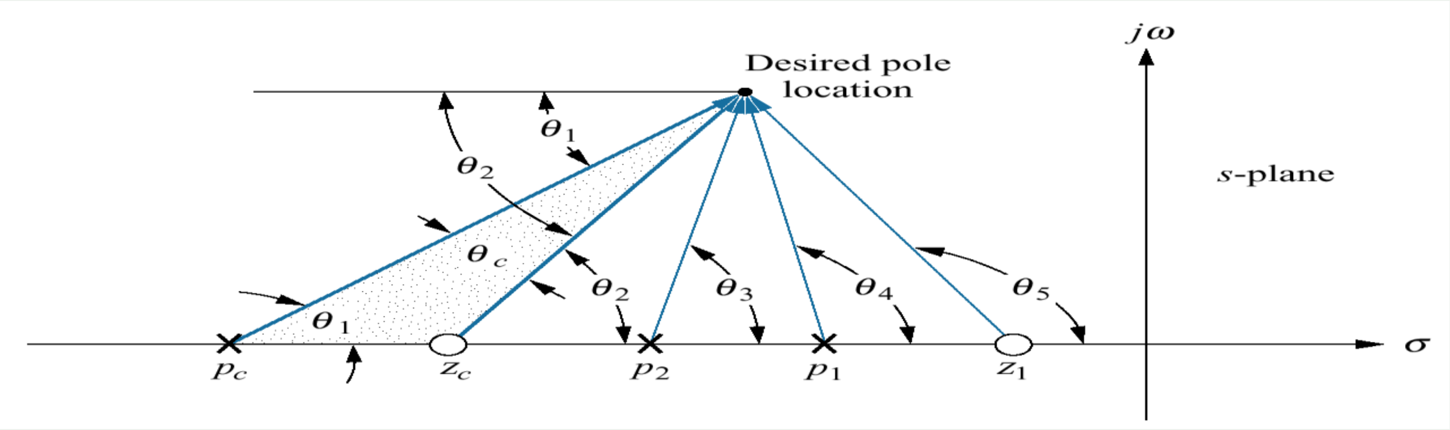
$$G_c(s) = K_2 s + K_1 = K_2\left(s + \frac{K_1}{K_2}\right)$$

K_2 is chosen to contribute to the required loop-gain value. And K_1/K_2 is chosen to equal the negative of the compensator zero.

Geometry of lead compensation

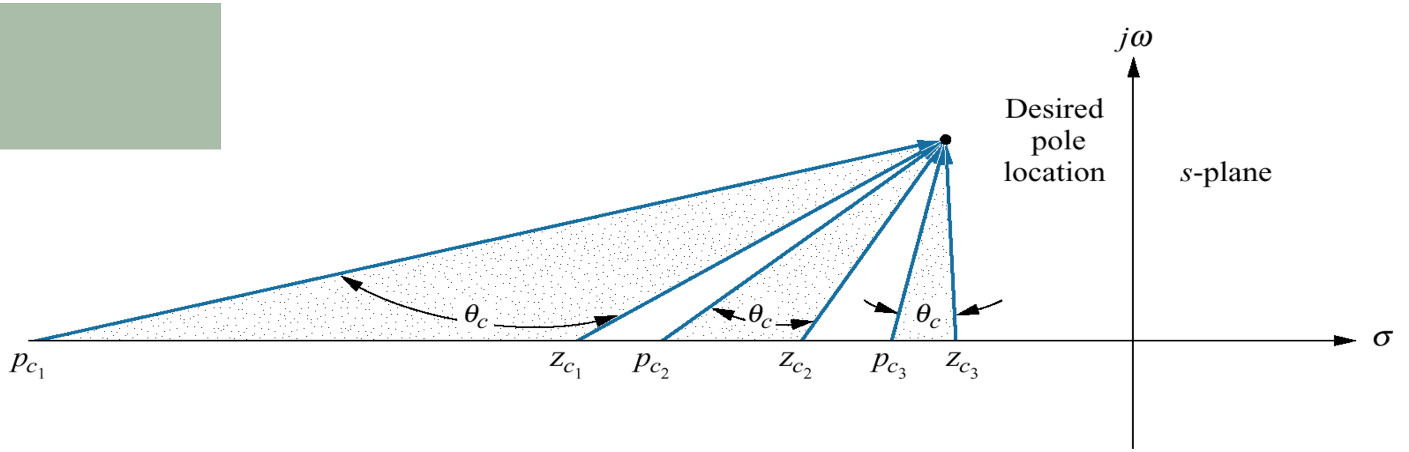
Advantages of a passive lead network over an active PD controller:

1) no need for additional power supply
2) noise due to differentiation is reduced



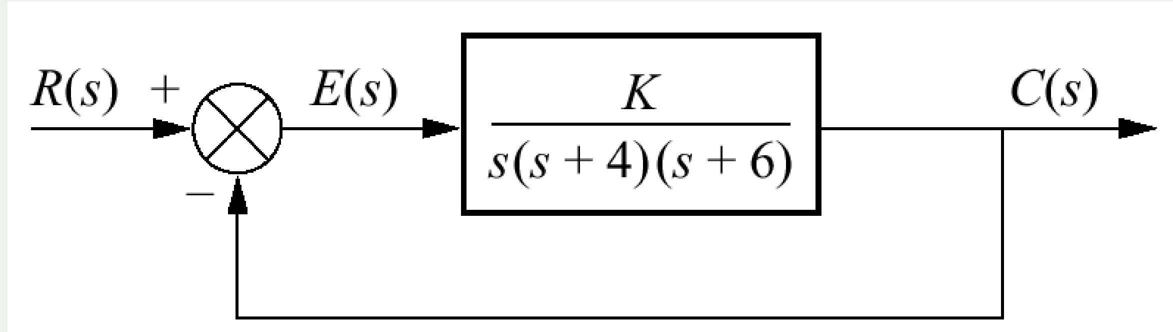
$$\theta_2 - \theta_1 - \theta_3 - \theta_4 + \theta_5 = (2k+1)180^\circ \quad \text{Note } (\theta_2 - \theta_1) = \theta_c$$

Three of the infinite possible lead compensator solutions



Lead compensator design, Example 9.4

Problem: Design 3 lead compensators for the system in figure that will reduce the settling time by a factor of 2 while maintaining 30% overshoot.



Solution: The uncompensated settling time is $T_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{1.007} = 3.972$

To find the design point, new settling time is $T_s = \dfrac{3.972}{2} = 1.986$

From which the real part of the desired pole location is $\sigma = \dfrac{4}{T_s} = \dfrac{4}{1.986} = 2.014$

And the imaginary part is

$$\omega_d = 2.014\tan(110.98^\circ) = 5.252$$