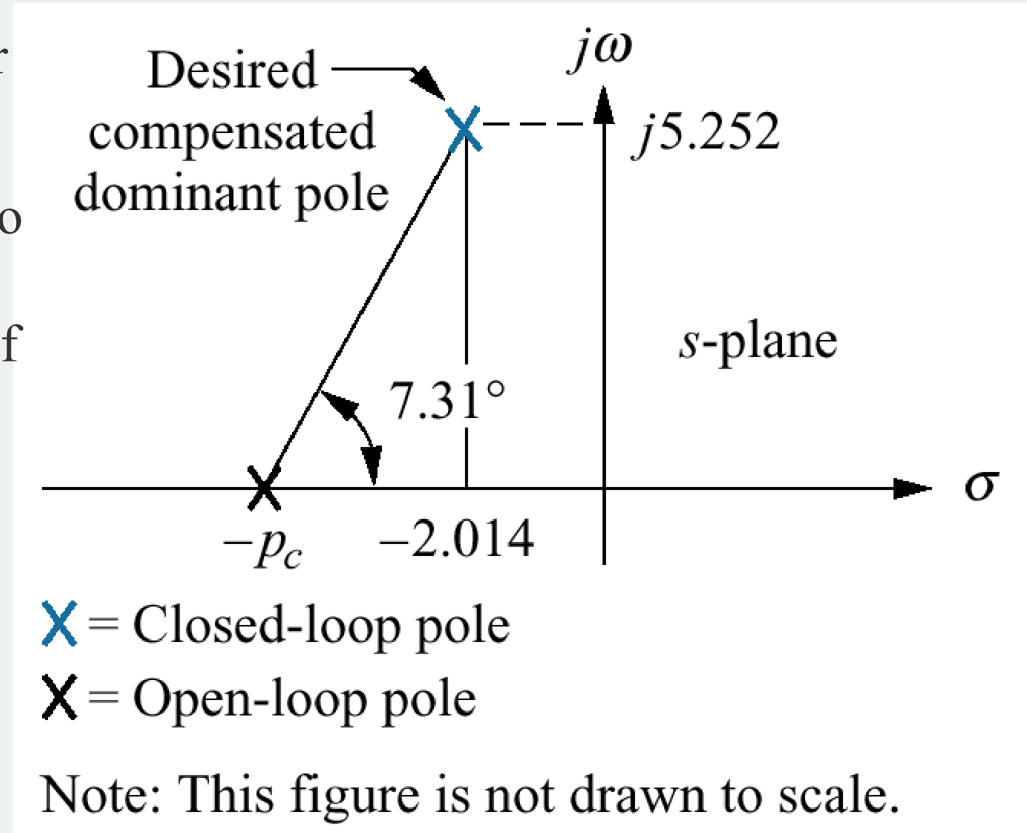
S-plane picture used to calculate the location of the compensator pole for Example 9.4

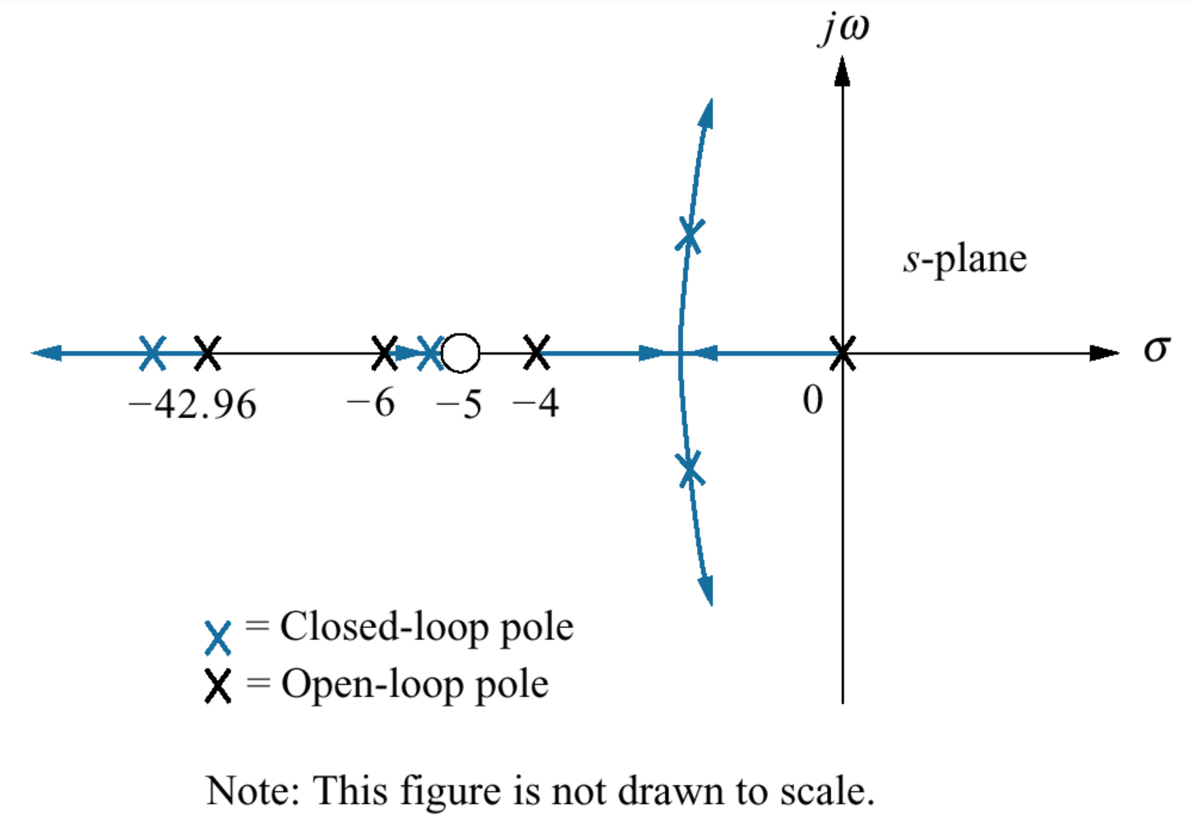
Arbitrarily assume a compensator zero at -5 on the real axis as possible solution. Then we find the compensator pole location as shown in figure.

Note sum of angles of compensator zero and all uncompensated poles and zeros is -172,69 so the angular contribution of the compensator pole is -7.31.

$$\frac{5.252}{p_c - 2.014} = \tan 7.31^\circ \quad \text{and} \quad p_c = 42.96$$



Compensated system root locus



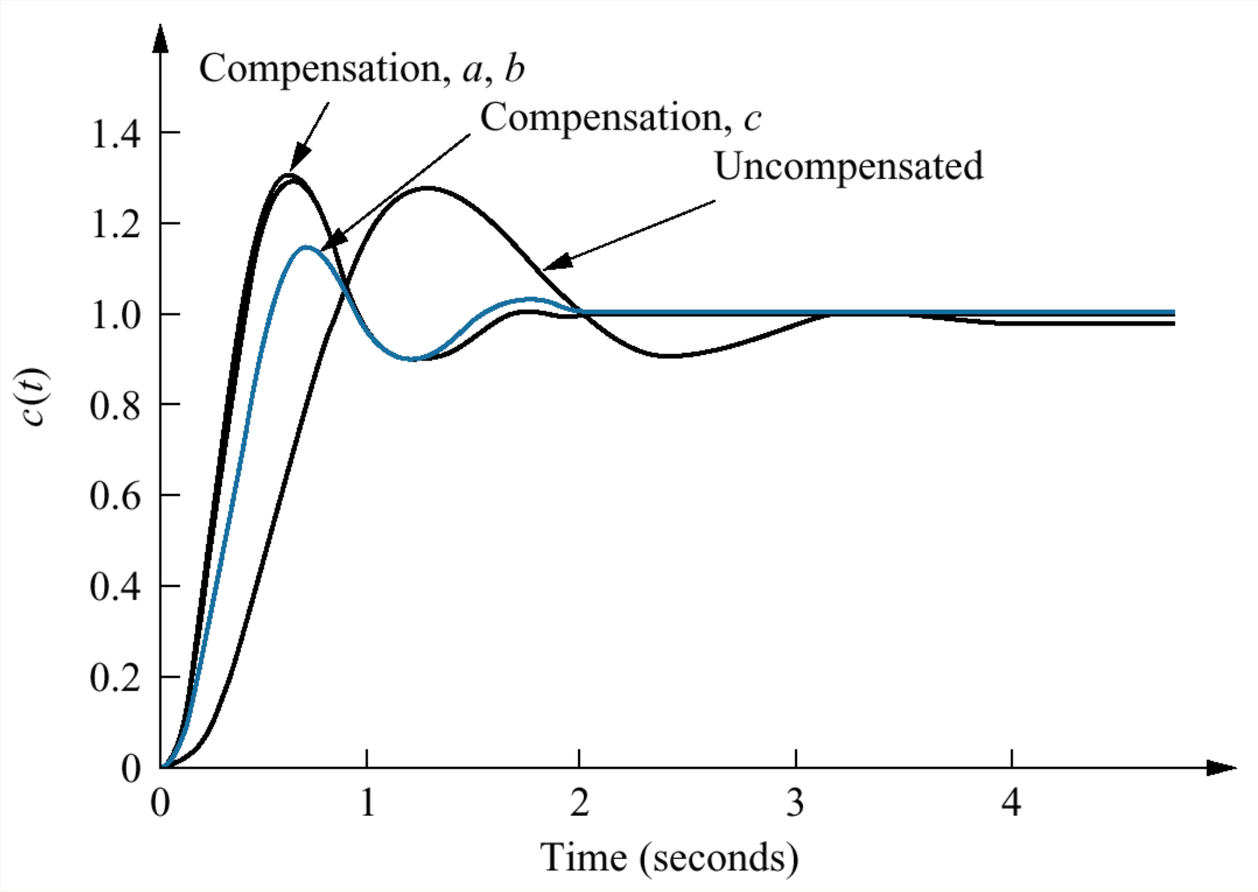
Comparison of lead compensation designs for Example 9.4

Table 9.4 Comparison of lead compensation designs for Example 9.4

	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
$\%OS^*$	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
T_s^*	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
T_p^*	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	-7.986	-43.8, -5.134	-22.06	-13.3, -1.642
Zero	None	-5	None	-2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

* Simulation results are shown in parentheses.

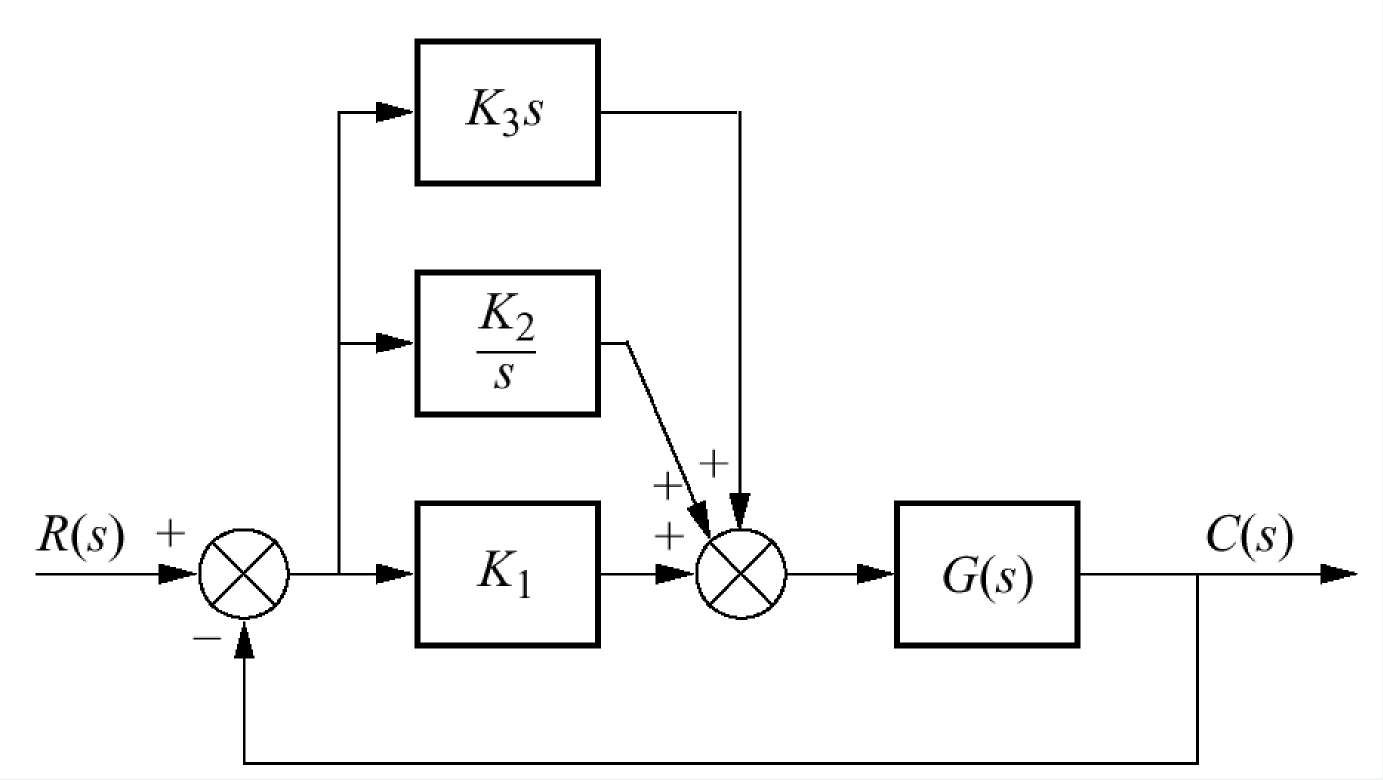
Uncompensated system and lead compensation responses for Example 9.4



Improving Steady-State Error and Transient Response

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 + K_2 + K_3 s^2}{s} = \frac{K_3(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3})}{s}$$

PID controller or using passive network it’s called lag-lad compensator



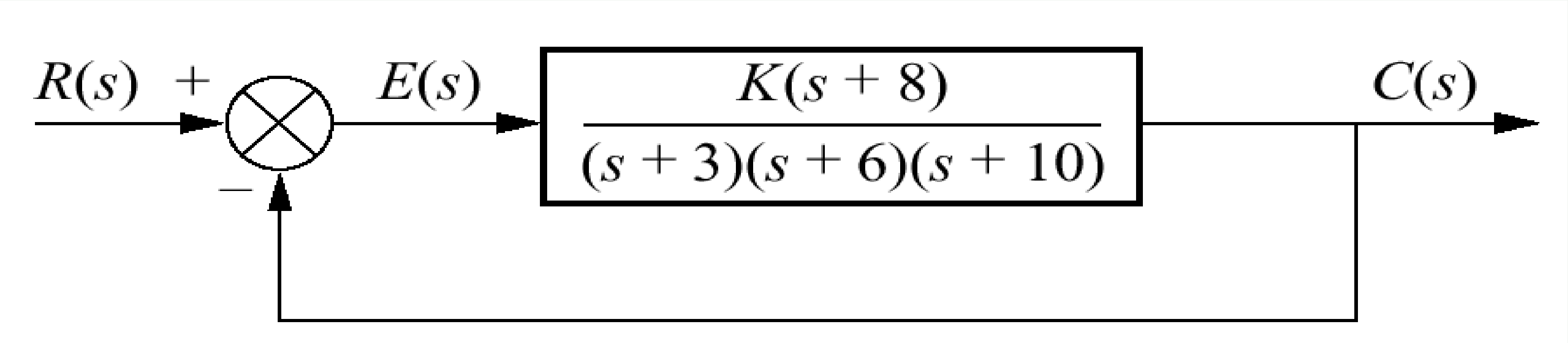
PID controller design

Design Steps:

- Evaluate the performance of the uncompensated system to determine how much improvement is required in transient response
- Design the PD controller to meet the transient response specifications. The design includes the zero location and the loop gain.
- Simulate the system to be sure all requirements have been met.
- Redesign if the simulation shows that requirements have not been met.
- Design the PI controller to yield the required steady-sate error.
- Determine the gains, K1, K2, and K3 shown in previous figure.
- Simulate the system to be sure all requirements have been met.
- Redesign if simulation shows that requirements have not been met.

PID controller design Example 9.5

Problem: **Using the system in the Figure, Design a PID controller so that the system can operate with a peak time that is 2/3 that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input**



Solution: **The uncompensated system operating at 20% overshoot has dominant poles at -5.415+j10.57 with gain 121.5, and a third pole at -8.169. The complete performance is shown in next table.**

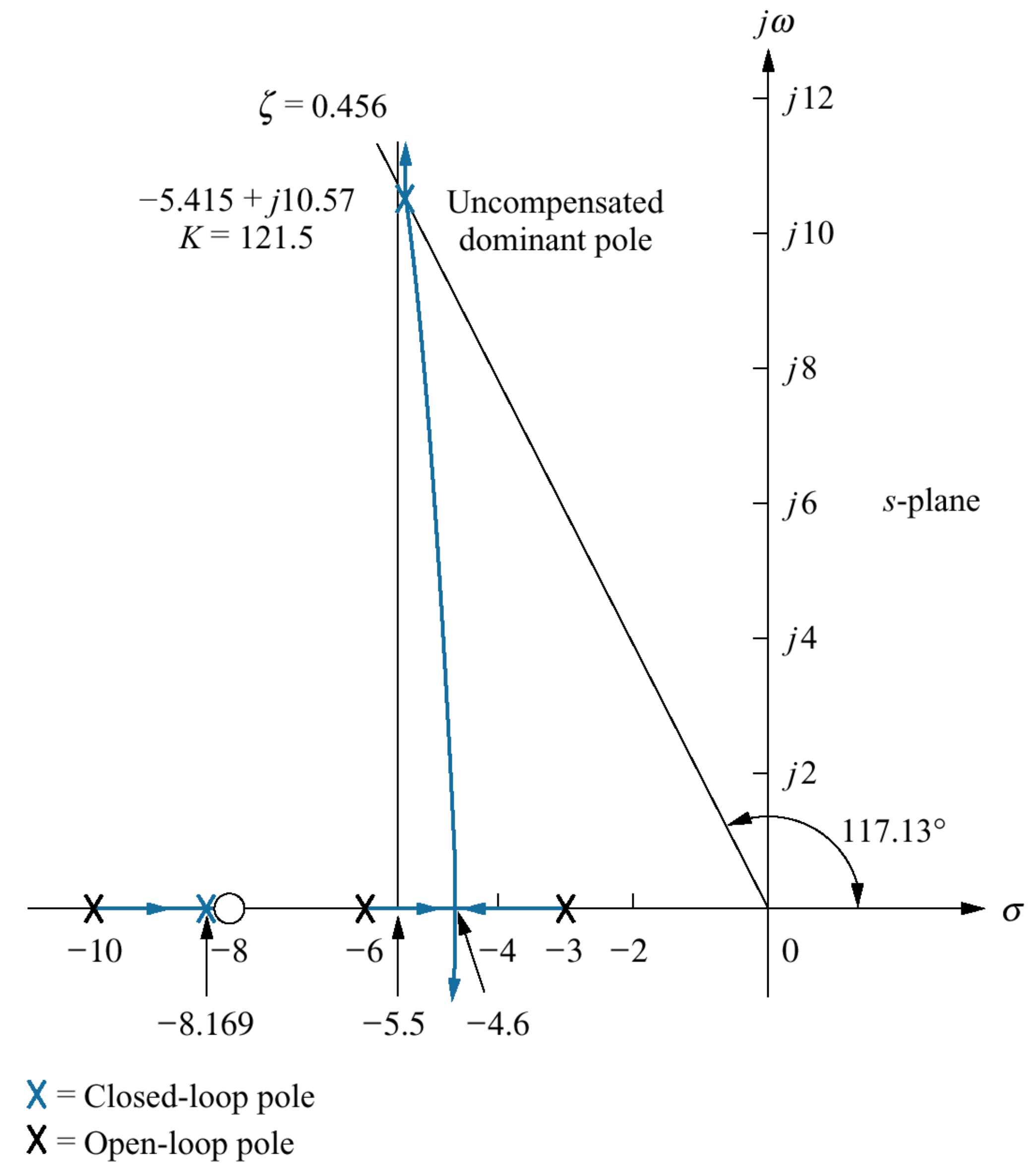
Root locus for the uncompensated system of Example 9.5

To compensate the system to reduce the peak time to 2/3 of original, we must find the compensated system dominant pole location. The imaginary part of the dominant pole is

$$\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$$

Thus the real part is

$$\sigma = \frac{\omega_d}{\tan 117.13^\circ} = -8.13$$



Predicted characteristics of uncompensated, PD- , and PID-compensated systems of Example 9.5

	Uncompensated	PD-compensated	PID-compensated
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	-8.099, -0.468
Zeros	-8	-8, -55.92	-8, -55.92, -0.5
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

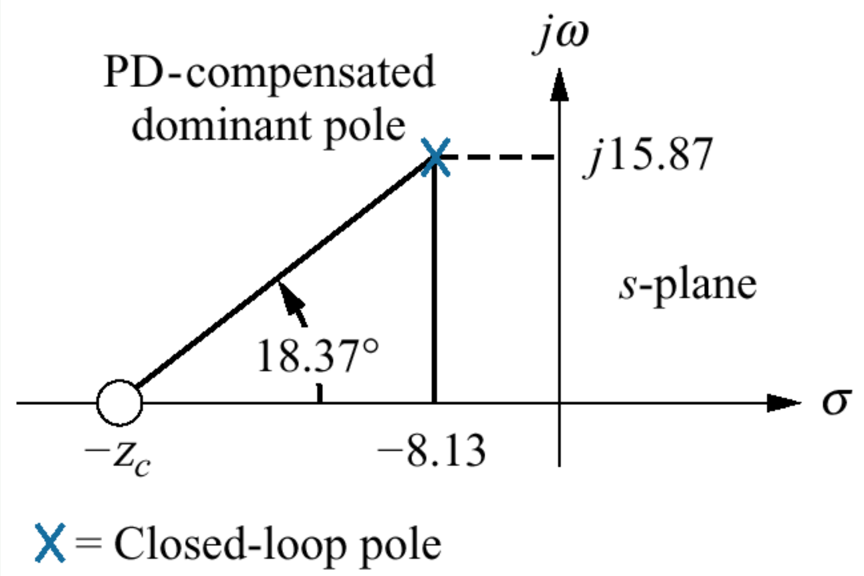
Calculating the PD compensator zero for Example 9.5

To design the compensator, we find the sum of angles from the uncompensated system's poles and zeros to the desired compensated dominant pole to be -198.37. Thus the contribution required from the compensator zero is 198.37-180=18.37. Then we calculate the location of the zero as:

$$\frac{15.87}{z_c - 8.13} = \tan 18.37^\circ \quad \text{and } z_c = 55.92$$

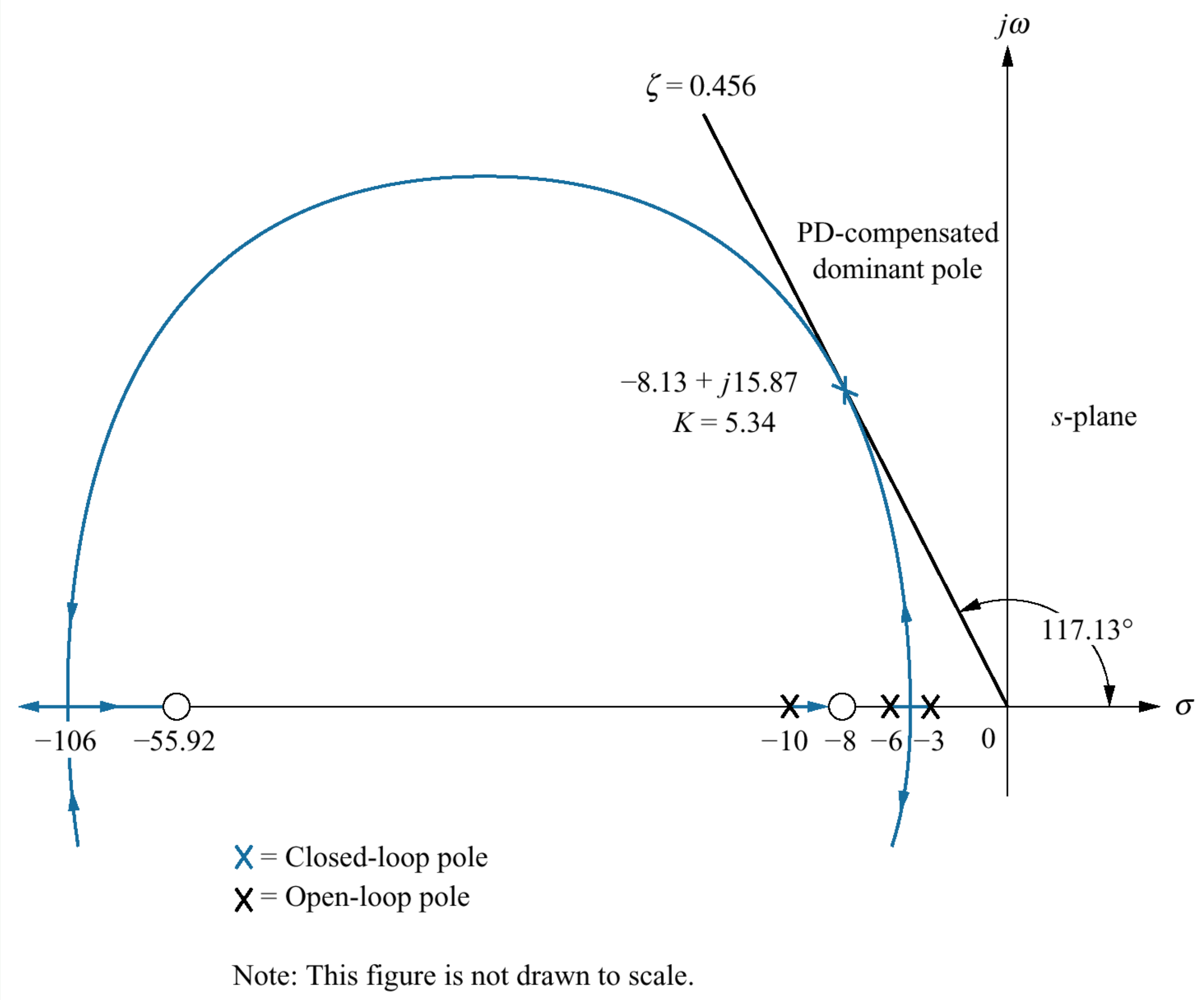
Thus the PD controller is $G_{PD}(s)$ = (s+55.92)

The complete root locus sketch is shown in next slide. Using program the gain at the design point is 5.34

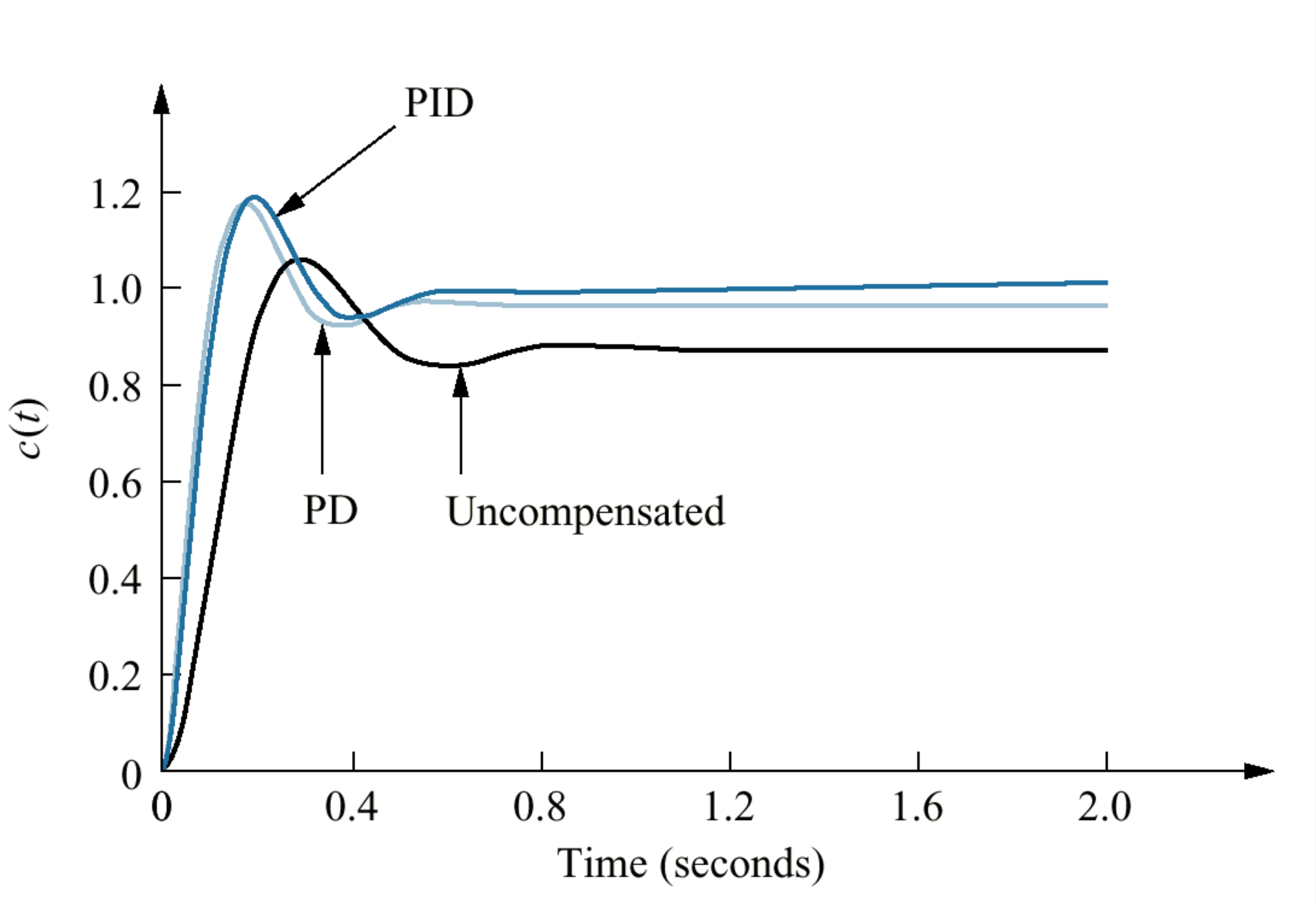


Note: This figure is not drawn to scale.

Root locus for PD-compensated system of Example 9.5



Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5

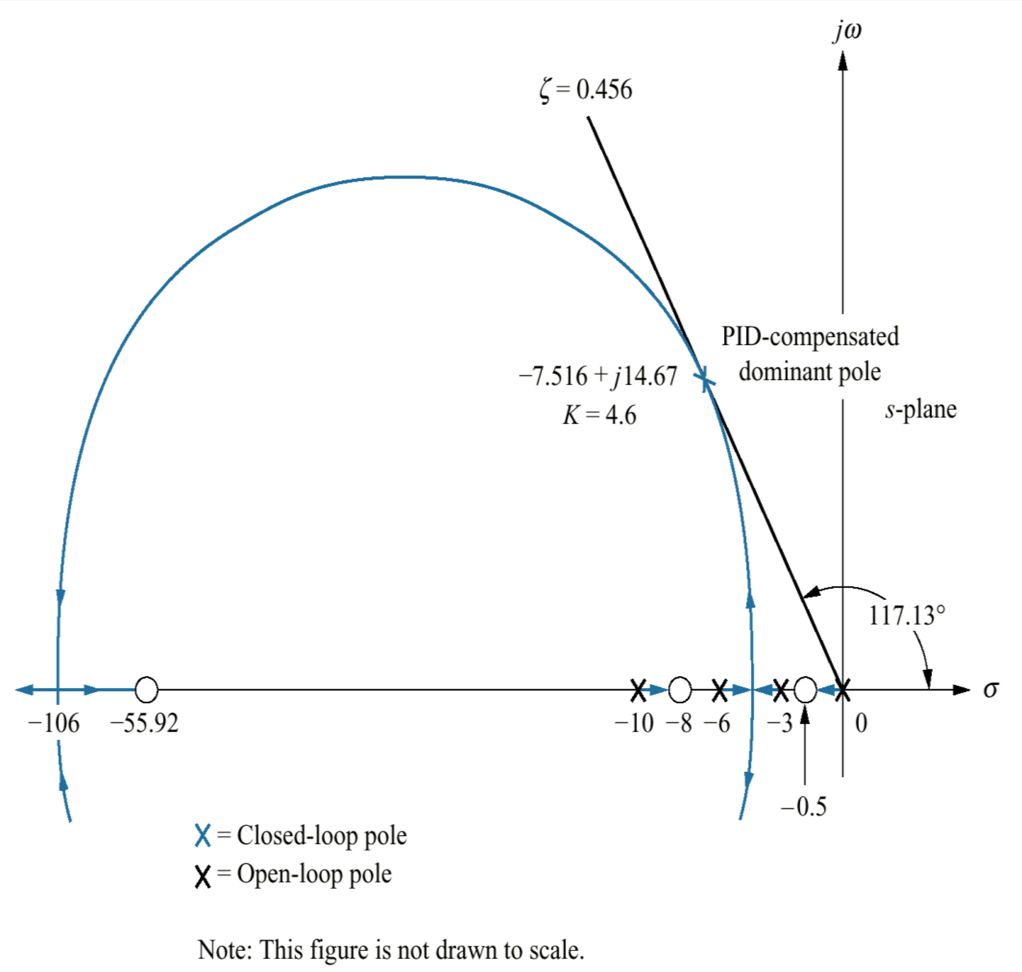


Root locus for PID-compensated system of Example 9.5

Choosing the ideal integral compensator to be $G_{PI}(s) = \dfrac{s+0.5}{s}$

And sketching the root locus for the PID-compensated system as shown. Searching the 0.456 damping ratio line, we find the dominant poles at -7.516+j14.67

The characteristics of the PID compensated system are shown in table.



Predicted characteristics of uncompensated, PD- , and PID-compensated systems of Example 9.5

	Uncompensated	**PD-compensated**	**PID-compensated**
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	$-8.099,\ -0.468$
Zeros	-8	$-8,\ -55.92$	$-8,\ -55.92,\ -0.5$
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

Improving Steady-State Error and Transient Response

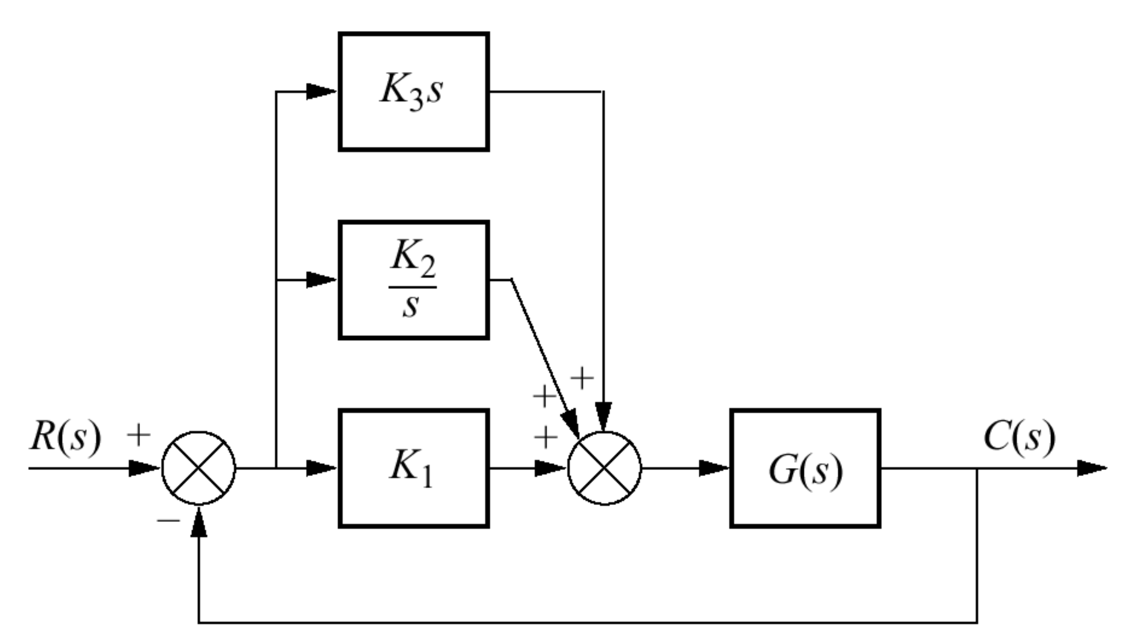
Finally to implement the compensator and find the K's, using the PD and PI compensators

$$G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s^2+56.42s+27.96)}{s}$$

and compare to

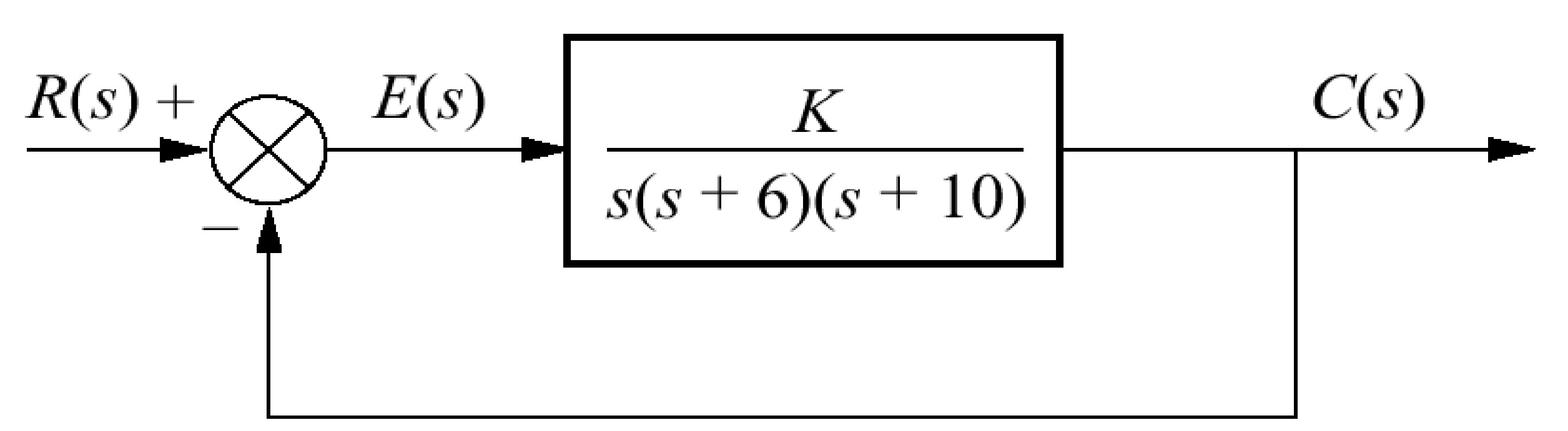
$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 + K_2 + K_3 s^2}{s} = \frac{K_3(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3})}{s}$$

we find K_1= 259.5, K_2=128.6, and K_3=4.6



Lag-Lead Compensator Design Example 9.6

Problem: Using the system in the Figure, Design a lag-lead compensator so that the system can operate with a twofold reduction in settling time, and 20% overshoot and a tenfold improvement in steady-state error for a ramp input



Solution: The uncompensated system operating at 20% overshoot has dominant poles at -1.794+j3.501 with gain 192.1, and a third pole at -12.41. The complete performance is shown in next table.

Root locus for uncompensated system of Example 9.6

To compensate the system to realize a twofold reduction in settling time, the real part of the dominant poles must be increased by a factor of 2, thus,

$$-\xi\omega_n = -2(1.794) = -3.588$$

And the imaginary part is $\omega_d = \xi\omega_n \tan 117.13^\circ = 3.588 \tan 117.13^\circ = 7.003$

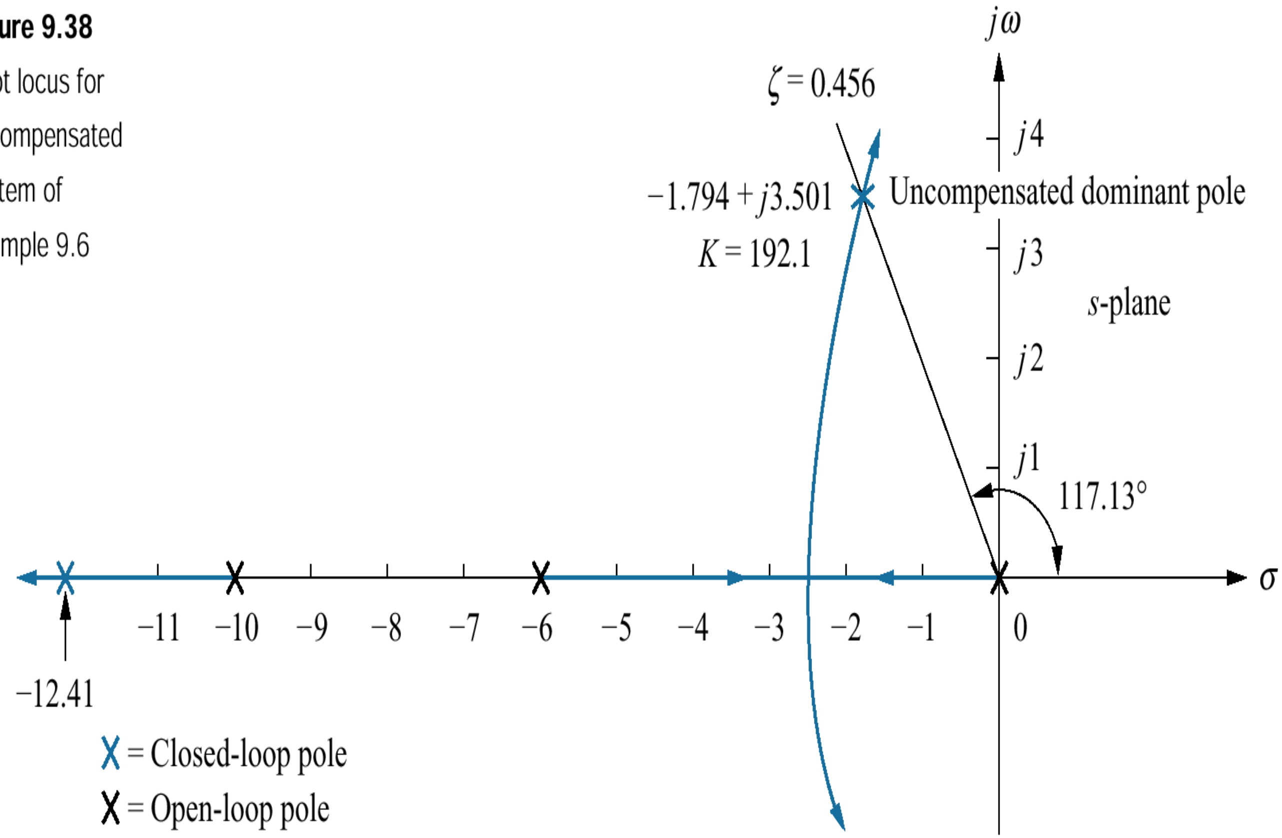


Figure 9.38 Root locus for uncompensated system of Example 9.6

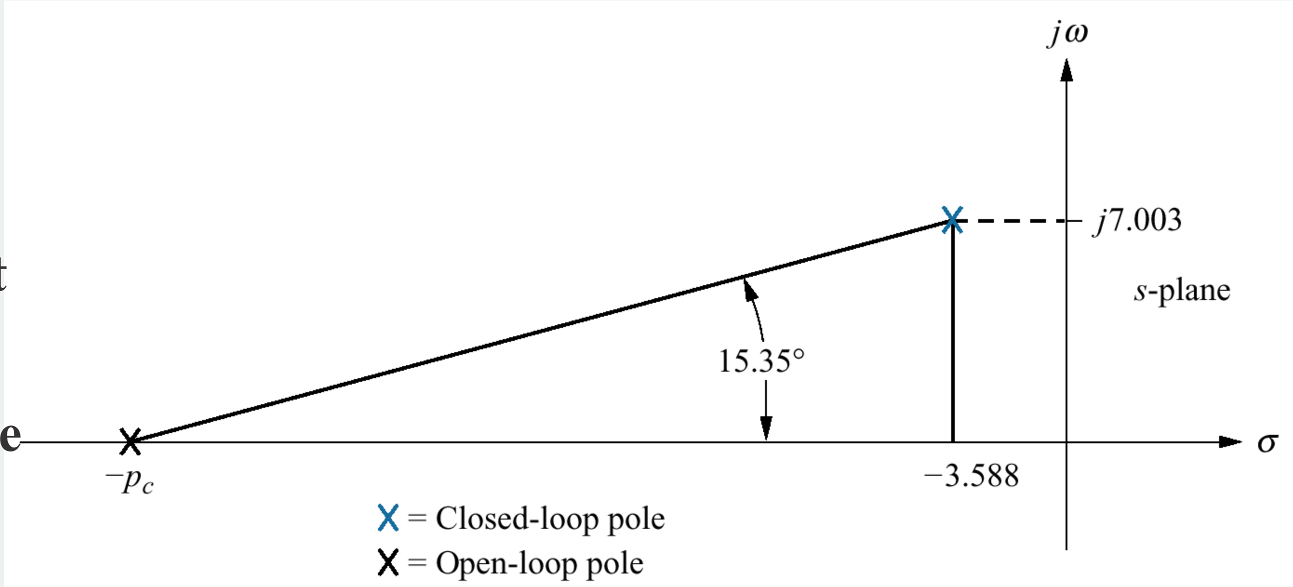
Predicted characteristics of uncompensated, lead-compensated, and lag-lead- compensated systems of Example 9.6

	Uncompensated	**Lead-compensated**	**Lag-lead-compensated**
Plant and compensator	$\dfrac{K}{s(s+6)(s+10)}$	$\dfrac{K}{s(s+10)(s+29.1)}$	$\dfrac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
$\%OS$	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	−12.41	−31.92	−31.91, −0.0474
Zero	None	None	−0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Evaluating the compensator pole for Example 9.6

Now to design the lead compensator, arbitrarily select a location for the lead compensator zero at -6, to cancel the pole.

To find the location of the compensator pole. Using program sum the angles to get -164.65. and the contribution of the pole is -15.35 we find the location of the pole from the figure as

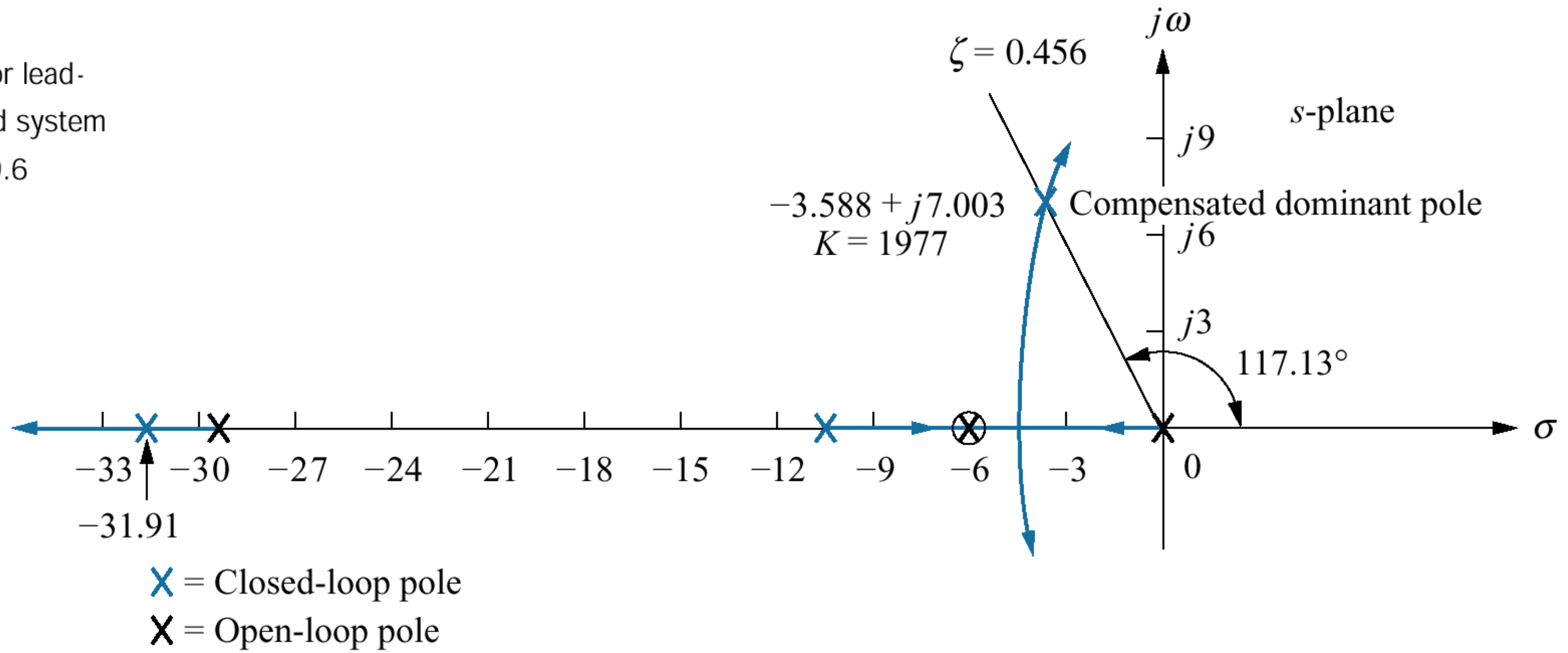


$$\frac{7.003}{p_c - 3.588} = \tan 15.35^\circ \quad \text{and } p_c = -29.1$$

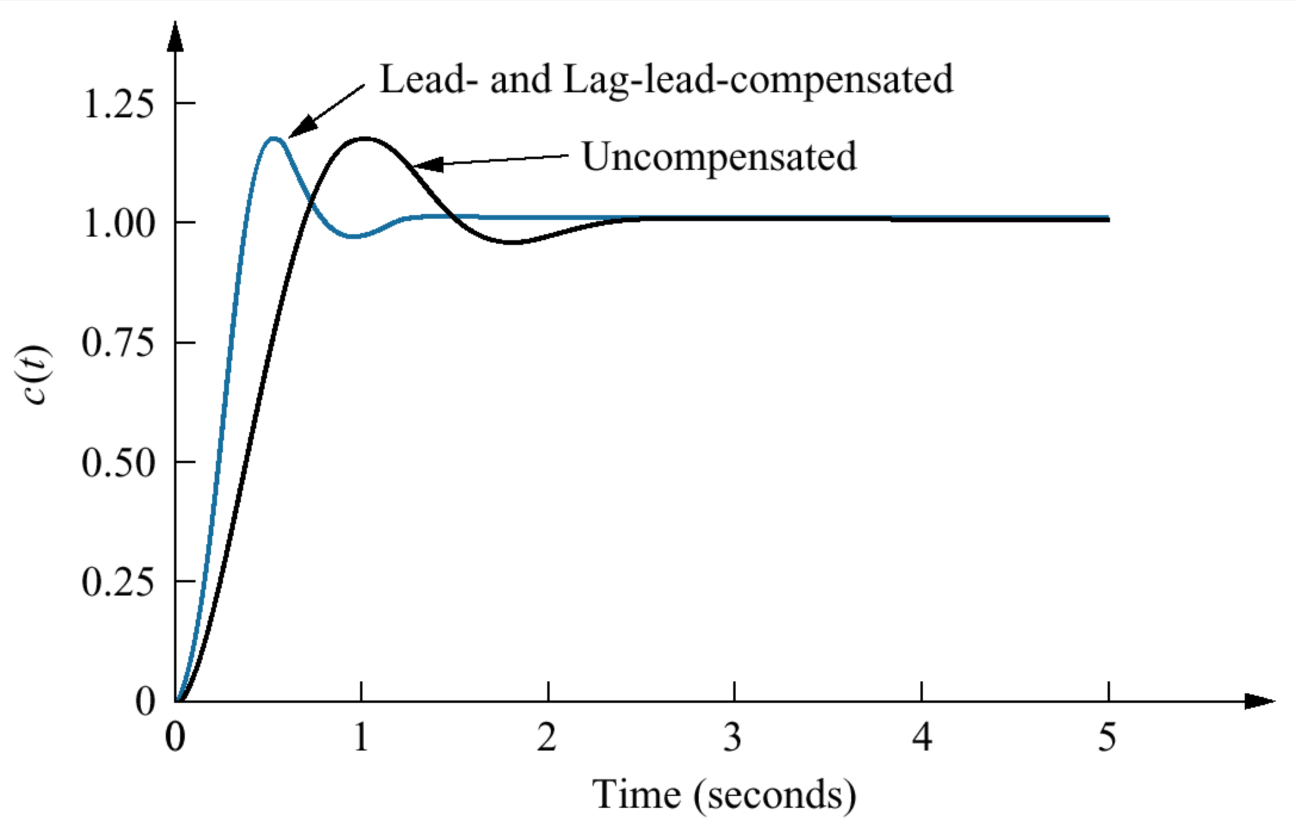
Results are satisfactory see results in next slide

Root locus for lead-compensated system of Example 9.6

Figure 9.40
Root locus for lead-compensated system of Example 9.6



Improvement in step response for lag-lead- compensated system of Example 9.6



Root locus for lag-lead- compensated system of Example 9.6

Since the uncompensated system's open-loop transfer function is

$$G(s) = \frac{192.1}{s(s+6)(s+10)}$$

The static error constant of the uncompensated system is 3.201

Since the open-loop transfer function of the lead-compensated system is

$$G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)}$$

the static error constant of the lead-compensated system is 6.794, so we have improvement by a factor of 2.122.

To improve the original system error by a factor of 10, the lag compensator must be designed to improve the error by a factor of 10/2.122 = 4.713

Root locus for lag-lead- compensated system of Example 9.6

We arbitrarily choose the lag compensator pole at 0.01, which then places the zero at 0.04713 yielding

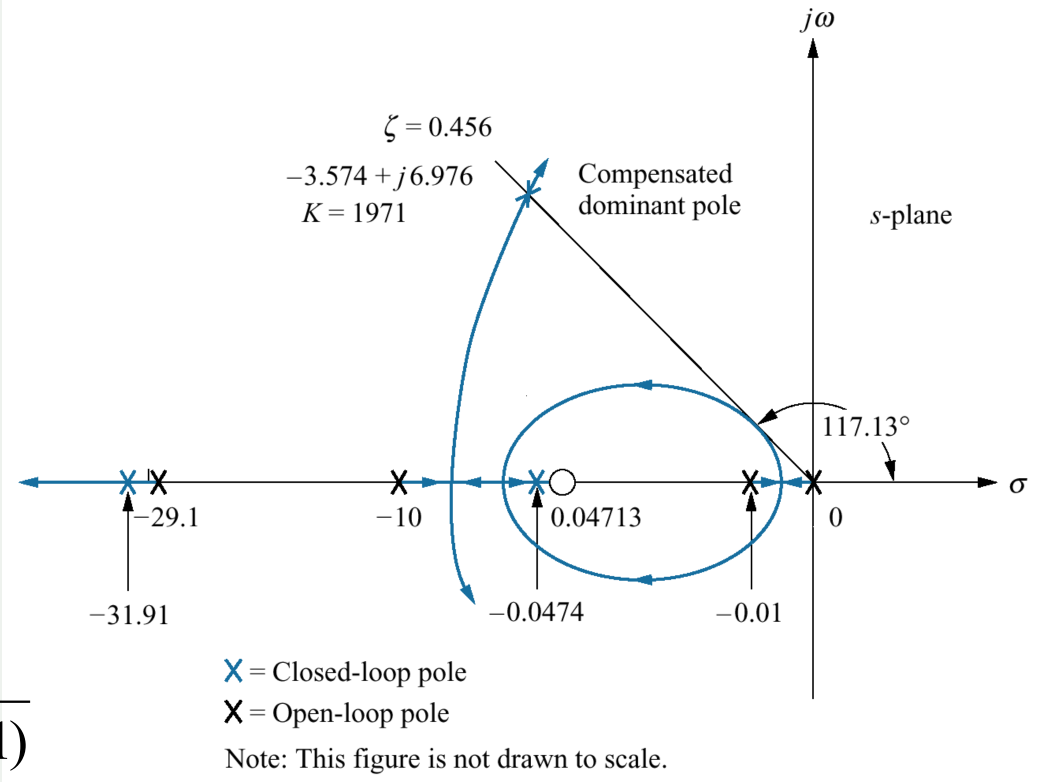
$$G_{lag}(s) = \frac{(s + 0.04713)}{(s + 0.01)}$$

as a lag compensator

and

$$G_{LLC}(s) = \frac{K(s + 0.04713)}{s(s + 10)(s + 29.1)(s + 0.01)}$$

as lag-lead-compensated system open-loop transfer function



Predicted characteristics of uncompensated, lead-compensated, and lag-lead- compensated systems of Example 9.6

	Uncompensated	**Lead-compensated**	**Lag-lead-compensated**
Plant and compensator	$\dfrac{K}{s(s+6)(s+10)}$	$\dfrac{K}{s(s+10)(s+29.1)}$	$\dfrac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
$\%OS$	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	-12.41	-31.92	-31.91, -0.0474
Zero	None	None	-0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Improvement in step response for lag-lead- compensated system of Example 9.6