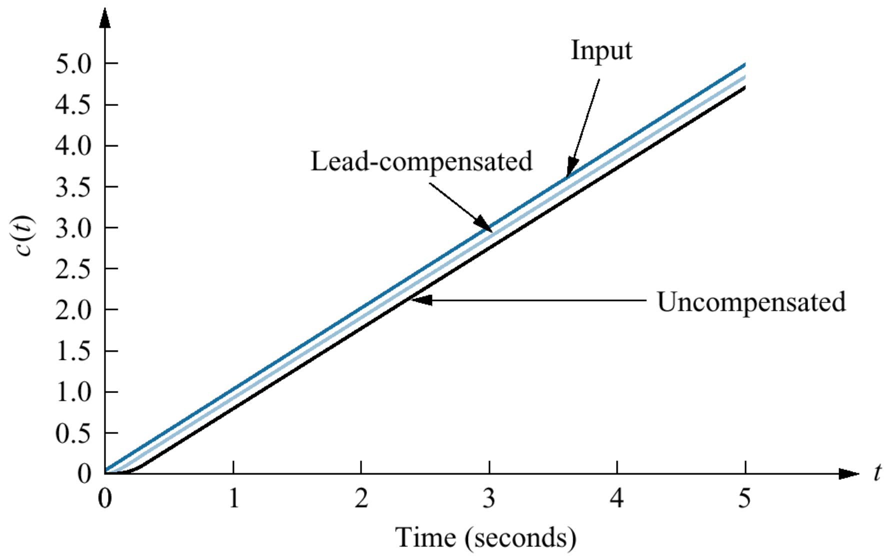
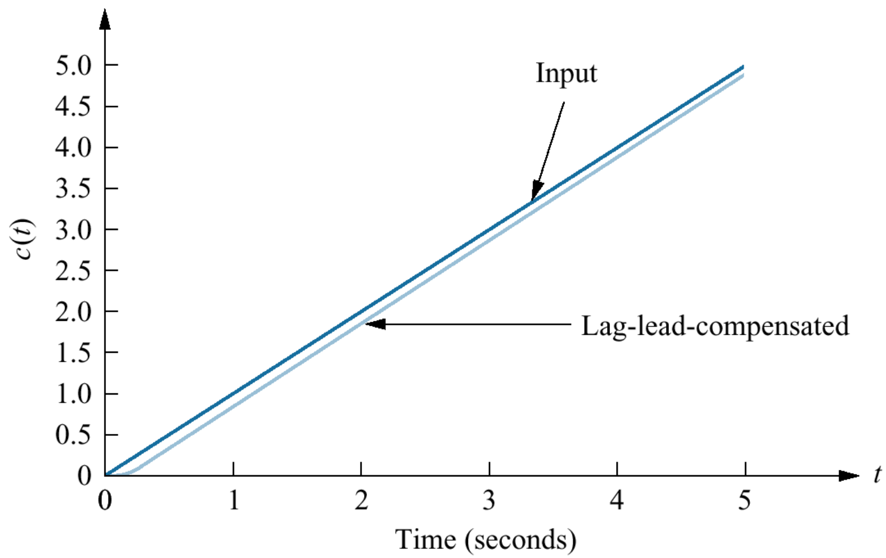
Improvement in ramp response error for the system of Example 9.6:

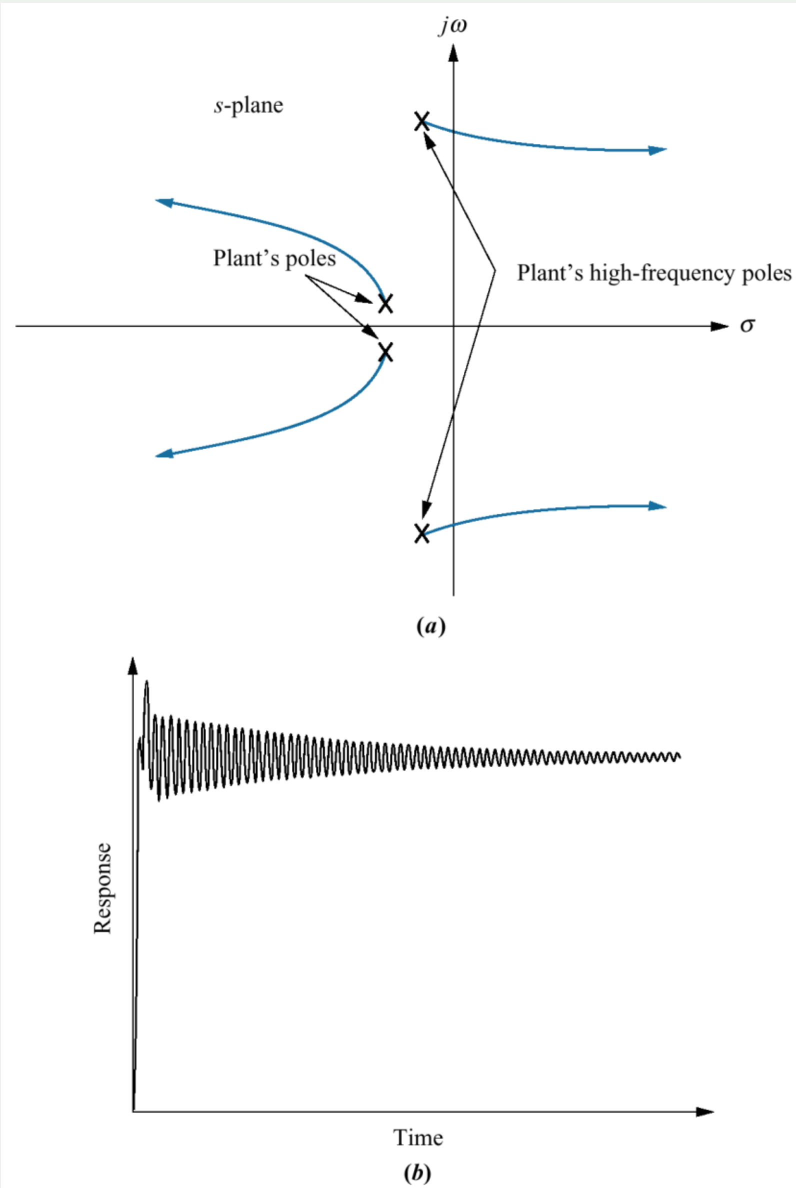


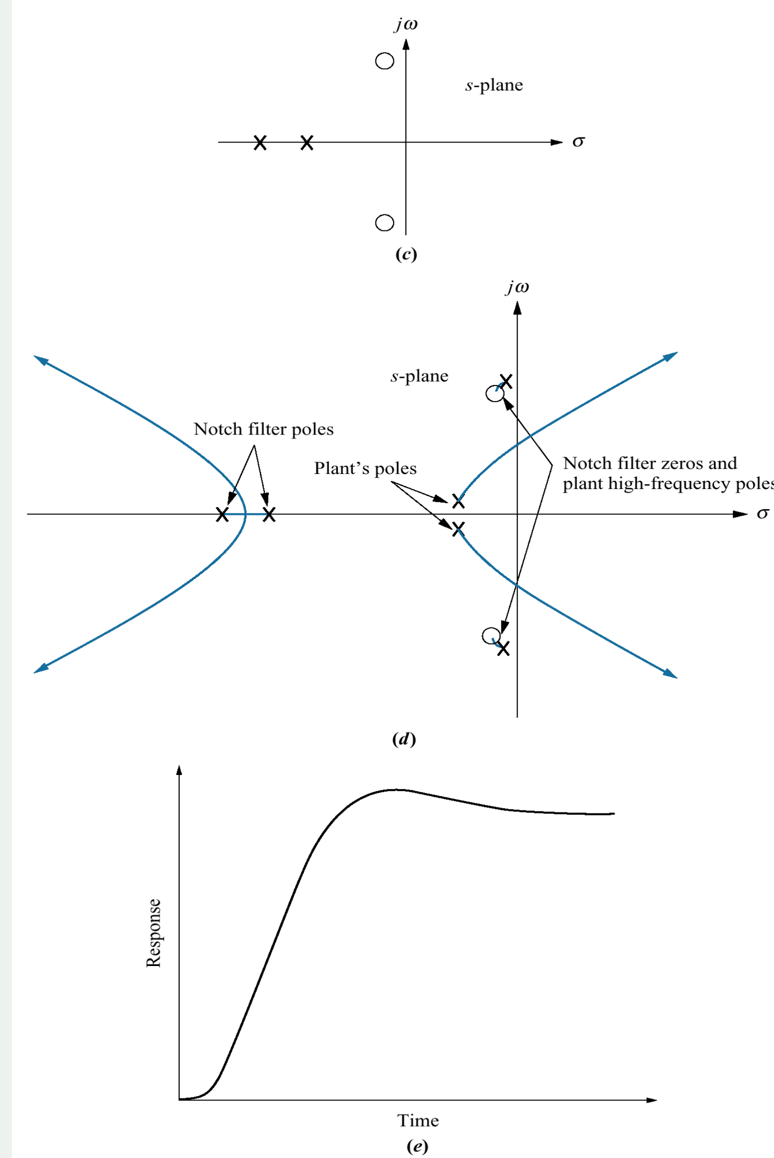
(a)



(b)

a. Root locus before cascading notch filter;
b. typical closed-loop step response before cascading notch filter;





c. pole-zero plot of a notch filter;
d. root locus after cascading notch filter;
e. closed-loop step response after cascading notch filter.

Types of cascade compensators

Table 9.7 Types of cascade compensators

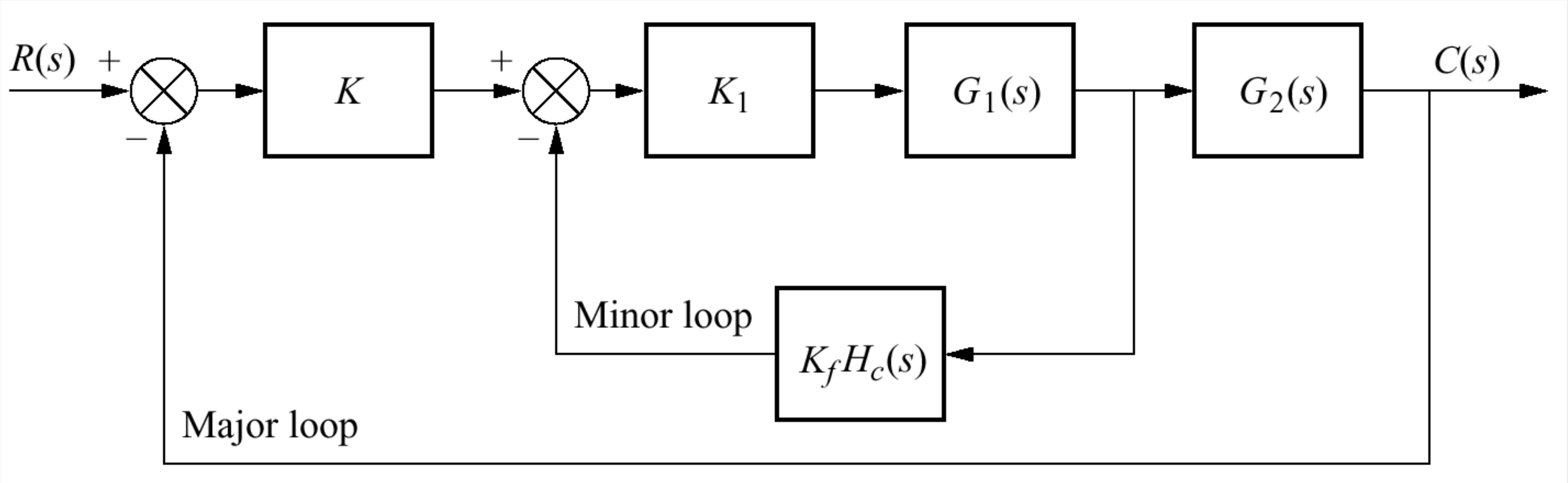
Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	1. Increases system type. 2. Error becomes zero. 3. Zero at $-z_c$ is small and negative. 4. Active circuits are required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	1. Error is improved but not driven to zero. 2. Pole at $-p_c$ is small and negative. 3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. 4. Active circuits are not required to implement.
Improve transient response	PD	$K(s+z_c)$	1. Zero at $-z_c$ is selected to put design point on root locus. 2. Active circuits are required to implement. 3. Can cause noise and saturation; implement with rate feedback or with a pole (lead).
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. 2. Pole at $-p_c$ is more negative than zero at $-z_c$. 3. Active circuits are not required to implement.

Types of cascade compensators

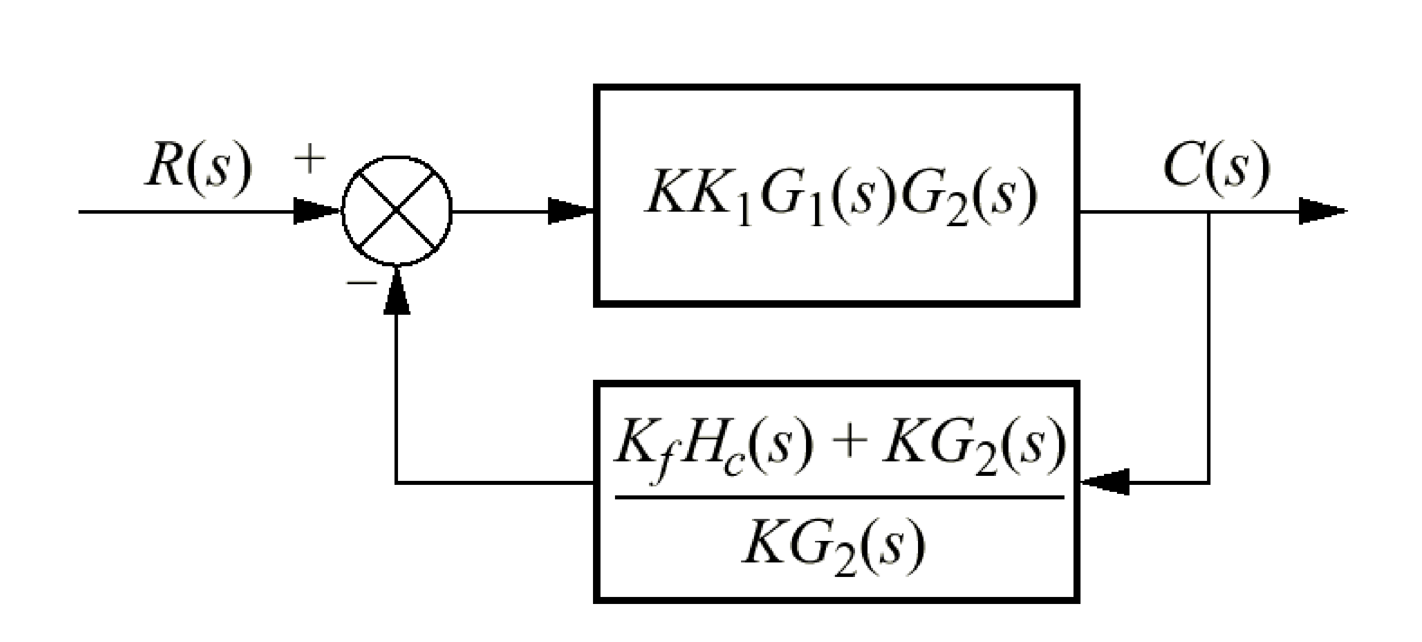
Table 9.7 Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	PID	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{s}$	1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error. 2. Lead zero at $-z_{\text{lead}}$ improves transient response. 3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin. 4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus. 5. Active circuits required to implement. 6. Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-lead	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. 2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. 3. Lag pole at $-p_{\text{lag}}$ is small and negative. 4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. 5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. 6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. 7. Active circuits are not required to implement.

Generic control system with feedback compensation



Equivalent block diagram



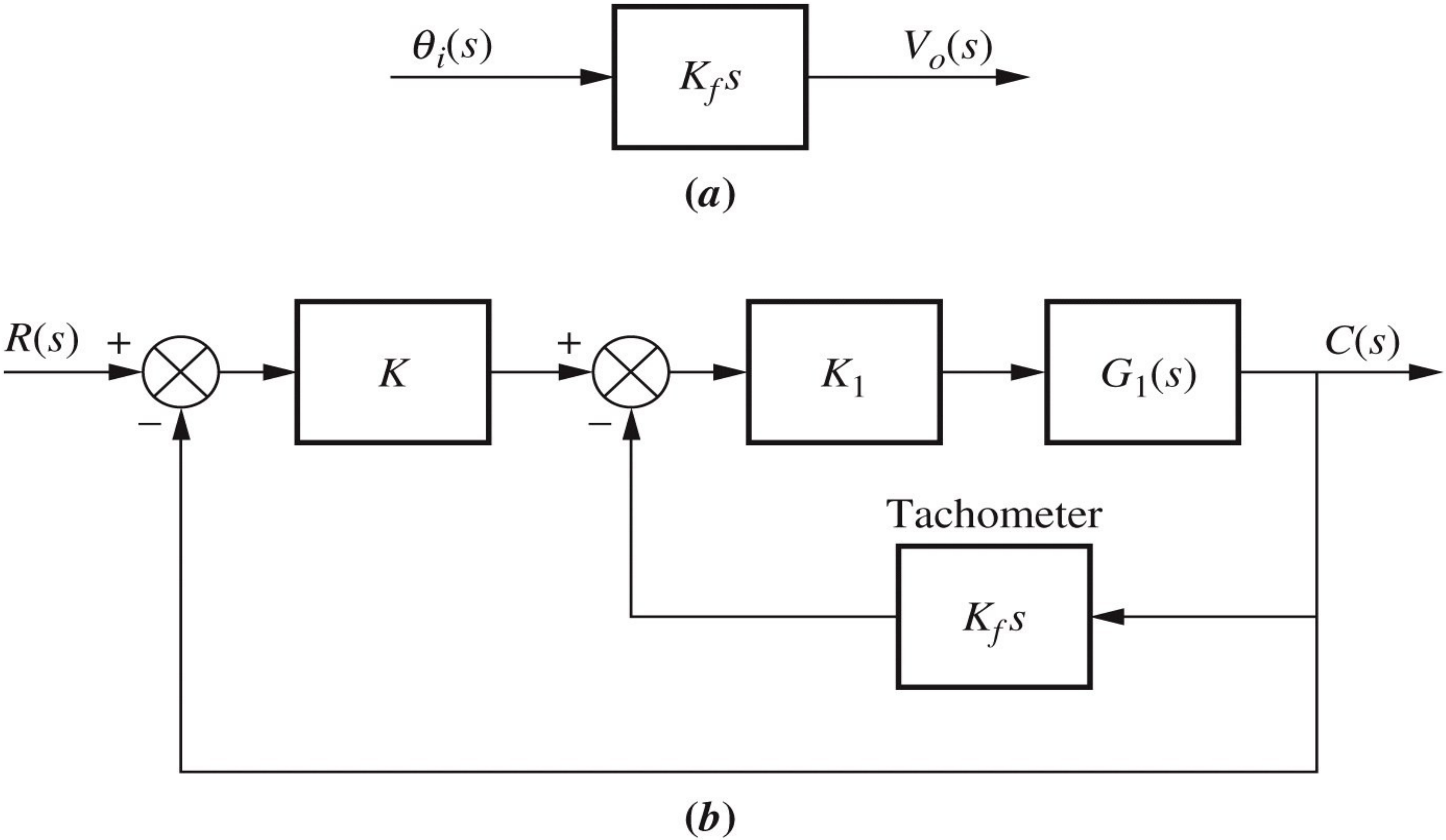
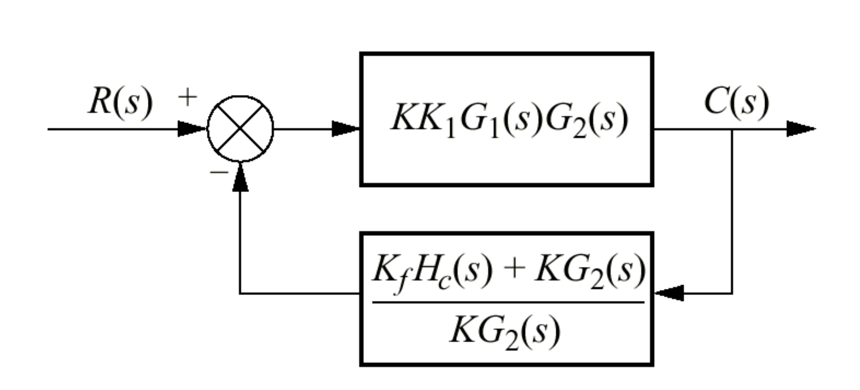


Figure 9.47

a. Transfer function of a tachometer

b. Tachometer feed-back compensation

Equivalent block diagram for the feedback compensator



The Figure shows that the loop gain, G(s)H(s), is

$G(s)H(s) = K_1G_1(s)\{K_fH_c(s)+KG_2(s)\}$

Without feedback, $K_fH_c(s)$, the loop gain is

$G(s)H(s) = KK_1G_1(s)G_2(s)$

Thus, the effect of adding feedback is to replace the poles and zeros of $G_2(s)$ with the poles and zeros of $[K_fH_c(s) + KG_2(s)]$. Hence, this method is similar to cascade compensation in that we add new poles and zeros via H(s) to reshape the root locus to go through the design point. However, one must remember that zeros of the equivalent feedback shown in the Figure, $H(s) = [K_fH_c(s) +KG_2(s)]/KG_2(s)$, are not closed-loop zeros.

For example, if $G_2(s) = 1$ and the minor-loop feedback, $K_fH_c(s)$, is a rate sensor, $= K_fS$, then the loop gain is $G(s)H(s) = K_fK_1G_1(s)(s+K/K_f)$

Thus, a zero at $-K/K_f$ is added to the existing open-loop poles and zeros. This zero reshapes the root locus to go through the desired design point. Again, this zero is not a closed-loop zero.

Example 9.7 Compensating Zero via Rate Feedback

PROBLEM: **Given the system of Figure 9.49(a), design rate feedback compensation, as shown in Figure 9.49(b), to reduce the settling time by a factor of 4 while continuing to operate the system with 20% overshoot.**

SOLUTION: First design a PD compensator. For the uncompensated system, search along the 20% overshoot line (ξ = 0.456) and find that the dominant poles are at -1.809 ±j3.531, as shown in Figure in next slide.

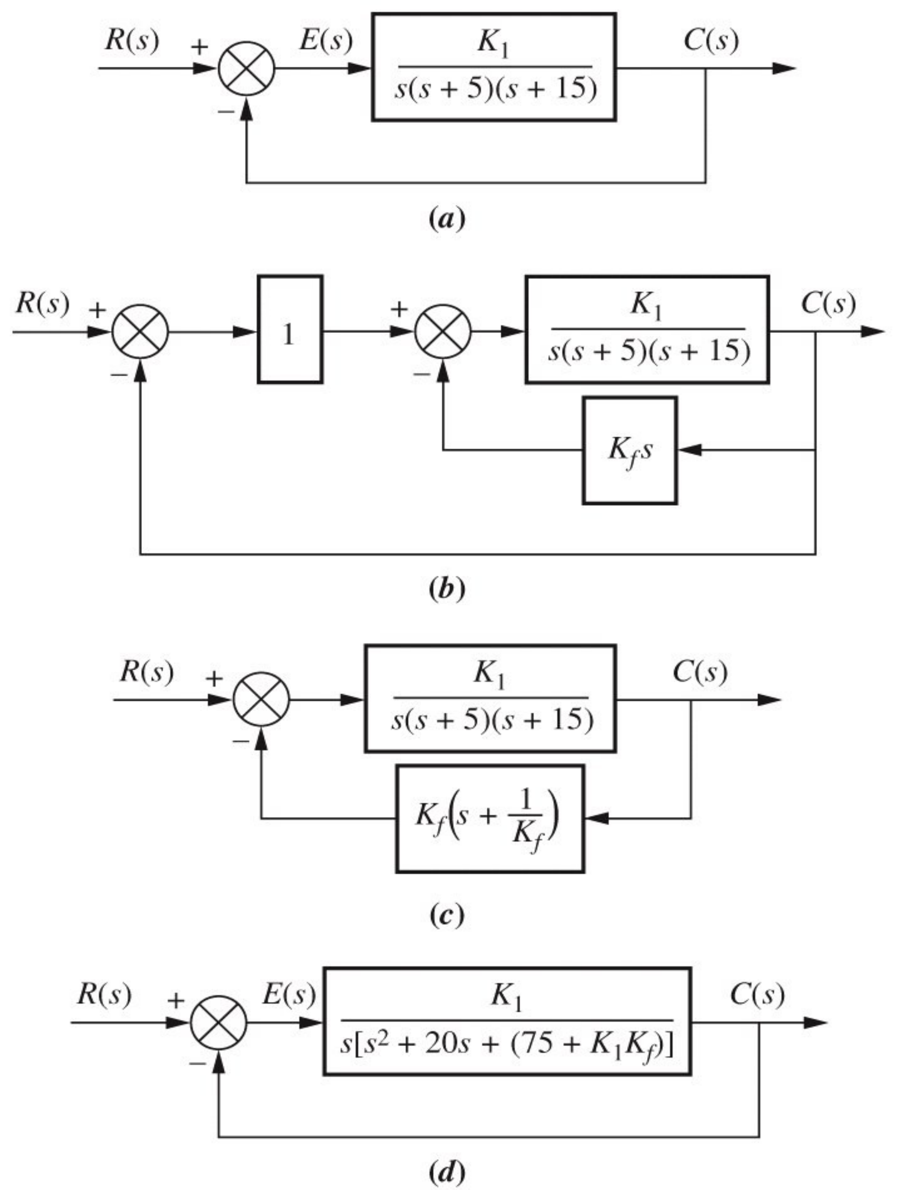


Figure 9.49

Root locus for uncompensated system

The settling time is 2.21 seconds and must be reduced by a factor of 4 to 0.55 second. Next determine the location of the dominant poles for the compensated system. To achieve a fourfold decrease in the settling time, the real part of the pole must be increased by a factor of 4. Thus, the compensated pole has a real part of 4(-1.809) = -7.236. The imaginary part is then

$\omega_d = -7.236 \tan 117.13° = 14.12$

where 117.13° is the angle of the 20% overshoot line.

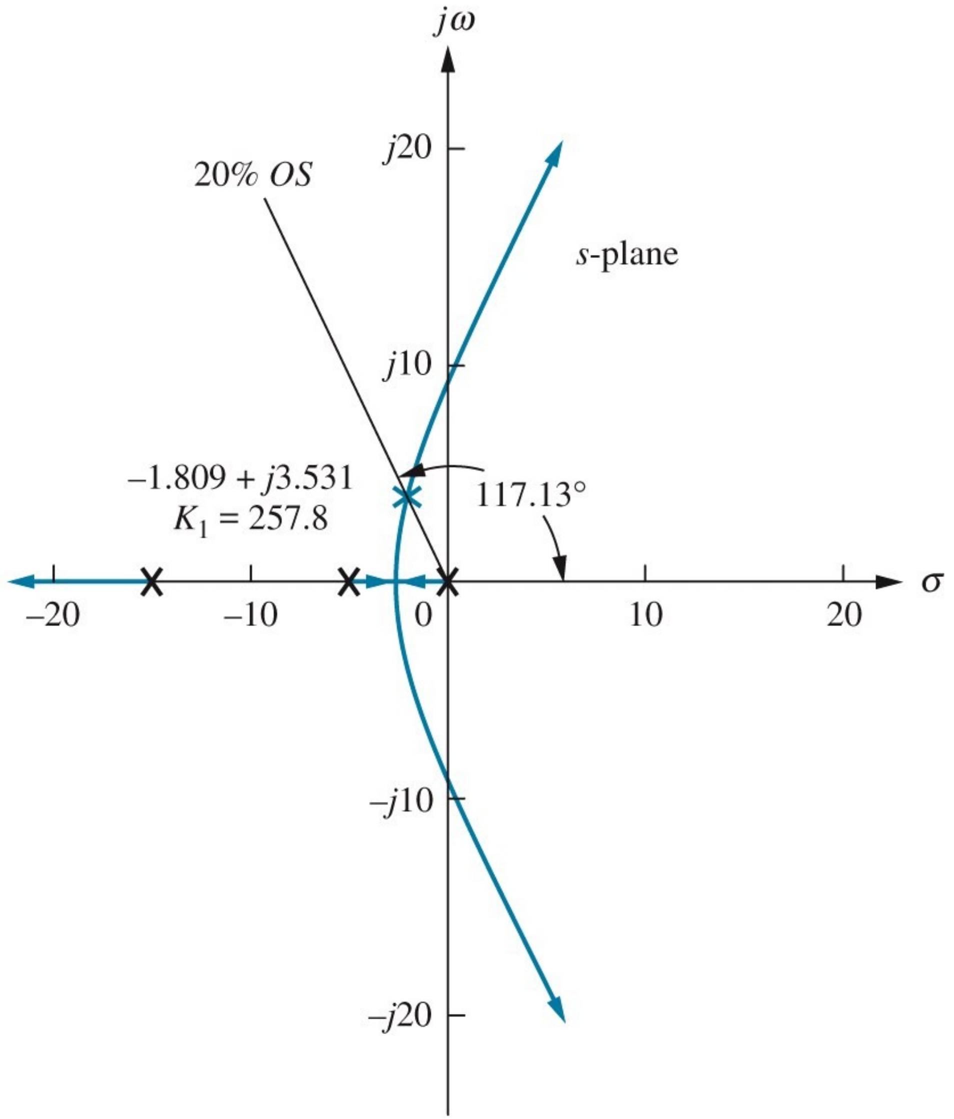


Figure 9.50

Step response for uncompensated system of Example

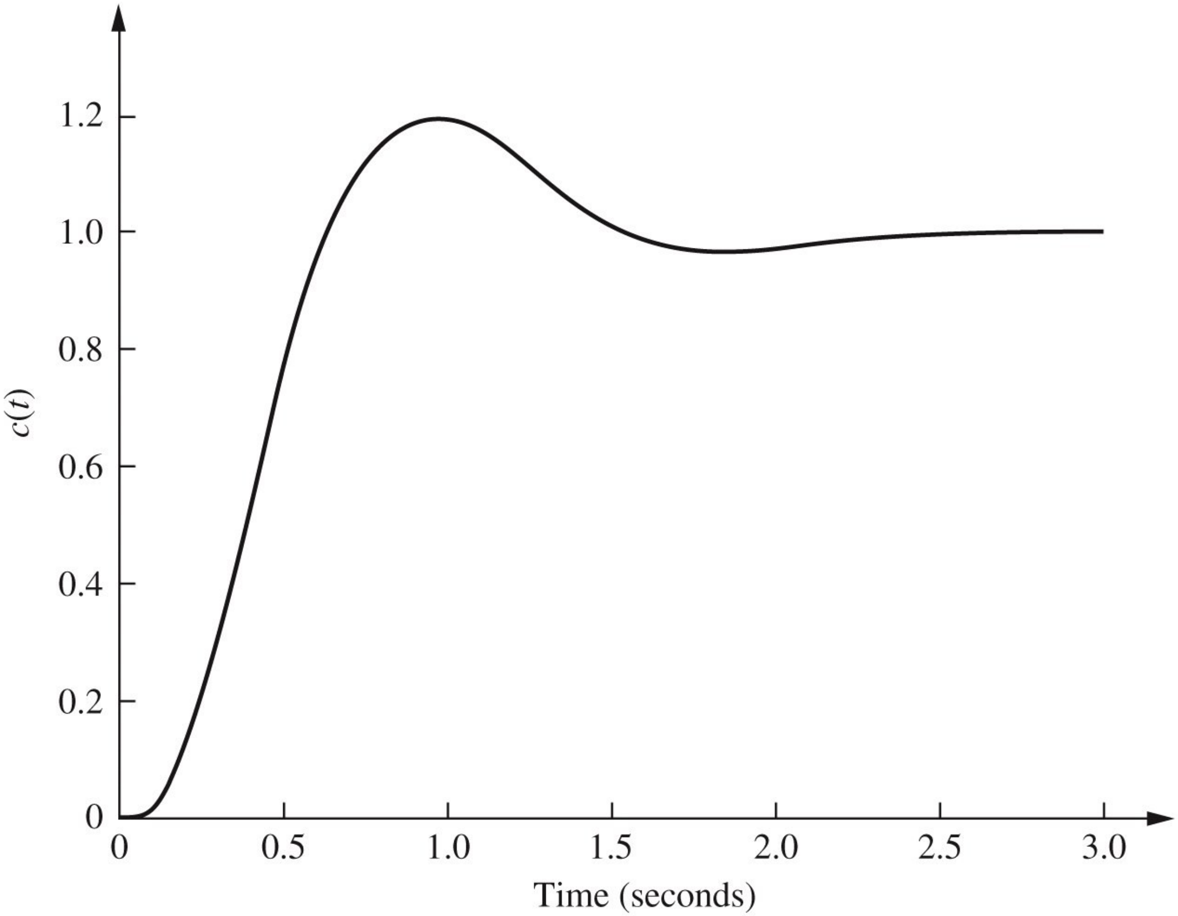


Figure 9.51

Predicted characteristics of uncompensated and compensated systems of Example

TABLE 9.8 Predicted characteristics of uncompensated and compensated systems of Example 9.7

	Uncompensated	Compensated
Plant and compensator	$\frac{K_1}{s(s+5)(s+15)}$	$\frac{K_1}{s(s+5)(s+15)}$
Feedback	1	$0.185(s+5.42)$
Dominant poles	$-1.809 \pm j3.531$	$-7.236 \pm j14.12$
K_1	257.8	1388
ζ	0.456	0.456
ω_n	3.97	15.87
$\%OS$	20	20
T_s	2.21	0.55
T_p	0.89	0.22
K_v	3.44	4.18
$e(\infty)$ (ramp)	0.29	0.24
Other poles	−16.4	−5.53
Zero	None	None
Comments	Second-order approx. OK	Simulate

Table 9.8

Finding the compensator zero in Example

Using the compensated dominant pole position of -7.236 ±j14.12, we sum the angles from the uncompensated system's poles and obtain -277.33°. This angle requires a compensator zero contribution of +97.33° to yield 180° at the design point. The geometry shown in the Figure leads to the calculation of the compensator's zero location. Hence, 14.12/ $(7.236 – z_c)$ = tan(180° - 97.33°); from which z_c = 5.42.

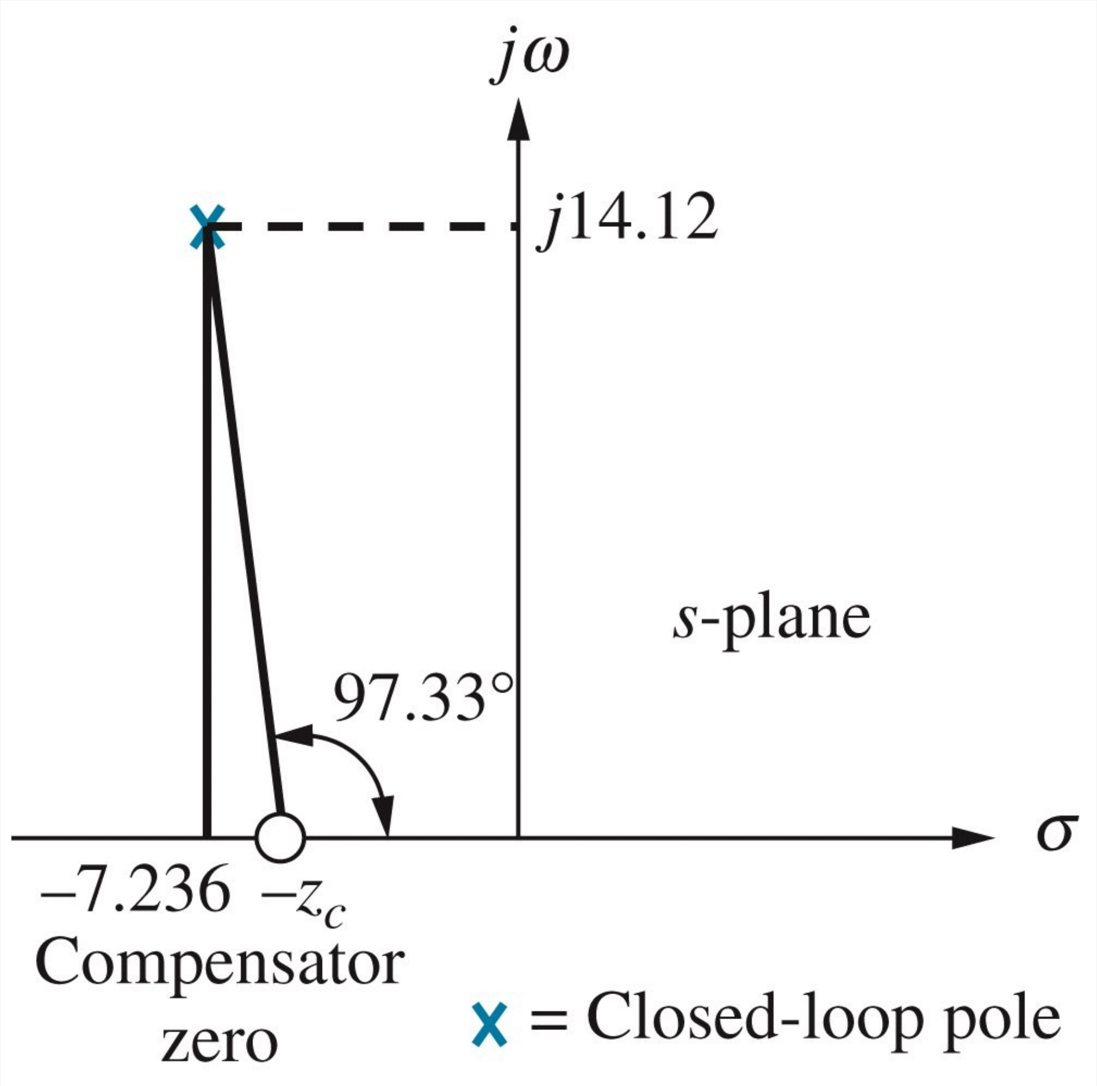


Figure 9.52

Root locus for the compensated system of Example

The root locus for the equivalent compensated system is shown in the Figure. The gain at the design point, which is K_1K_f from Figure 9.49(c), is found to be 256.7. Since K_f is the reciprocal of the compensator zero, K_f = 0.185. Thus, K_1 = 1388.

In order to evaluate the steady-state error characteristic, K_v is found from Figure 9.49(d) to be
$Kv = K_1 /(75 + K_1K_f) = 4.18$

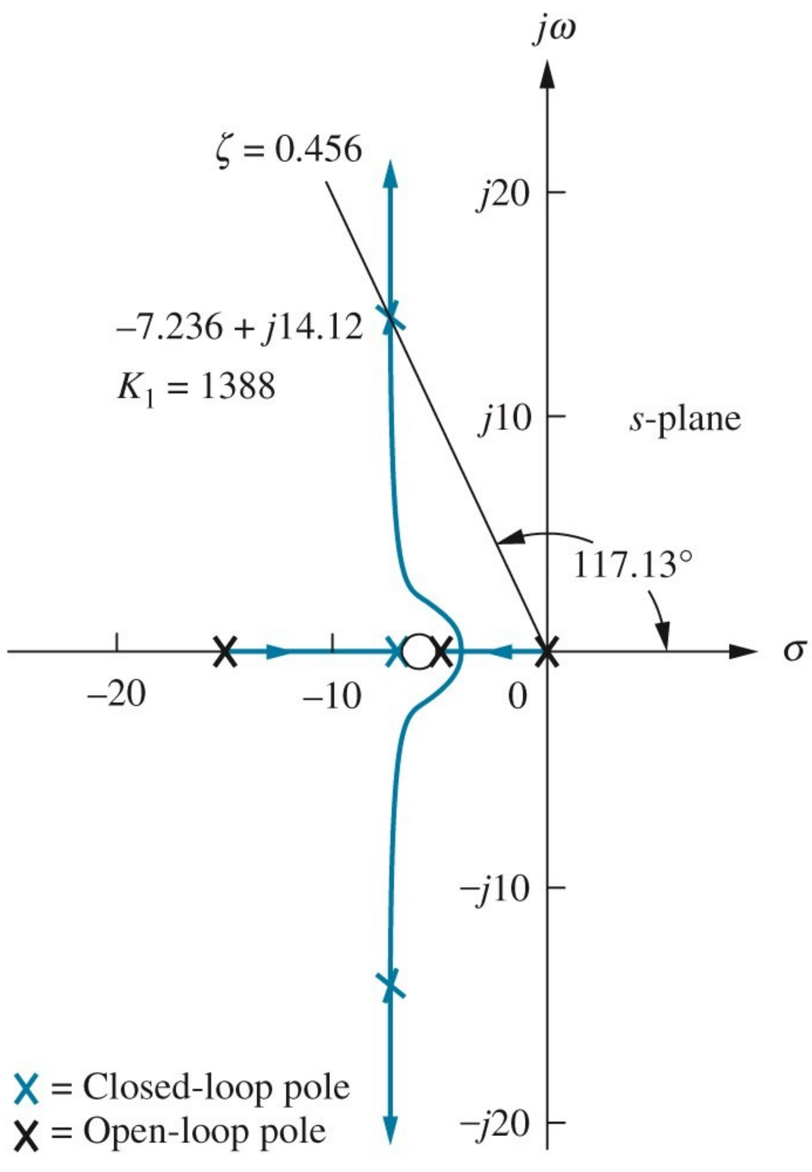


Figure 9.53

Step response for the compensated system of Example

we see that the closed-loop transfer function is $T(s) = G(s) / \{1 + G(s)H(s)\} = K_1 / (s^3 + 20s^2 + (75 + K_1K_f)s + K_1)$

Thus, as predicted, the open-loop zero is not a closed-loop zero, and there is no pole-zero cancellation. Hence, the design must be checked by simulation.

The results of the simulation are shown in Figure and show an over-damped response with a settling time of 0.75 second, compared to the uncompensated system's settling time of approximately 2.2 seconds

Although not meeting the design requirements, the response still represents an improvement over the uncompensated system. Typically, less overshoot is acceptable. The system should be redesigned for further reduction in settling time.

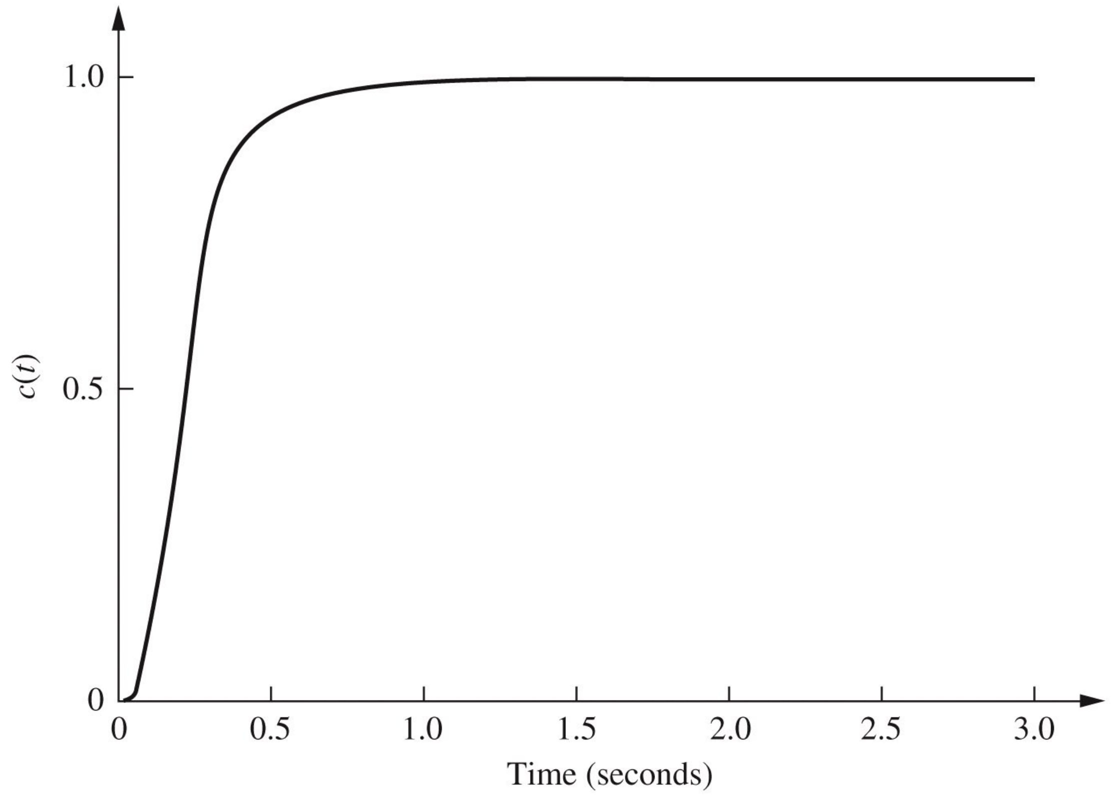


Figure 9.54

Physical Realization of Compensation -- Active-Circuit Realization

$$V_o(s)/V_i(s) = Z_2(s)/Z_1(s)$$

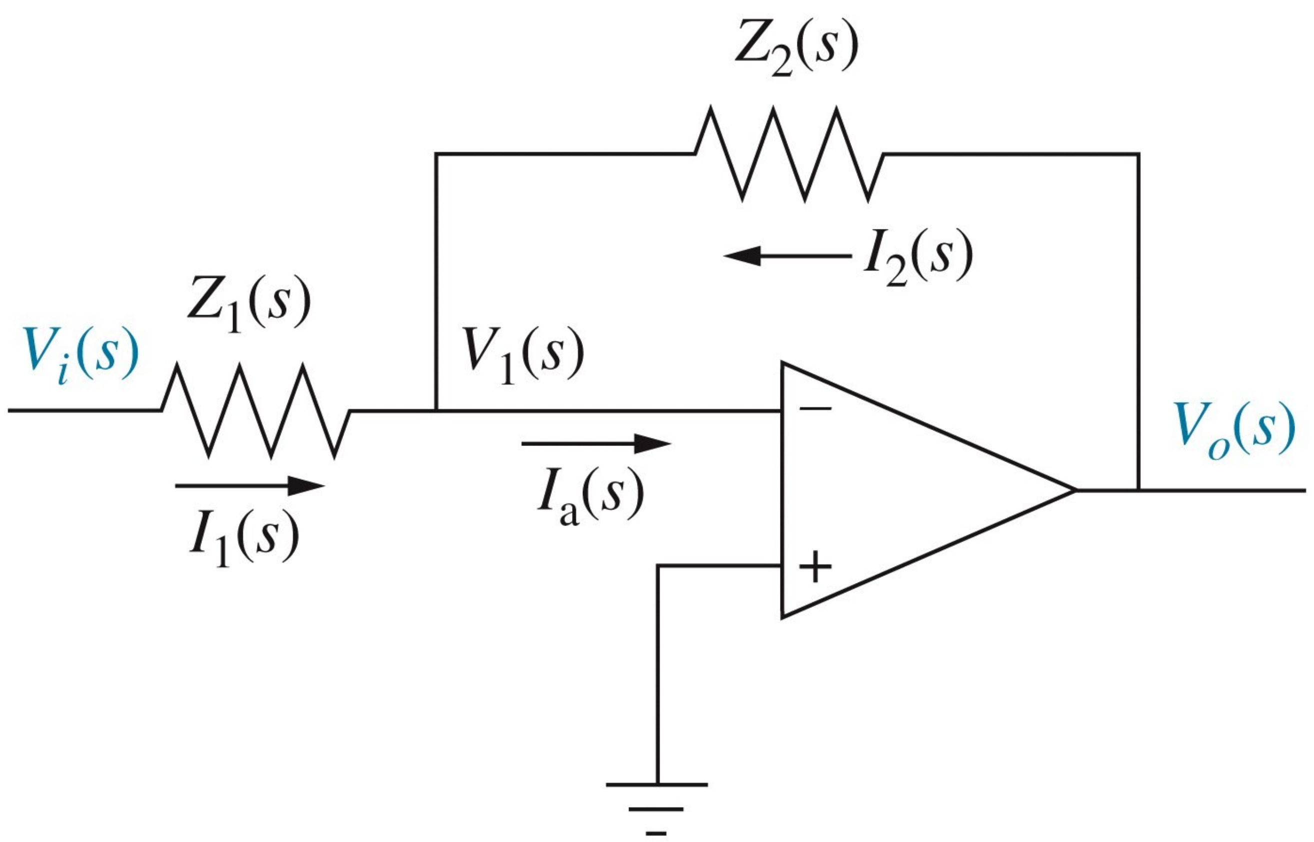


Figure 9.60

Operational amplifier configured for transfer function realization

TABLE 9.10 Active realization of controllers and compensators, using an operational amplifier

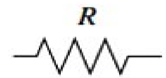
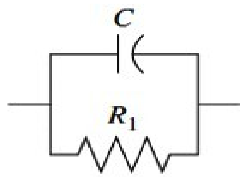
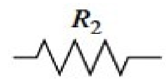
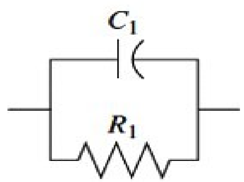
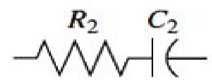
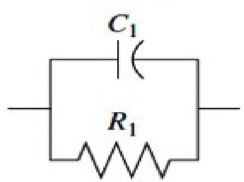
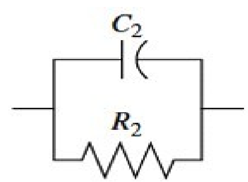
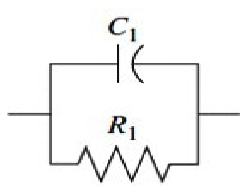
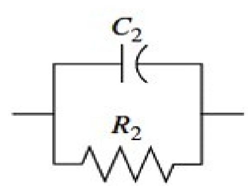
Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain			$-\frac{R_2}{R_1}$
Integration			$-\frac{\frac{1}{RC}}{s}$
Differentiation			$-RCs$
PI controller			$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller			$-R_2C\left(s+\frac{1}{R_1C}\right)$
PID controller			$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$
Lag compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

Table 9.10

Lag-lead compensator implemented with operational amplifiers

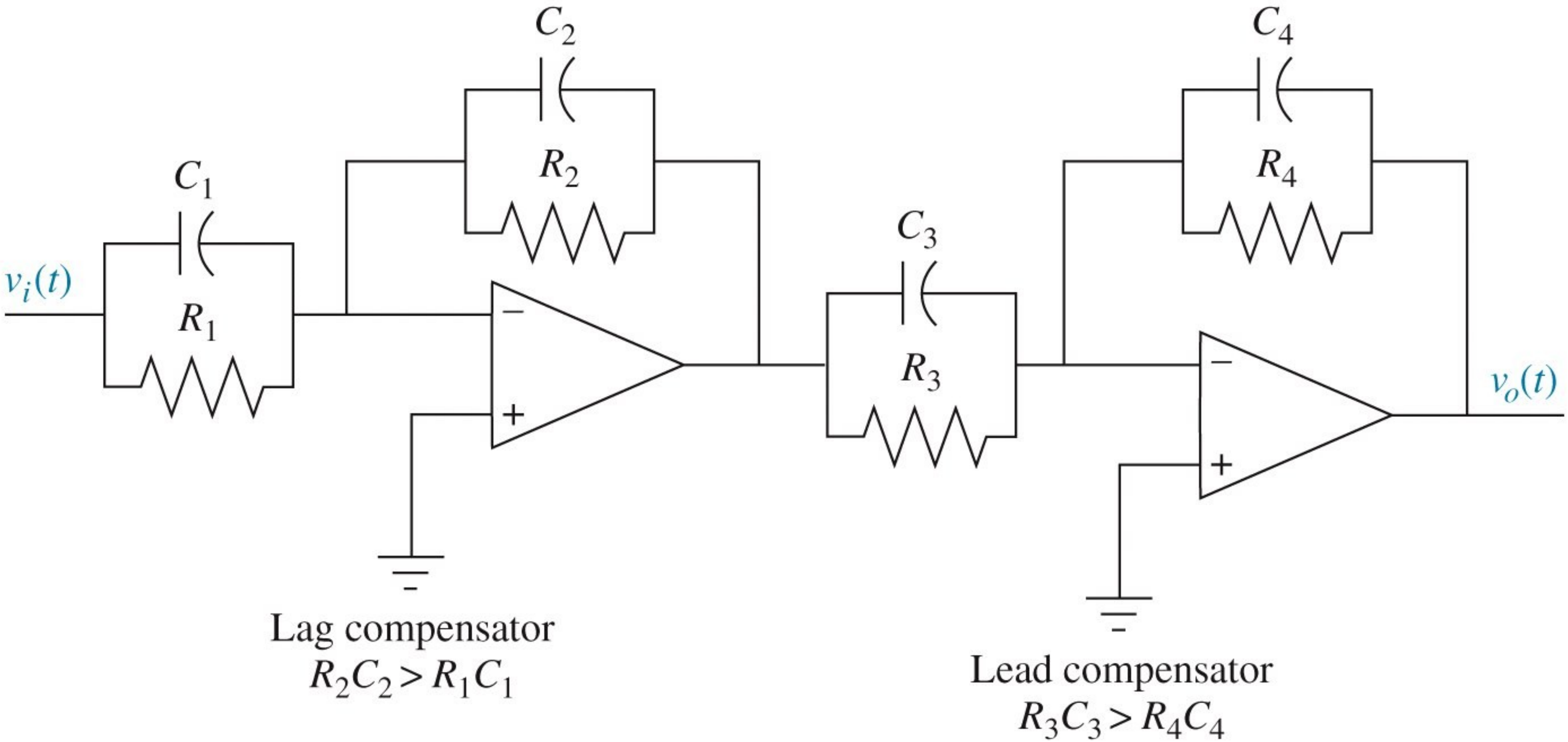


Figure 9.61

Example 9.9 Implementing a PID Controller

PROBLEM: Implement the PID controller of Example 9.5.

SOLUTION: The transfer function of the PID controller is
$G_c(s) = (s + 55.92)(s + 0.5)/s$
which can be put in the form $G_c(s) = s + 56.42 + 27.96/s$
Comparing the PID controller in Table 9.10 with previous equation we obtain the following three relationships:

$R_2/R_1+C_1/C_2 = 56.42$
$R_2C_1 = 1$
$1/R_1C_2 = 27.96$
(Since there are four unknowns and three equations, we arbitrarily select a practical value for one of the elements. Selecting $C_2 = 0.1\mu F$, the remaining values are found to be $R_1 = 357.65$ kΩ, $R_2 = 178{,}891$ kΩ, and $C_1 = 5.59$ μF.
The complete circuit is shown in Figure 9.62, where the circuit element values have been rounded off.

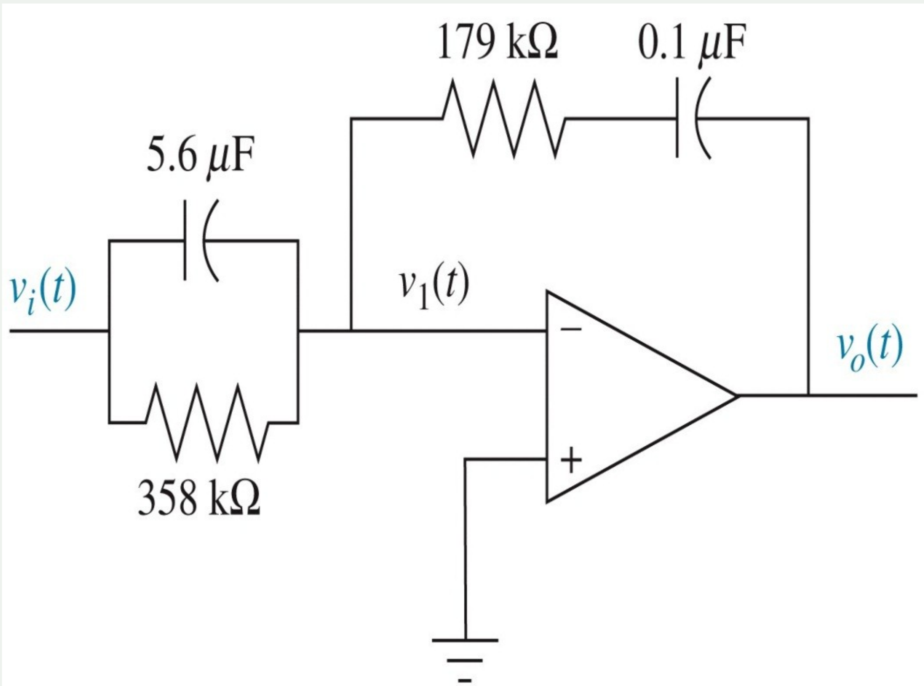
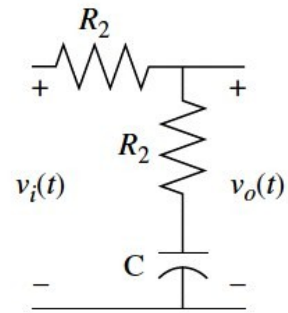
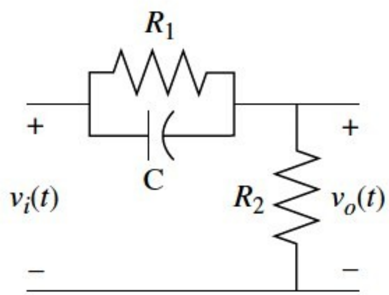
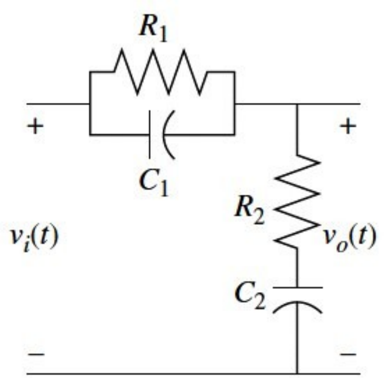


Figure 9.62

TABLE 9.11 Passive realization of compensators

Function	Network	Transfer function, $\dfrac{V_o(s)}{V_i(s)}$
Lag compensation		$\dfrac{R_2}{R_1+R_2}\dfrac{s+\dfrac{1}{R_2C}}{s+\dfrac{1}{(R_1+R_2)C}}$
Lead compensation		$\dfrac{s+\dfrac{1}{R_1C}}{s+\dfrac{1}{R_1C}+\dfrac{1}{R_2C}}$
Lag-lead compensation		$\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_2}+\dfrac{1}{R_2C_1}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$

able 9.11