FEMT
Unit-2

Conductor, Semiconductor, Dielectric and Boundary Conditions

Review (1)

- The *electric potential difference* V_{ba} is a work done by an external force to move a charge from point *a* to point *b* in an electric field divided by the amount of charge moved.

$$V_{ba} = \frac{W}{Q} = -\int_a^b \vec{E} \cdot d\vec{L}$$

- The electric potential is the same no matter which routes are used. Only displacement distance (shortest route) matters

Review (2)

- Conductors and Ohm's law
  - **Current, I** is defined as the amount of charge that passes through a reference plane in a given amount of time.

$$I = \frac{dQ}{dt} \quad \text{Ampere.}$$

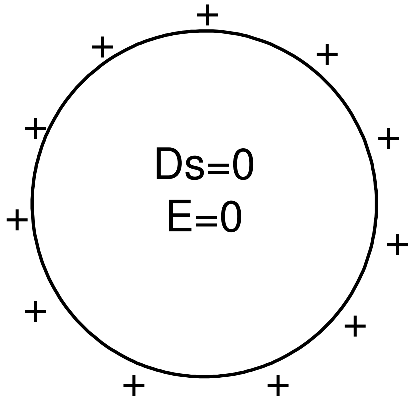
  - **Current density, J** is defined as the amount of current per unit area $\Delta I = \vec{J} \cdot \Delta \vec{S}$ A/m²
  - the relationship between I and J, $I = \int_s \vec{J} \cdot \Delta \vec{S}$
  - convection current $\vec{J} = \rho_v \vec{v}$
  - conduction current $\vec{J} = \sigma \vec{E}$

Outline

- Conductor and boundary conditions
- Semiconductor and insulator

Conductors and boundary conditions

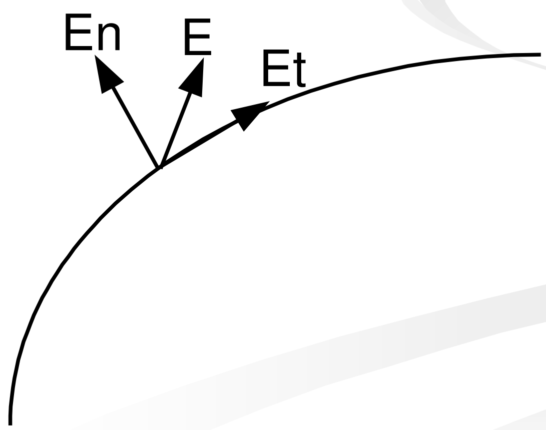
- charge density is zero inside a conductor
- Surface charge density D_S is on the conductor surface
- An electric field inside a conductor is zero



outside charges
cause and electric field.

Tangential and normal fields.

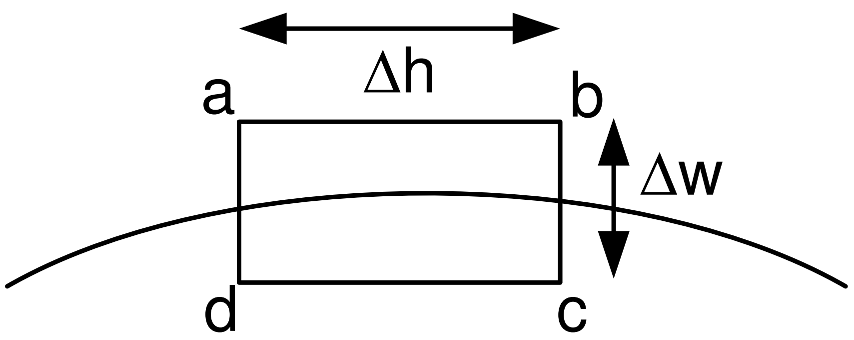
- The electric field on the surface can be divided into two components.
  - tangential electric field, E_t, = 0 for an equipotential surface
  - normal electric field, E_n



Boundary conditions (1)

- Consider a conductor-free space boundary

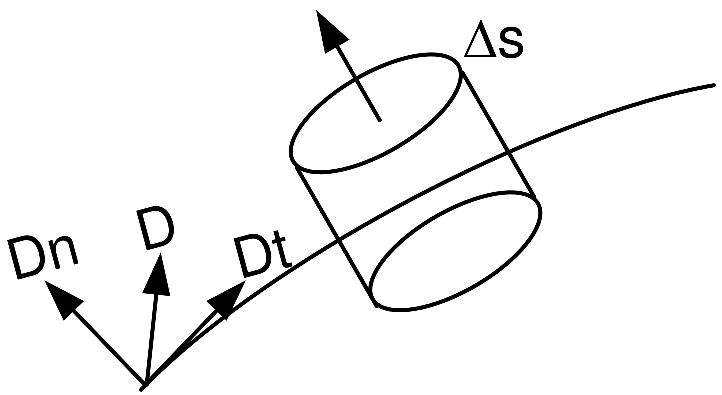
From $$\oint \vec{E} \cdot d\vec{l} = 0$$



Boundary conditions (3)

- Consider Gauss's law

From $$\oint \vec{D} \cdot d\vec{S} = Q$$



Boundary conditions (3)

- For conductor-free space boundary conditions (B.Cs.)

$$D_t = E_t = 0$$

$$D_n = \varepsilon_0 E_n = \rho_s$$

Ex1 Let $V = 100e^{-5x}\sin(3y)\cos(4z)$ **V and let a point** ***P*****(0.1, π/12, π/24) locate at the conductor-free space boundary. At point** ***P*****, determine**

a) V

b) E

c) E_n

d) E_t

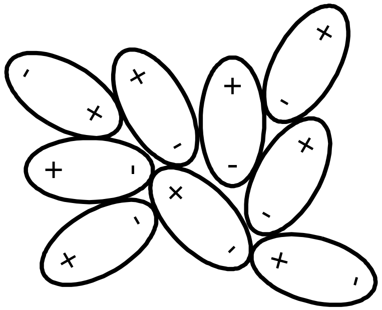
e) ρ_S

Semiconductors

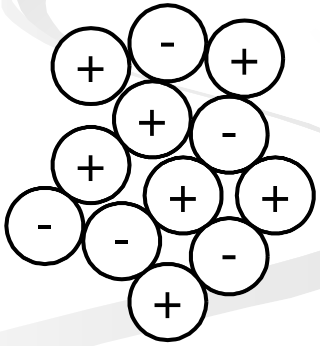
- Electron and hole currents
- conductivity $\sigma = -\rho_e \mu_e + \rho_h \mu_h$
- mobility is 10-100 times higher than conductor.
- electron and hole density depend on temperature.
- Doping is the process of adding impurities to a semiconductor to alter the polarity.

Dielectric or insulator

- no free charge
- microscopic electric dipoles
- energy stored capability
- polar and non-polar molecules

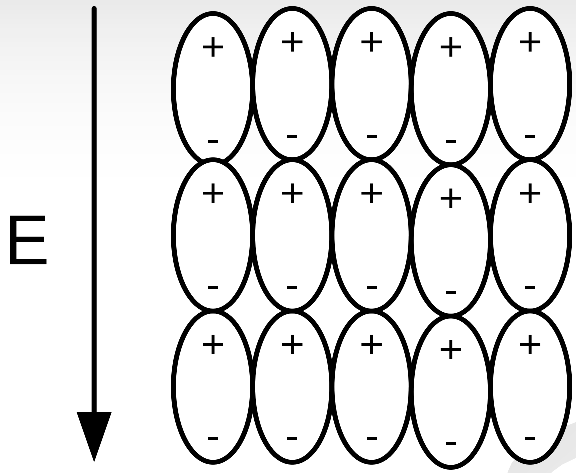


polar molecules



non-polar molecules

Alignment of dipoles with E field



Polarization

Each dipole has its dipole moment, $\vec{p}$

$$\vec{p} = Q\vec{d} \qquad C \cdot m$$

where Q is the positive one of the two bound charges
$\vec{d}$ is the vector from the negative to the positive charge.

$$\vec{p}_{total} = \sum_{i=1}^{n\Delta v} p_i$$

where n = number of dipoles per volume.

Polarization is dipole moment per unit volume,

$$\vec{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} p_i \qquad C/m^2$$

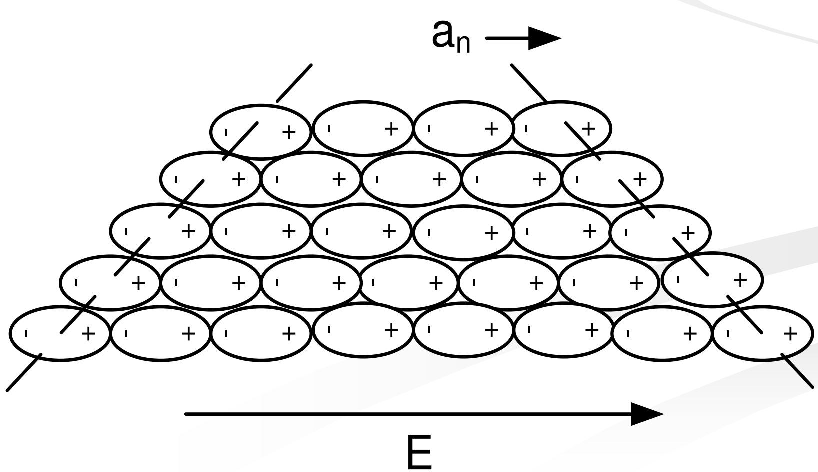
Equivalent polarization

- The movement of bound charges when induced by an electric field causes changes in surface and volume charge densities.

1. Equivalent polarization surface charge density, $\rho_{\rho s}$

$$\rho_{\rho s} = \vec{P} \cdot a_n \qquad C/m^2$$

2. Equivalent polarization volume charge density, $\rho_{\rho v}$



Electric flux density in a dielectric material (1)

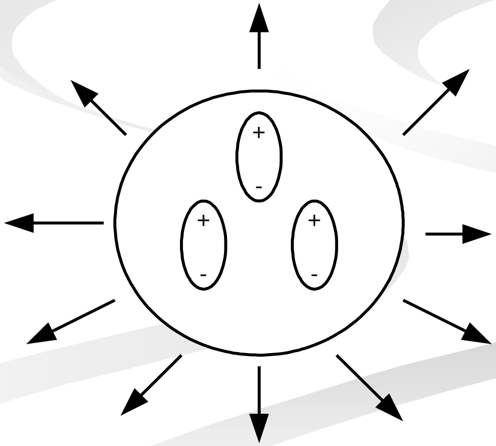
- $\vec{D}$ can be calculated from free and bound charges.

Net bound charges flowing out of the closed surfaces,

$$Q_b = -\oint_S \vec{P} \cdot d\vec{S} \qquad C$$

Let $$Q_T = Q + Q_b$$

where Q_T = total charge
Q = free charge



Electric flux density in a dielectric material (2)

From $$Q_T = \oint_s \varepsilon_0 \vec{E} \cdot d\vec{S}$$

then $$Q = Q_T - Q_b = \oint_s (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S}.$$

Let $$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

we can write $$Q = \oint_s \vec{D} \cdot d\vec{S}$$

where $\vec{D}$ is an electric flux density in a dielectric material.

Equivalent divergence relationships

From $$Q_b = -\oint_s \vec{P} \cdot a_n d\vec{S},$$

use a divergence theorem, we have $$-\oint_s \vec{P} \cdot d\vec{S} = \oint_v -\nabla \cdot \vec{P} dv.$$

Since $$Q_b = \int \rho_{\rho v} dv$$

then $$\nabla \cdot \vec{P} = -\rho_{\rho v} = -\rho_b.$$

We can also show that $$\nabla \cdot \varepsilon_0 \vec{E} = \rho_T,$$

therefore $$\nabla \cdot \vec{D} = \rho_T - \rho_b = \rho_v.$$

Electric flux density in dielectric medium (1)

- If the dielectric material is linear and isotropic, the polarization $\vec{P}$ is proportional to the electric field $\vec{E}$.

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where χ_e is an electric susceptibility.

Then $$\vec{D} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E}.$$

Let $$\varepsilon_r = \chi_e + 1$$

where ε_r is a relative permittivity or a dielectric constant

Electric flux density in dielectric medium (2)

So we can write

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$$

or

$$\vec{D} = \varepsilon \vec{E}$$

where

$$\varepsilon = \varepsilon_r \varepsilon_0 \quad \text{F/m.}$$

Ex2 **A dielectric material has an electric susceptibility χ_e =0.12 and has a uniform electric flux density D = 1.6 nC/m^2, determine**

a) *E*

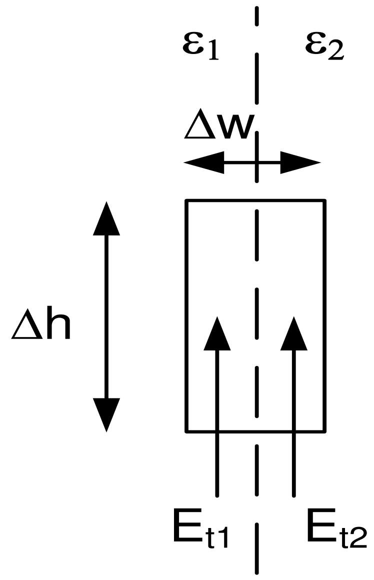
b) *P*

c) Average dipole moment if there are 2×10^{19} dipoles/m^3

Boundary conditions for dielectric materials: tangential fields

- It is useful to determine the electric field in a dielectric medium.

Dielectric-Dielectric



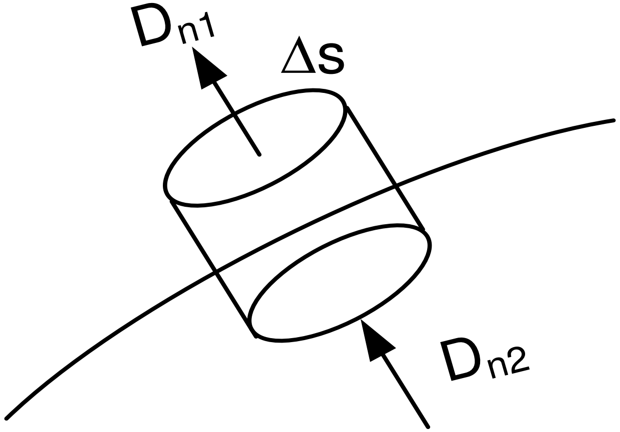
$$E_{t1} = E_{t2}$$

and

$$\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

Boundary conditions for dielectric materials: normal fields

Dielectric-Dielectric



$$a_{21} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_S$$

For a free of charge boundary,

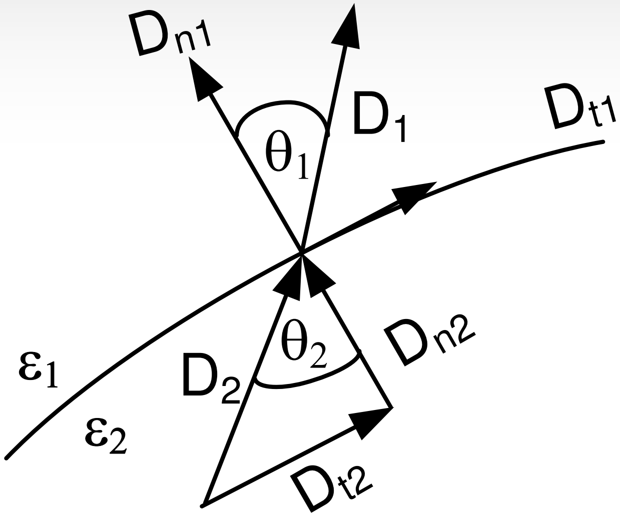
$$D_{n1} = D_{n2}$$

and

$$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}.$$

a_{21} is the unit vector pointing from medium 2 to medium 1

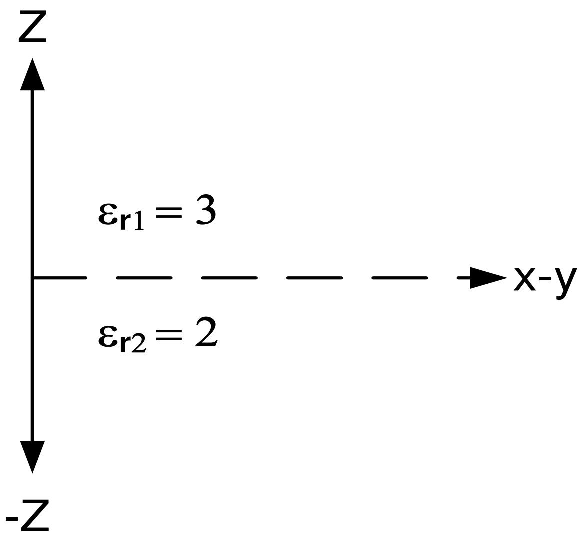
Use B.C.s to determine magnitude of $\vec{D}$ and $\vec{E}$



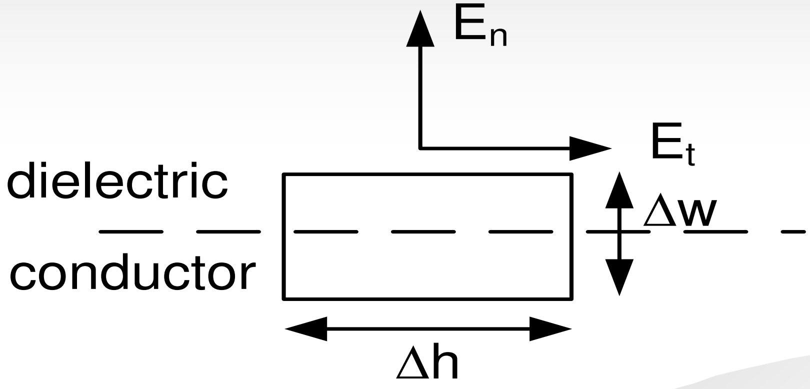
$$D_2 = D_1\sqrt{\cos^2\theta_1 + \frac{\varepsilon_2^2}{\varepsilon_1^2}\sin^2\theta_1}$$

$$E_2 = E_1\sqrt{\sin^2\theta_1 + \frac{\varepsilon_1^2}{\varepsilon_2^2}\cos^2\theta_1}$$

<u>Ex3</u> The isotropic dielectric medium with $\varepsilon_{r1} = 3$ and $\varepsilon_{r2} = 2$ is connected as shown. Given $\vec{E}_1 = a_x - 5a_y - 4a_z$ V/m, determine $\vec{E}_2$ and its magnitude, $\vec{D}_2$ and its magnitude, θ_1, and θ_2.



Boundary conditions for dielectric-conductor



$$D_t = E_t = 0$$

$$D_n = \varepsilon E_n = \rho_s$$

Ex4 **Between a dielectric-conductor interface has a surface charge density of ρ_s = 2x10^{-9} C/m^2. Given** $\vec{E}_1 = -30a_x + 50a_y + 70a_z$ **V/m, determine** $\vec{E}_2$.