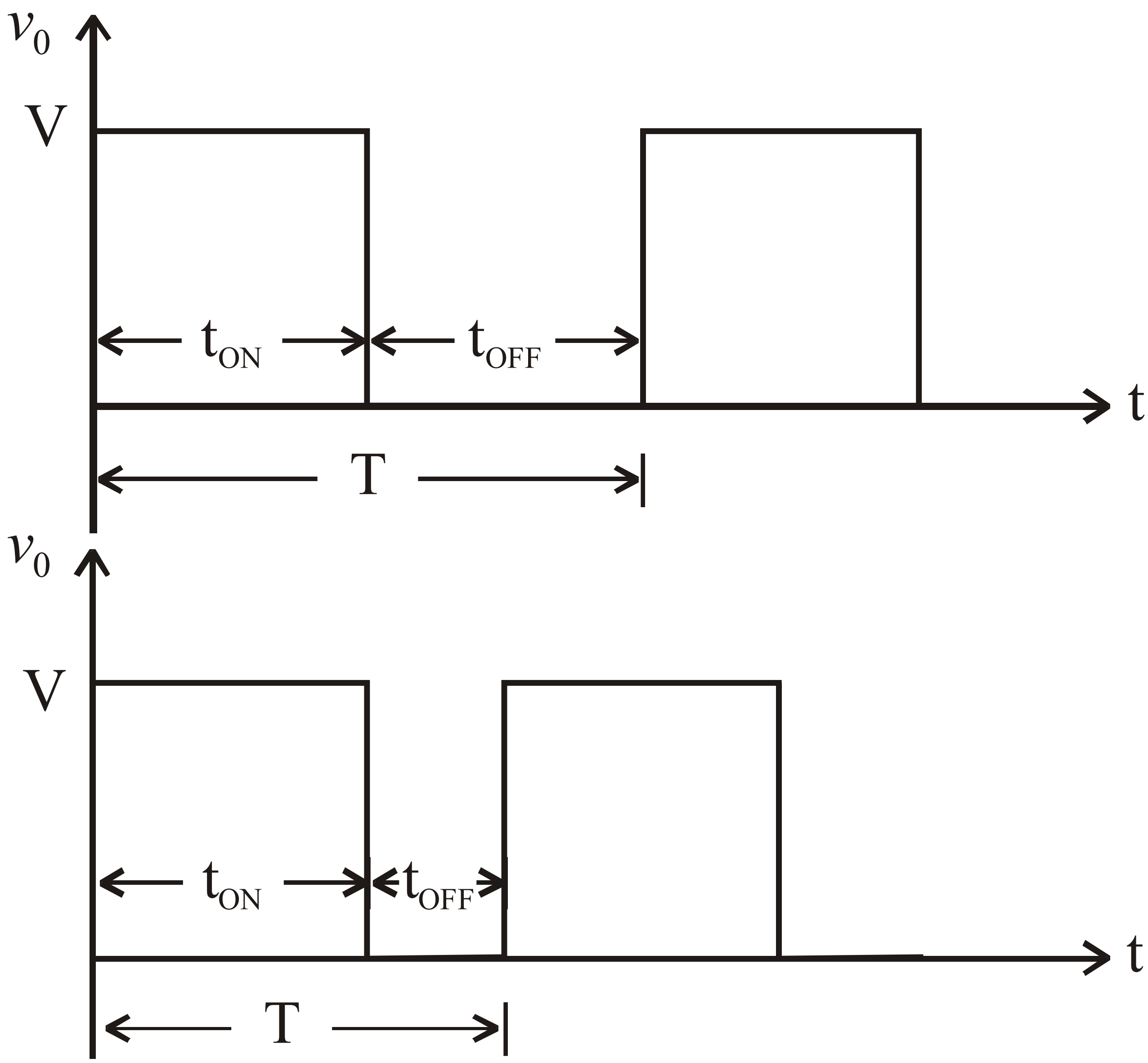
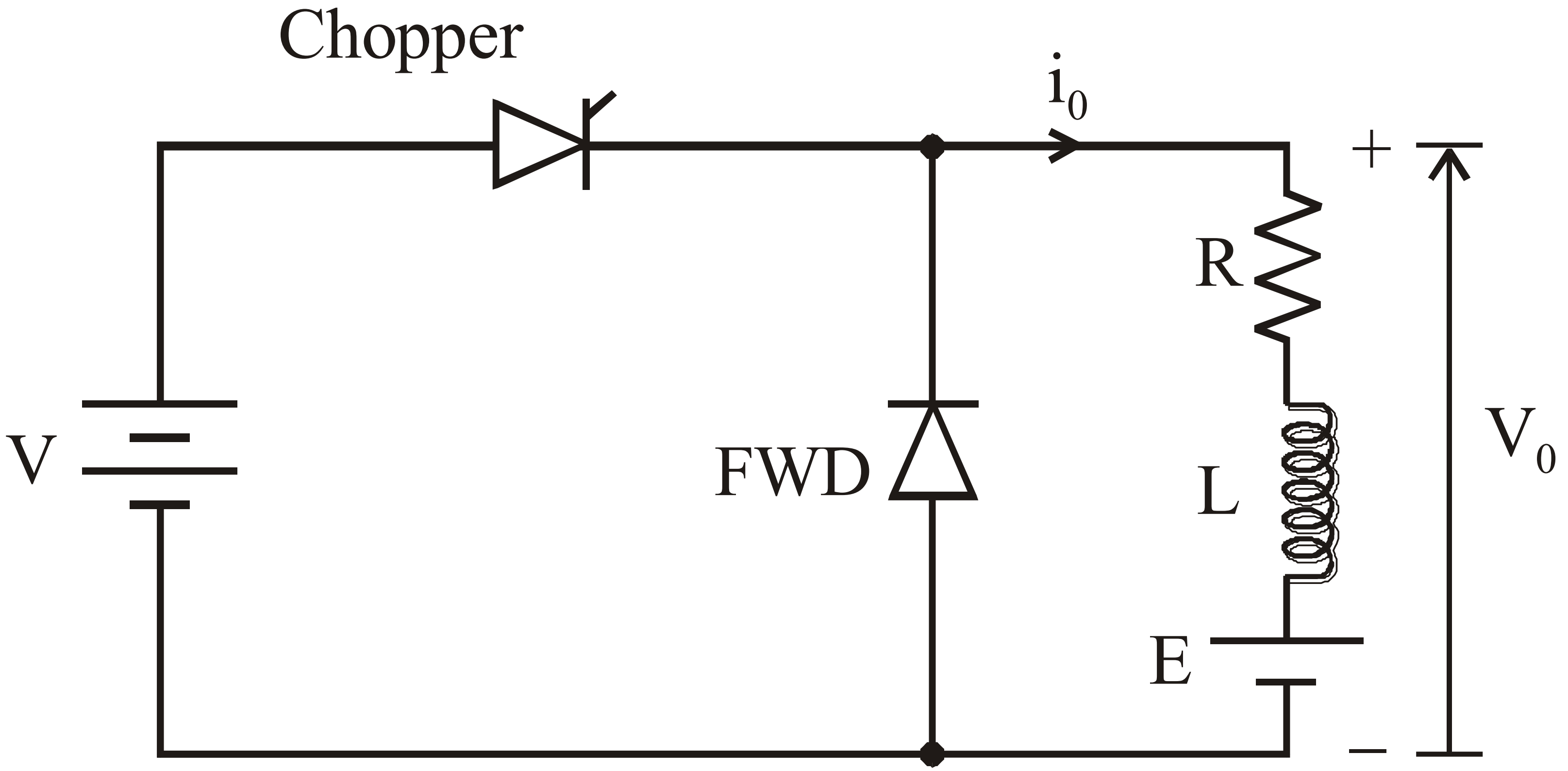
Variable Frequency Control

- Chopping frequency 'f' is varied keeping either t_{ON} or t_{OFF} constant.
- To obtain full output voltage range, frequency has to be varied over a wide range.
- This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous

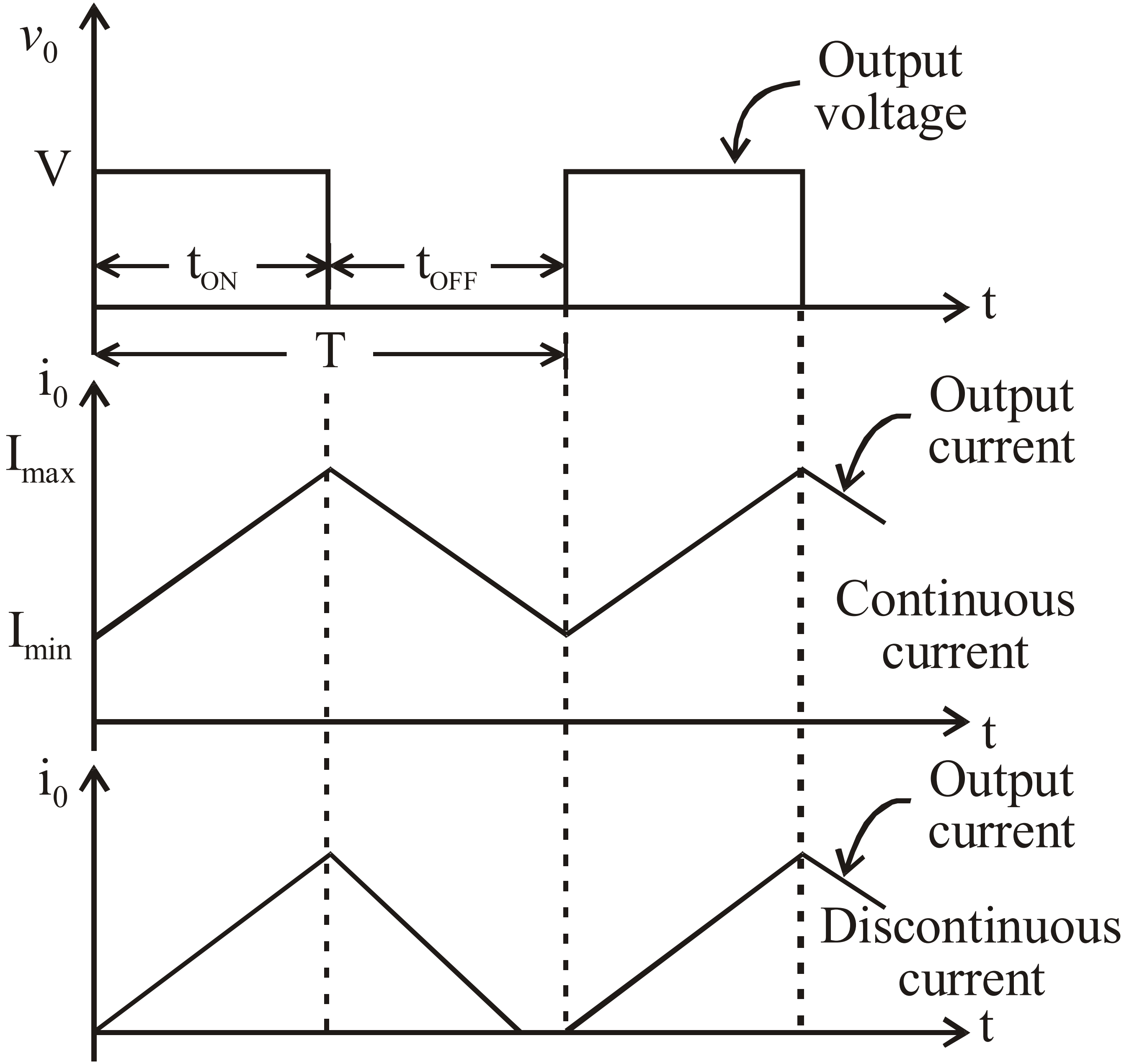

v_0
V
t_{ON}
t_{OFF}
t
T
v_0
V
t_{ON}
t_{OFF}
t
T

Step-down Chopper
With R-L Load



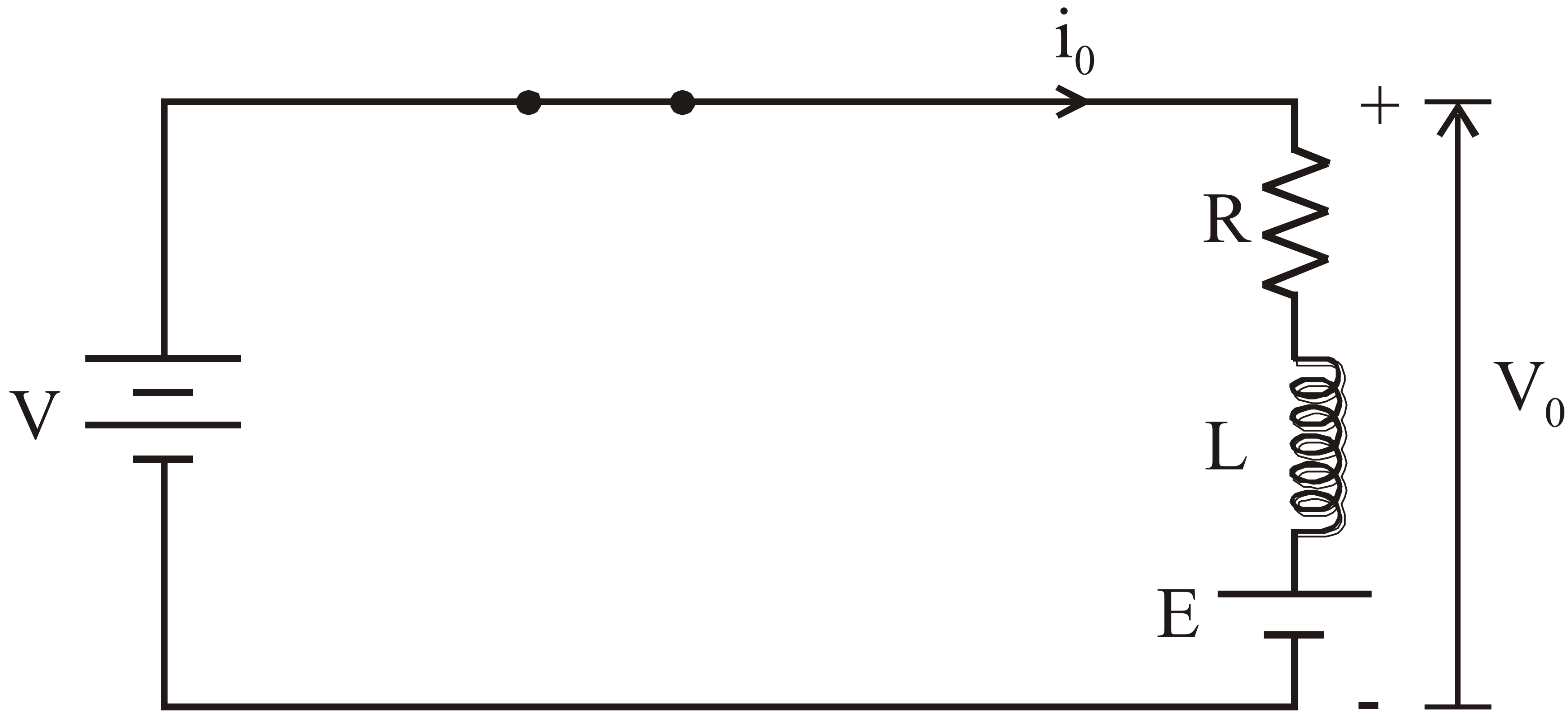
- When chopper is ON, supply is connected across load.
- Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor *'L'*.

- Load current can be continuous or discontinuous depending on the values of *'L'* and duty cycle *'d'*
- For a continuous current operation, load current varies between two limits I_{max} and I_{min}
- When current becomes equal to I_{max} the chopper is turned-off and it is turned-on when current reduces to I_{min}.



v_0
V
Output
voltage
t_{ON}
t_{OFF}
t
T
i_0
I_{max}
Output
current
I_{min}
Continuous
current
t
i_0
Output
current
Discontinuous
current
t

Expressions For
Load Current
i_O For Continuous Current Operation When
Chopper Is ON $(0 \leq t \leq t_{ON})$



$\dot{i}_0$
V
R
L
E
+
-
V_0

$$V = i_O R + L\frac{di_O}{dt} + E$$

Taking Laplace Transform

$$\frac{V}{S} = RI_O(S) + L\left[S.I_O(S) - i_O(0^-)\right] + \frac{E}{S}$$

At $t = 0$, initial current $i_O(0^-) = I_{\min}$

$$I_O(S) = \frac{V - E}{LS\left(S + \frac{R}{L}\right)} + \frac{I_{\min}}{S + \frac{R}{L}}$$

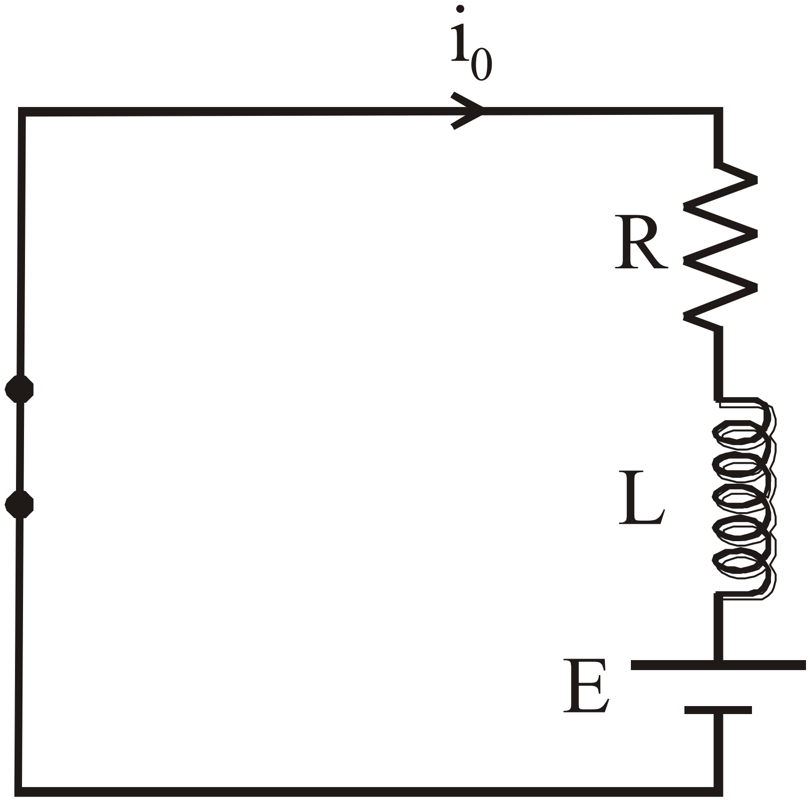
Taking Inverse Laplace Transform

$$i_O(t) = \frac{V-E}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right] + I_{\min} e^{-\left(\frac{R}{L}\right)t}$$

This expression is valid for $0 \le t \le t_{ON}$, i.e., during the period chopper is ON.

At the instant the chopper is turned off, load current is $i_O(t_{ON}) = I_{\max}$

When Chopper is OFF



When Chopper is OFF $\left(0 \le t \le t_{OFF}\right)$

$$0 = Ri_O + L\frac{di_O}{dt} + E$$

Talking Laplace transform

$$0 = RI_O\left(S\right) + L\left[SI_O\left(S\right) - i_O\left(0^-\right)\right] + \frac{E}{S}$$

Redefining time origin we have at $t = 0$,

initial current $i_O\left(0^-\right) = I_{\max}$

$$\therefore I_O(S) = \frac{I_{\max}}{S + \frac{R}{L}} - \frac{E}{LS\left(S + \frac{R}{L}\right)}$$

Taking Inverse Laplace Transform

$$i_O(t) = I_{\max} e^{-\frac{R}{L}t} - \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

The expression is valid for $0 \le t \le t_{OFF}$,
i.e., during the period chopper is OFF

At the instant the chopper is turned ON or at the end of the off period, the load current is

$$i_O\left(t_{OFF}\right) = I_{\min}$$

To Find $I_{\max}$ & $I_{\min}$

From equation

$$i_O(t) = \frac{V-E}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right] + I_{\min} e^{-\left(\frac{R}{L}\right)t}$$

At $\quad t = t_{ON} = dT, \quad i_O(t) = I_{\max}$

$$\therefore \quad I_{\max} = \frac{V-E}{R}\left[1-e^{-\frac{dRT}{L}}\right] + I_{\min} e^{-\frac{dRT}{L}}$$

From equation

$$i_O(t) = I_{\max} e^{-\frac{R}{L}t} - \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

At $t = t_{OFF} = T - t_{ON}, \quad i_O(t) = I_{\min}$

$$t = t_{OFF} = (1-d)T$$

$$\therefore \qquad I_{\min} = I_{\max} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R}\left[1 - e^{-\frac{(1-d)RT}{L}}\right]$$

Substituting for $I_{\min}$ in equation

$$I_{\max} = \frac{V-E}{R}\left[1 - e^{-\frac{dRT}{L}}\right] + I_{\min} e^{-\frac{dRT}{L}}$$

we get,

$$I_{\max} = \frac{V}{R}\left[\frac{1 - e^{-\frac{dRT}{L}}}{1 - e^{-\frac{RT}{L}}}\right] - \frac{E}{R}$$

Substituting for I_{max} in equation

$$I_{min} = I_{max} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R}\left[1 - e^{-\frac{(1-d)RT}{L}}\right]$$

we get,

$$I_{min} = \frac{V}{R}\left[\frac{e^{\frac{dRT}{L}} - 1}{e^{\frac{RT}{L}} - 1}\right] - \frac{E}{R}$$

$(I_{max} - I_{min})$ is known as the steady state ripple.

Therefore peak-to-peak ripple current

$$\Delta I = I_{\max} - I_{\min}$$

Average output voltage

$$V_{dc} = d.V$$

Average output current

$$I_{dc(approx)} = \frac{I_{\max} + I_{\min}}{2}$$

Assuming load current varies linearly from $I_{\min}$ to $I_{\max}$ instantaneous load current is given by

$$i_O = I_{\min} + \frac{(\Delta I).t}{dT} \quad for \;\; 0 \le t \le t_{ON}\,(dT)$$

$$i_O = I_{\min} + \left(\frac{I_{\max} - I_{\min}}{dT}\right)t$$

RMS value of load current

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT} i_0^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min} + \frac{(I_{\max} - I_{\min})t}{dT}\right]^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min}^2 + \left(\frac{I_{\max} - I_{\min}}{dT}\right)^2 t^2 + \frac{2I_{\min}(I_{\max} - I_{\min})t}{dT}\right] dt}$$

RMS value of output current

$$I_{O(RMS)} = \left[I_{\min}^2 + \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + I_{\min}\left(I_{\max} - I_{\min}\right) \right]^{\frac{1}{2}}$$

RMS chopper current

$$I_{CH} = \sqrt{\frac{1}{T}\int_0^{dT} i_0^2 \, dt}$$

$$I_{CH} = \sqrt{\frac{1}{T}\int_0^{dT} \left[I_{\min} + \left(\frac{I_{\max} - I_{\min}}{dT} \right) t \right]^2 dt}$$

$$I_{CH} = \sqrt{d}\left[I_{\min}^2 + \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + I_{\min}\left(I_{\max} - I_{\min}\right)\right]^{\frac{1}{2}}$$

$$I_{CH} = \sqrt{d}\, I_{O(RMS)}$$

Effective input resistance is

$$R_i = \frac{V}{I_S}$$

Where

$$I_S = \text{Average source current}$$

$$I_S = dI_{dc}$$

$$\therefore \qquad R_i = \frac{V}{dI_{dc}}$$