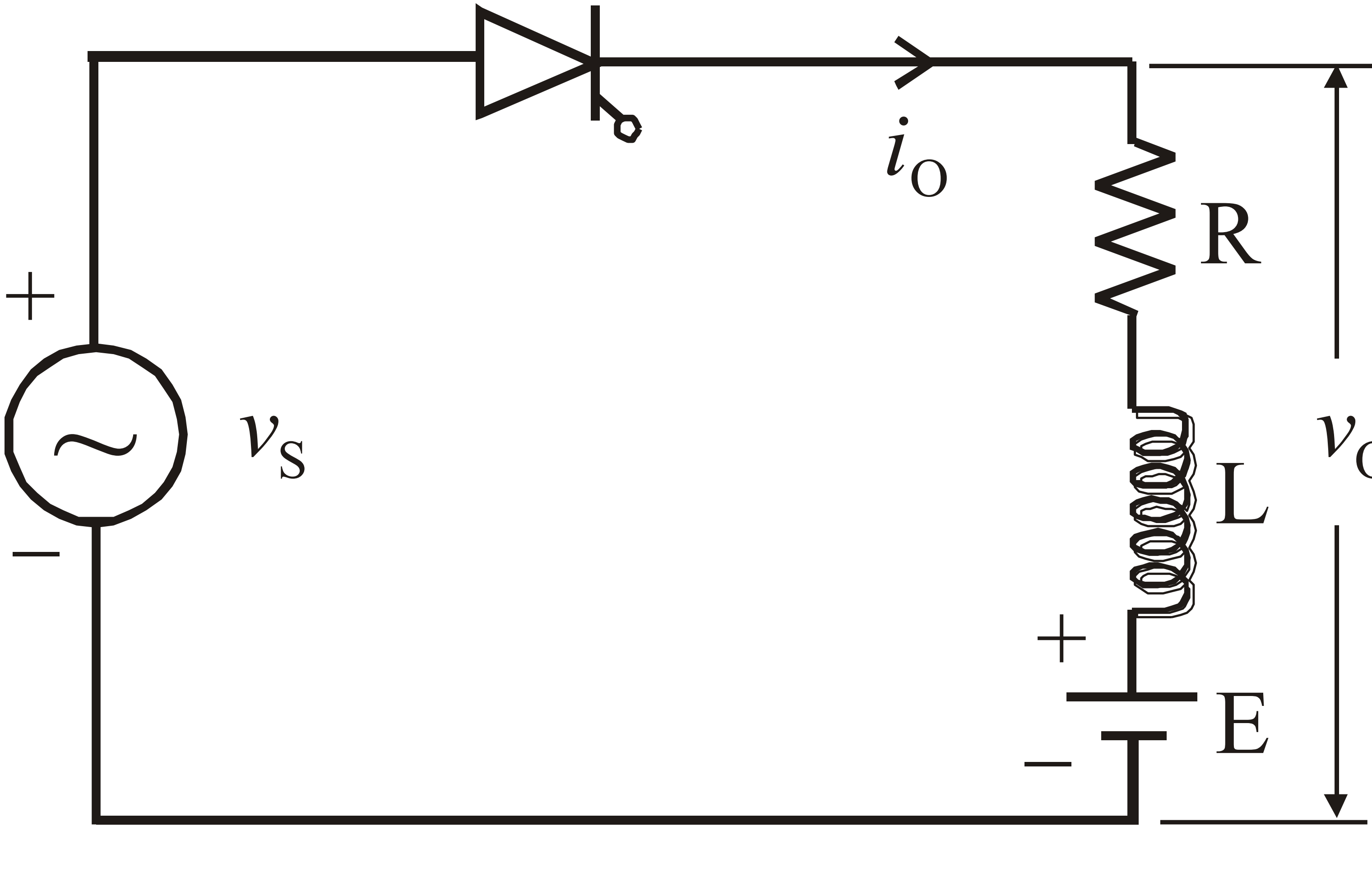
Single Phase Half Wave Controlled Rectifier With A General Load



+
−
v_S
i_O
R
L
+
E
−
v_O

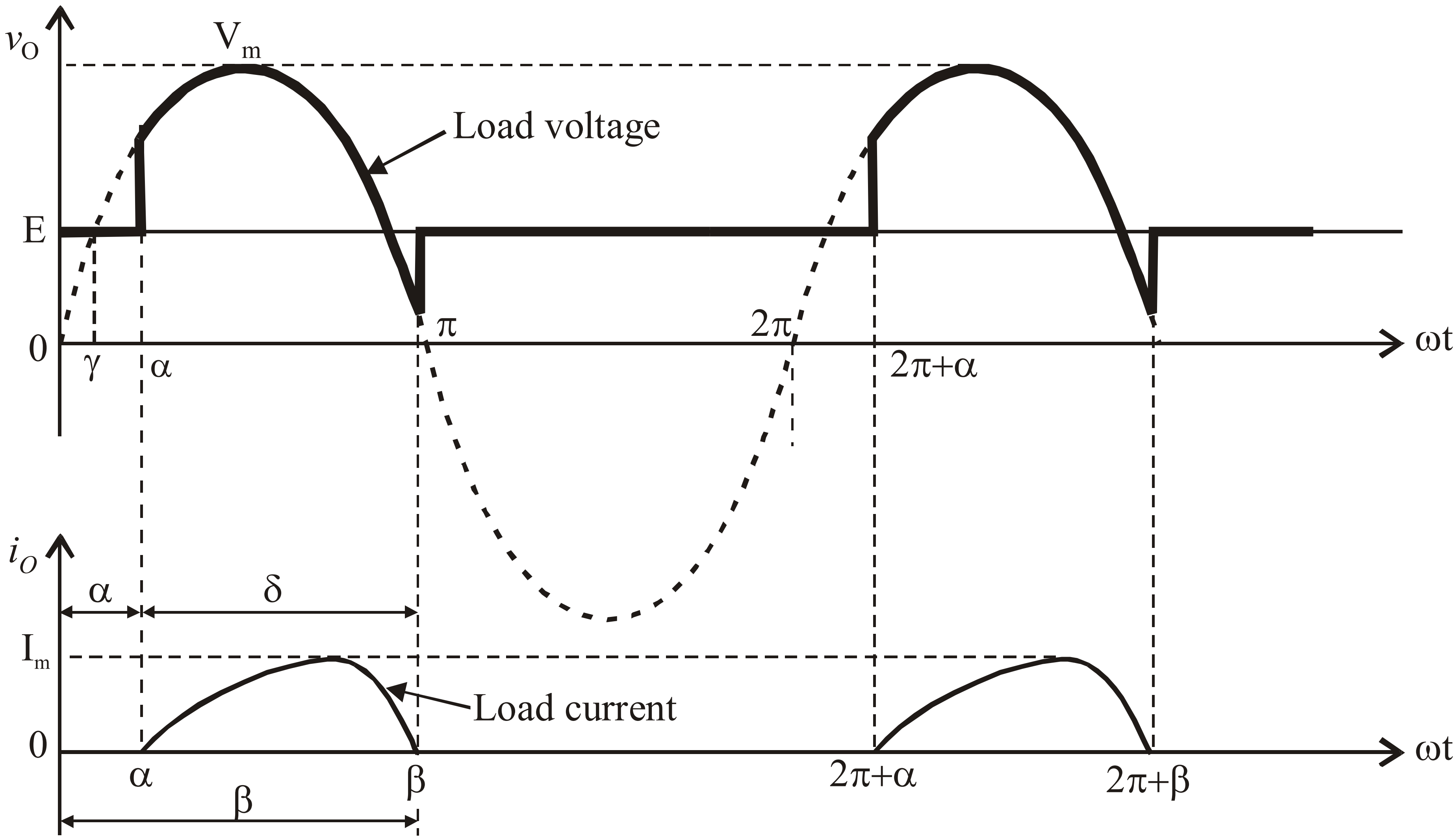
$$\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$$

For trigger angle $\alpha < \gamma$,

the Thyristor conducts from $\omega t = \gamma$ to β

For trigger angle $\alpha > \gamma$,

the Thyristor conducts from $\omega t = \alpha$ to β



v_O
V_m
Load voltage
E
0
γ
α
π
2π
ωt
2π+α
i_O
α
δ
I_m
Load current
0
α
β
β
2π+α
2π+β
ωt

Equations

$$v_S = V_m \sin \omega t = \text{ Input supply voltage.}$$

$$v_O = V_m \sin \omega t = \text{ o/p } (\text{load}) \text{ voltage}$$

$$\text{for } \omega t = \alpha \text{ to } \beta.$$

$$v_O = E \quad \text{for } \omega t = 0 \text{ to } \alpha \;\&$$

$$\text{for } \omega t = \beta \text{ to } 2\pi.$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of $\alpha > \gamma$, the eqn. for the circuit can be written as

$$V_m \sin \omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \text{E} \; ; \; \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z} \sin\left(\omega t - \phi\right) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

The general expression for the o/p current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$$

To find the value of the constant 'A' apply the initial conditions at $\omega t = \alpha$, load current $i_O = 0$, Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z} \sin\left(\alpha - \phi\right) \right] e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current; we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z} \sin\left(\omega t - \phi\right) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z} \sin\left(\alpha - \phi\right) \right] e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

To Derive
An
Expression For The Average
Or
DC Load Voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_{0}^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi} \right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta) \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta) \right] + \frac{E}{2\pi}\left[(2\pi - \beta + \alpha) \right]$$

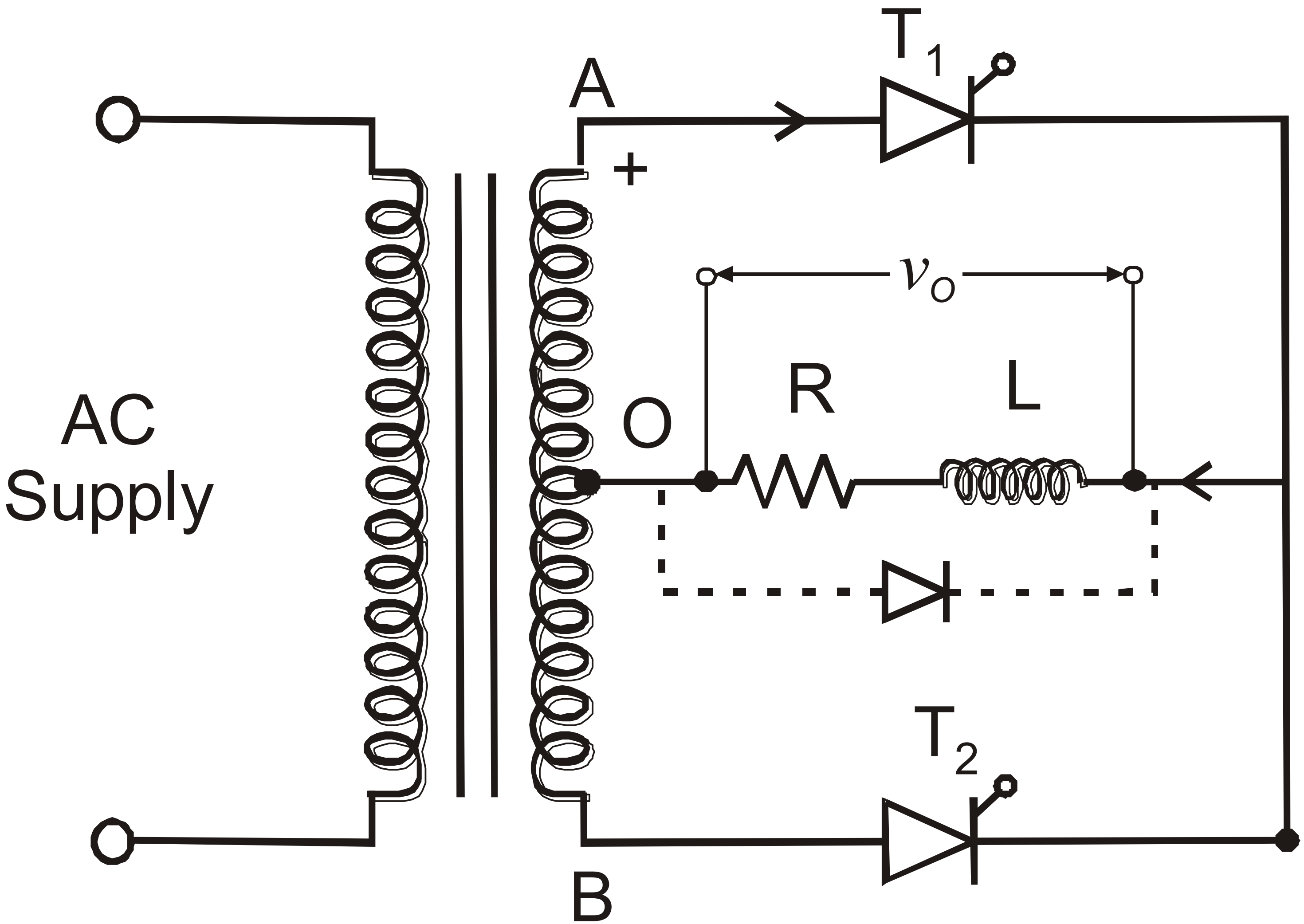
$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi} \right] E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

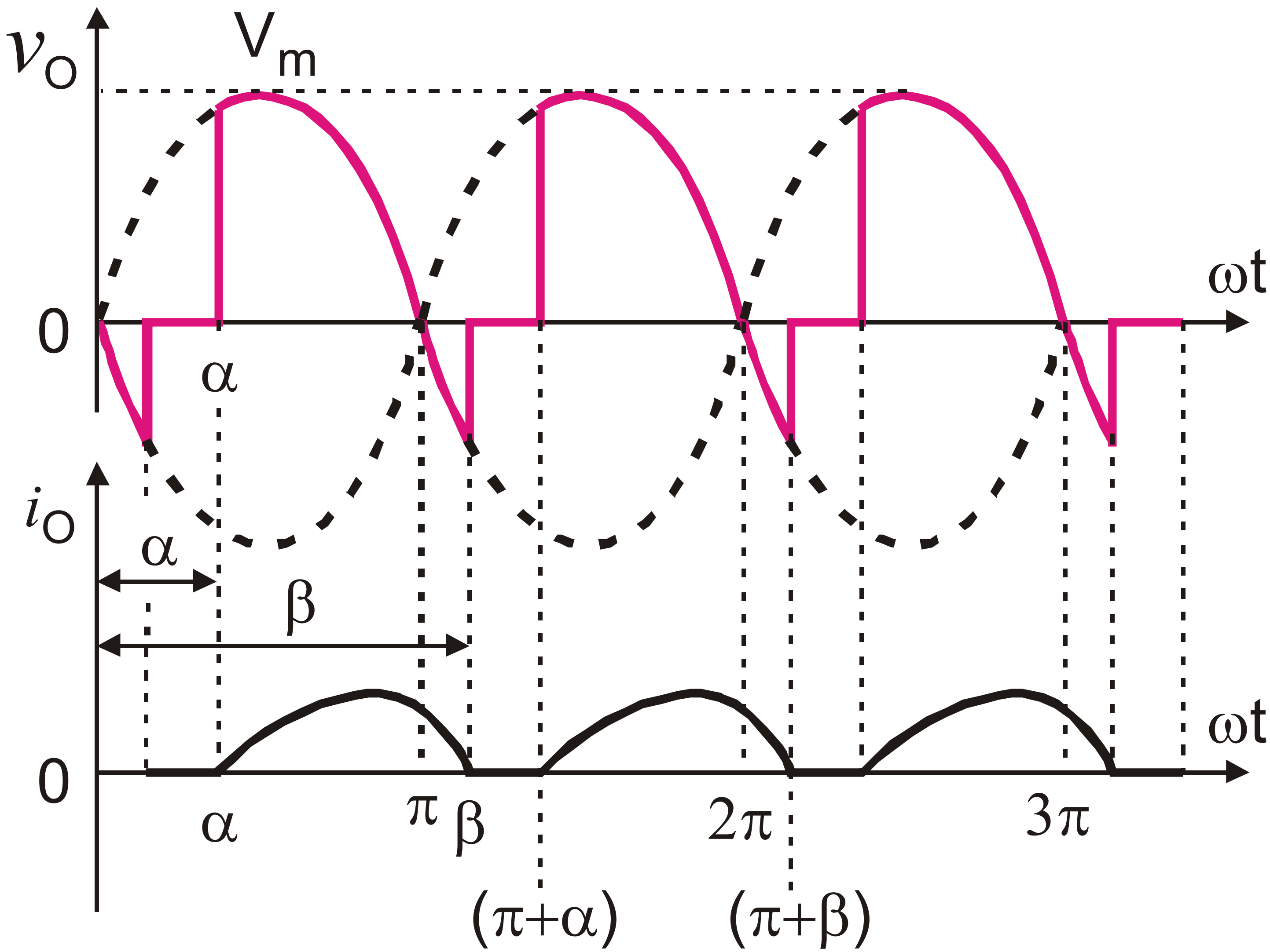
Single Phase
Full Wave Controlled Rectifier Using
A
Center Tapped Transformer



T1
A
+
vO
AC
Supply
O
R
L
T2
B

Discontinuous Load Current Operation without FWD for

$$\pi < \beta < (\pi + \alpha)$$



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
$(\pi+\alpha)$
$(\pi+\beta)$

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin\omega t \;\; ; \; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$V_m = \sqrt{2} V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

∴ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \; at \;\; \omega t = \beta$

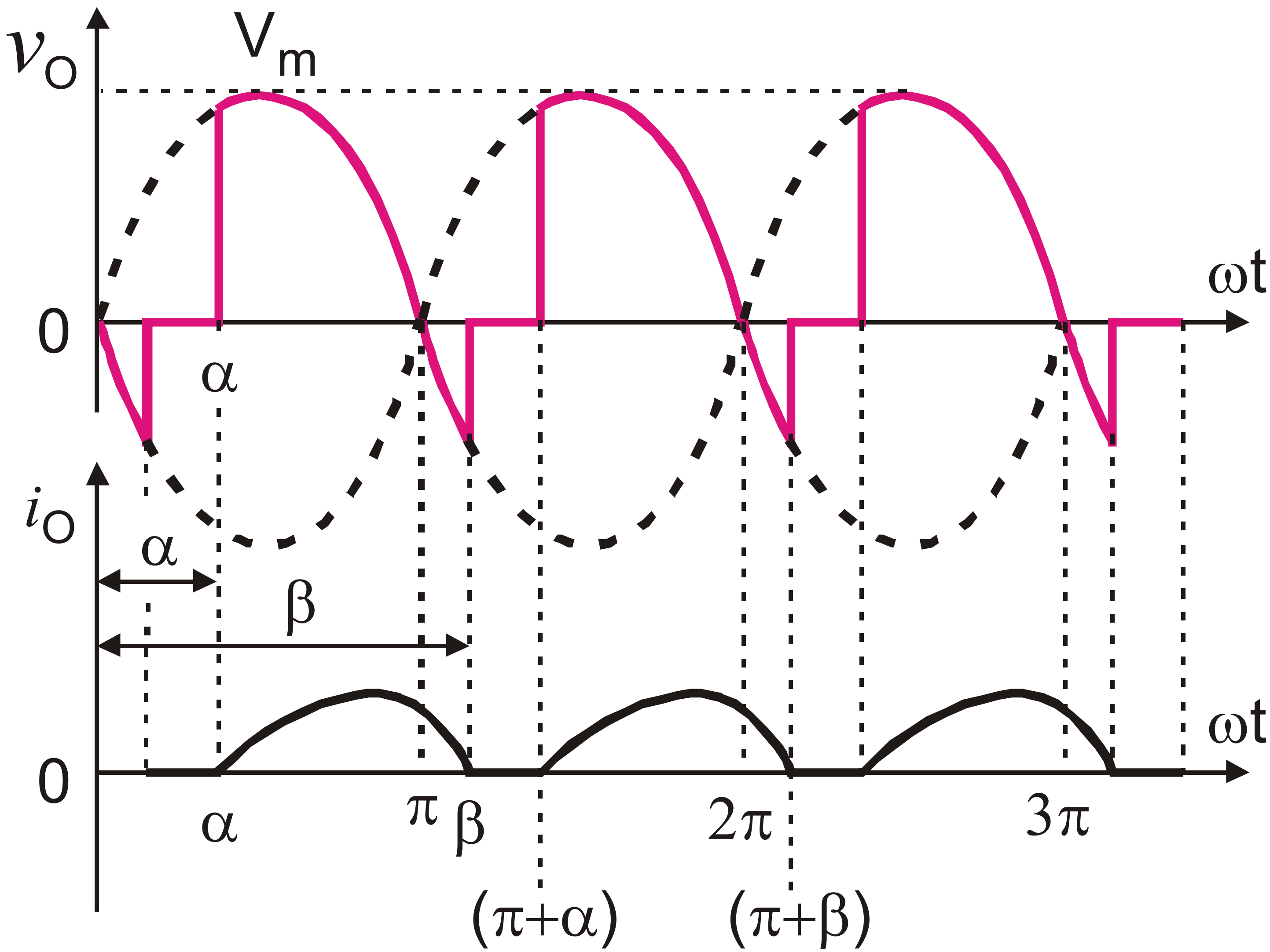
$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right] = 0$$

$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}\left(\beta - \alpha\right)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load

(Without FWD)



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible $\left(\text{ i.e., } L \approx 0\right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

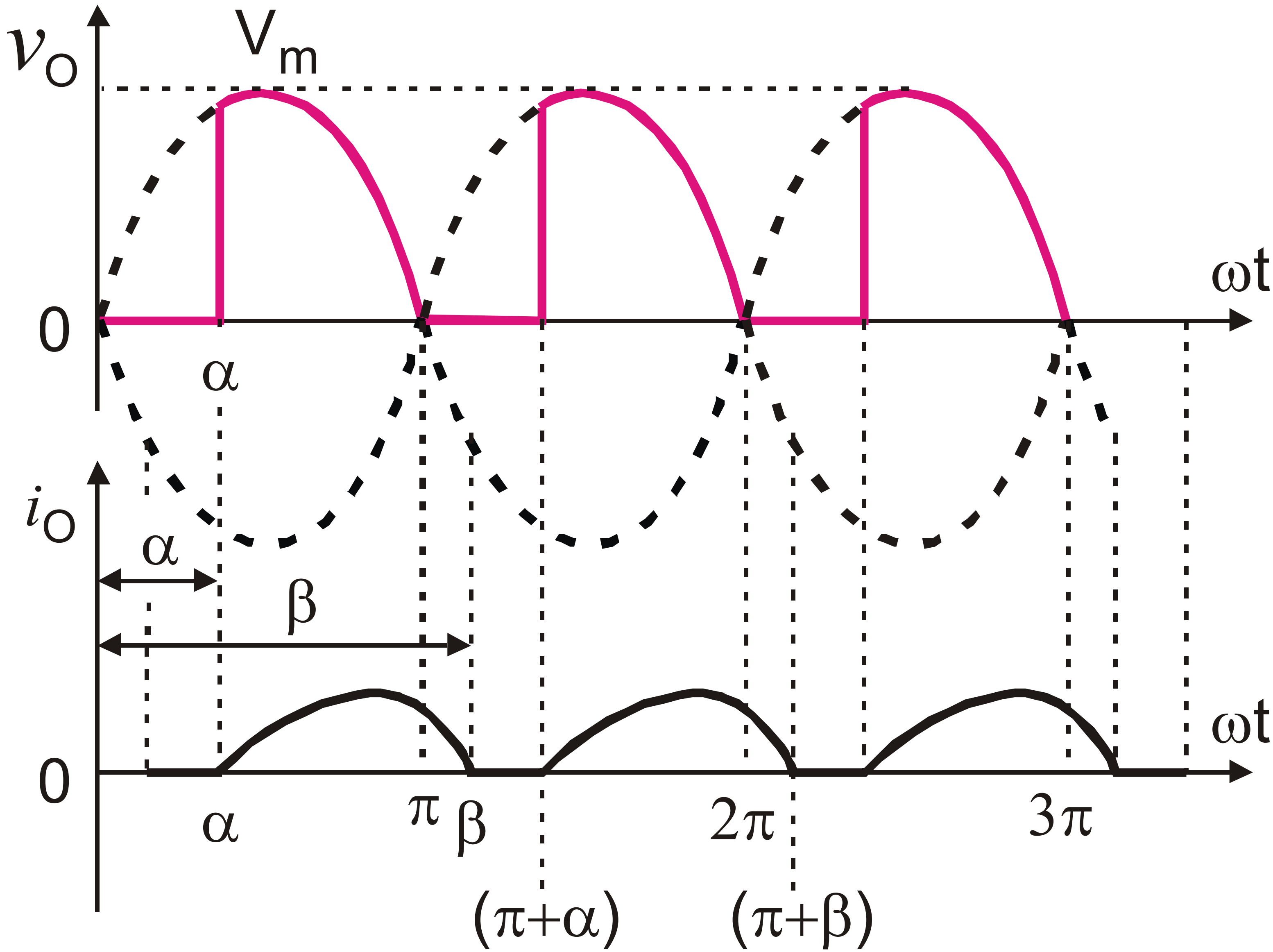
$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \left(-1\right)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

Discontinuous Load Current Operation with FWD



vO
Vm
ωt
0
α
iO
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \;\; ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.