Angle Modulation – Frequency Modulation

Consider again the general carrier $v_c(t) = V_c \cos(\omega_c t + \varphi_c)$

$(\omega_c t + \varphi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

a) By varying the frequency, ω_c – **Frequency Modulation.**
b) By varying the phase, ϕ_c – **Phase Modulation**

Frequency Modulation

In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Consider the carrier

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal $v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$,where the frequency deviation will depend on $m(t)$.

Given that the carrier frequency will change we may write for an instantaneous carrier signal

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\varphi_i)$$

where ϕ_i is the instantaneous angle = $\omega_i t = 2\pi f_i t$ and f_i is the instantaneous frequency.

Frequency Modulation

Since $\varphi_i = 2\pi f_i t$ then $\dfrac{d\varphi_i}{dt} = 2\pi f_i \quad \text{or} \quad f_i = \dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\varphi_i}{dt}$$

Δf_c is the peak deviation of the carrier.

Hence, we have $\dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$,*i.e.* $\dfrac{d\varphi_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$

Frequency Modulation

After integration *i.e.* $\int \left(\omega_c + 2\pi\Delta f_c \cos(\omega_m t)\right) dt$

$$\varphi_i = \omega_c t + \frac{2\pi\Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\varphi_i = \omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = V_c \cos(\varphi_i)$

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)\right)$$

Frequency Modulation

The ratio $\dfrac{\Delta f_c}{f_m}$ is called the **Modulation Index** denoted by β *i.e.*

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process – *i.e.* the principle of superposition does not apply. The FM signal for a message $m(t)$ as a band of signals is very complex. Hence, $m(t)$ is usually considered as a 'single tone modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

Frequency Modulation

The equation $v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t) \right)$ may be expressed as Bessel series (Bessel functions)

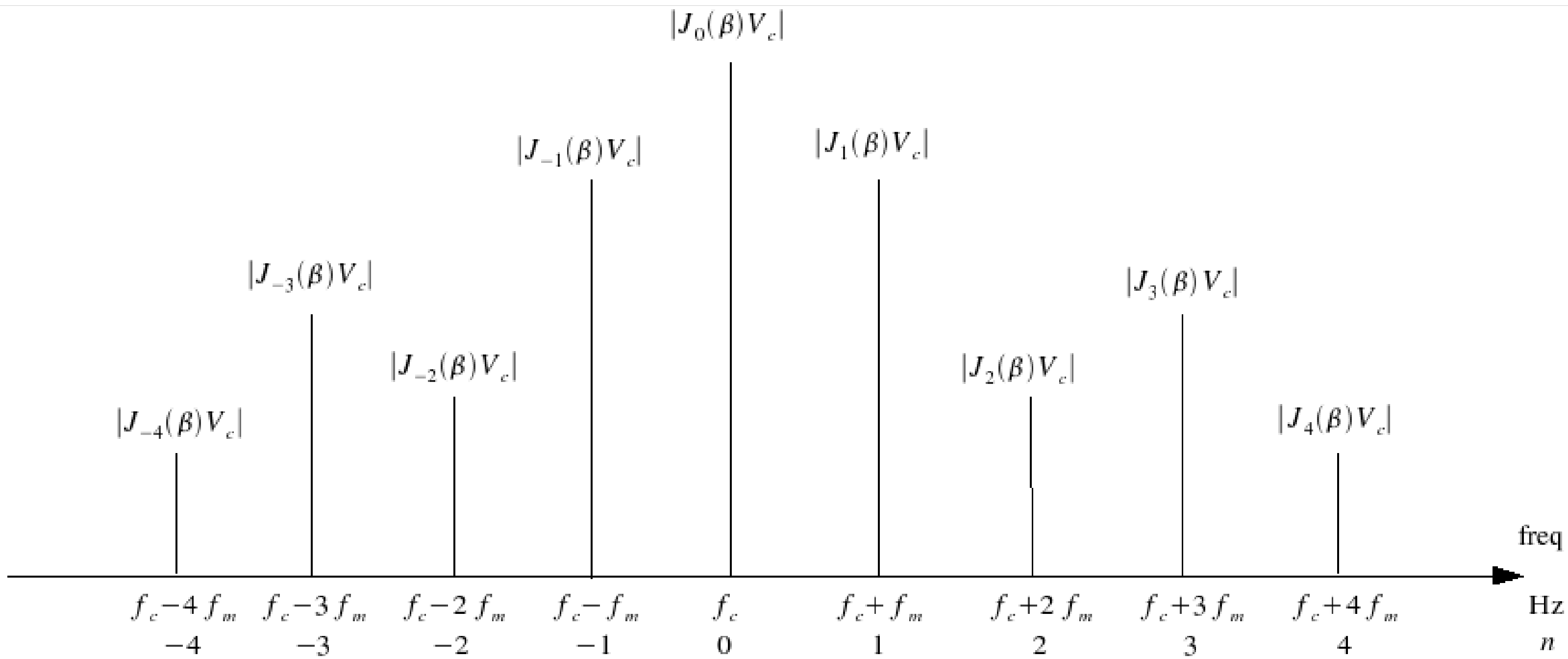
$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ are Bessel functions of the first kind. Expanding the equation for a few terms we have:

$$v_s(t) = \underbrace{V_c J_0(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c)}_{f_c} t + \underbrace{V_c J_1(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + \omega_m)}_{f_c + f_m} t + \underbrace{V_c J_{-1}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - \omega_m)}_{f_c - f_m} t$$

$$+ \underbrace{V_c J_2(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + 2\omega_m)}_{f_c + 2f_m} t + \underbrace{V_c J_{-2}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - 2\omega_m)}_{f_c - 2f_m} t + \cdots$$

FM Signal Spectrum.



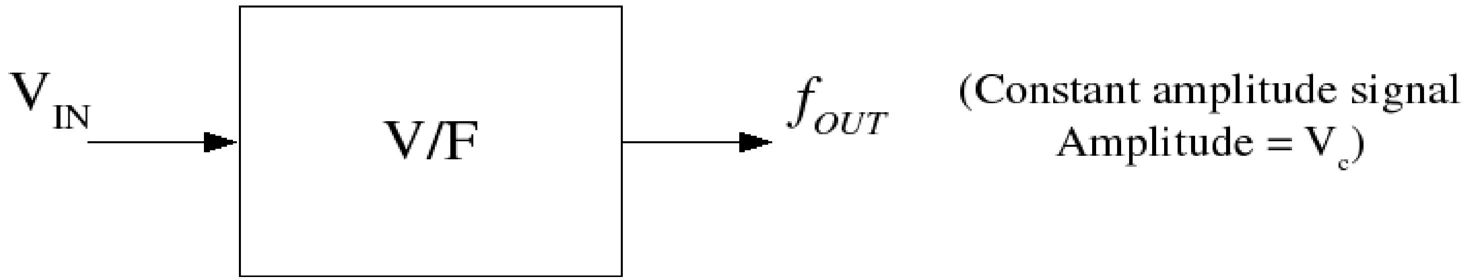
The amplitudes drawn are completely arbitrary, since we have not found any value for $J_n(\beta)$ – this sketch is only to illustrate the spectrum.

Generation of FM signals – Frequency Modulation.

An FM demodulator is:

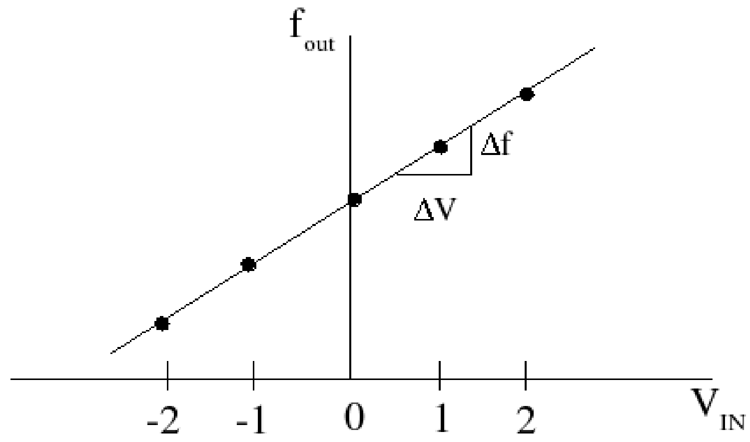
- a voltage-to-frequency converter V/F
- a voltage controlled oscillator VCO

In these devices (V/F or VCO), the output frequency is dependent on the input voltage amplitude.



V/F Characteristics.

Apply V_{IN} , *e.g.* 0 Volts, +1 Volts, +2 Volts, -1 Volts, -2 Volts, ... and measure the frequency output for each V_{IN} . The ideal V/F characteristic is a straight line as shown below.

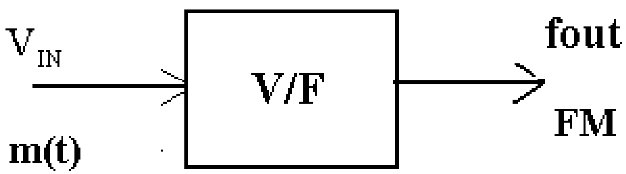


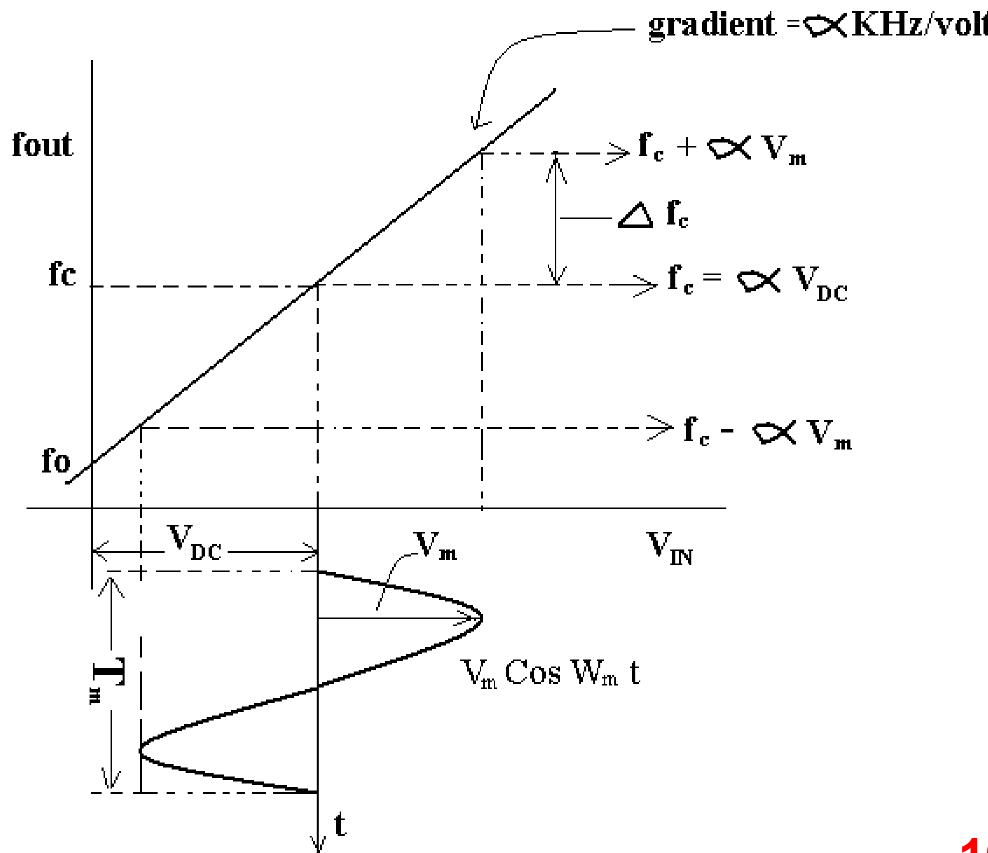
f_c, the frequency output when the input is zero is called the undeviated or nominal carrier frequency.

The gradient of the characteristic $\frac{\Delta f}{\Delta V}$ is called the **Frequency Conversion Factor**, denoted by α per Volt.

V/F Characteristics.

Consider now, an analogue message input, $m(t) = V_m \cos(\omega_m t)$





As the input *m*(*t*) varies from

$$+V_m \rightarrow 0 \rightarrow -V_m$$

the output frequency will vary from a maximum, through f_c, to a minimum frequency.

V/F Characteristics.

For a straight line, $y = c + mx$, where c = value of y when $x = 0$, m = gradient, hence we may say

$$f_{\mathrm{OUT}} = f_c + \alpha V_{\mathrm{IN}}$$

and when $V_{IN} = m(t)$ $f_{\mathrm{OUT}} = f_c + \alpha m(t)$,*i.e.* the deviation depends on $m(t)$.

Considering that maximum and minimum input amplitudes are $+V_m$ and $-V_m$ respectively, then

$$f_{\max} = f_c + \alpha V_m$$
$$f_{\min} = f_c - \alpha V_m$$

on the diagram on the previous slide.

The peak-to-peak deviation is $f_{max} - f_{min}$, but more importantly for FM the peak deviation Δf_c is

Peak Deviation, $\Delta f_c = \alpha V_m$ Hence, **Modulation Index**, $\beta = \frac{\Delta f_c}{f_m} = \frac{\alpha V_m}{f_m}$

Summary of the important points of FM

- In FM, the message signal $m(t)$ is assumed to be a single tone frequency,

$$m(t) = V_m \cos(\omega_m t)$$

- The FM signal $v_s(t)$ from which the spectrum may be obtained as

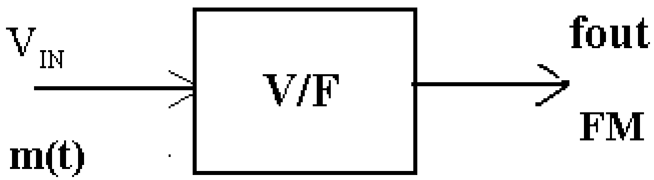
$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ are Bessel coefficients and **Modulation Index**, $\beta = \dfrac{\Delta f_c}{f_m} = \dfrac{\alpha V_m}{f_m}$

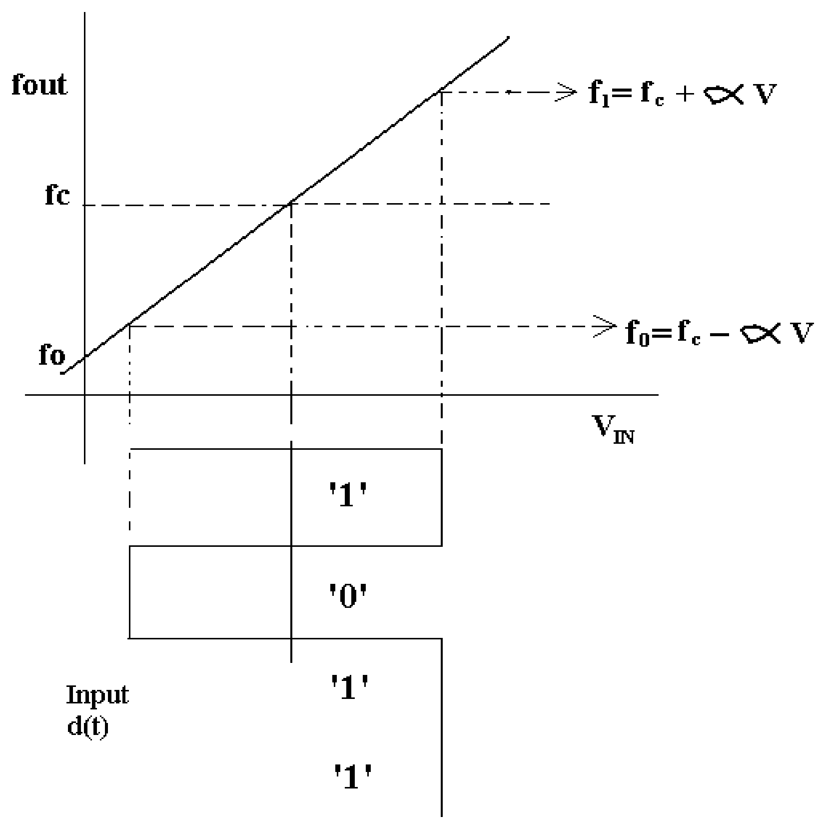
- α Hz per Volt is the V/F modulator, gradient or **Frequency Conversion Factor**, α per Volt
- α is a measure of the change in output frequency for a change in input amplitude.
- **Peak Deviation** (of the carrier frequency from f_c) $\Delta f_c = \alpha V_m$

FM Signal Waveforms.

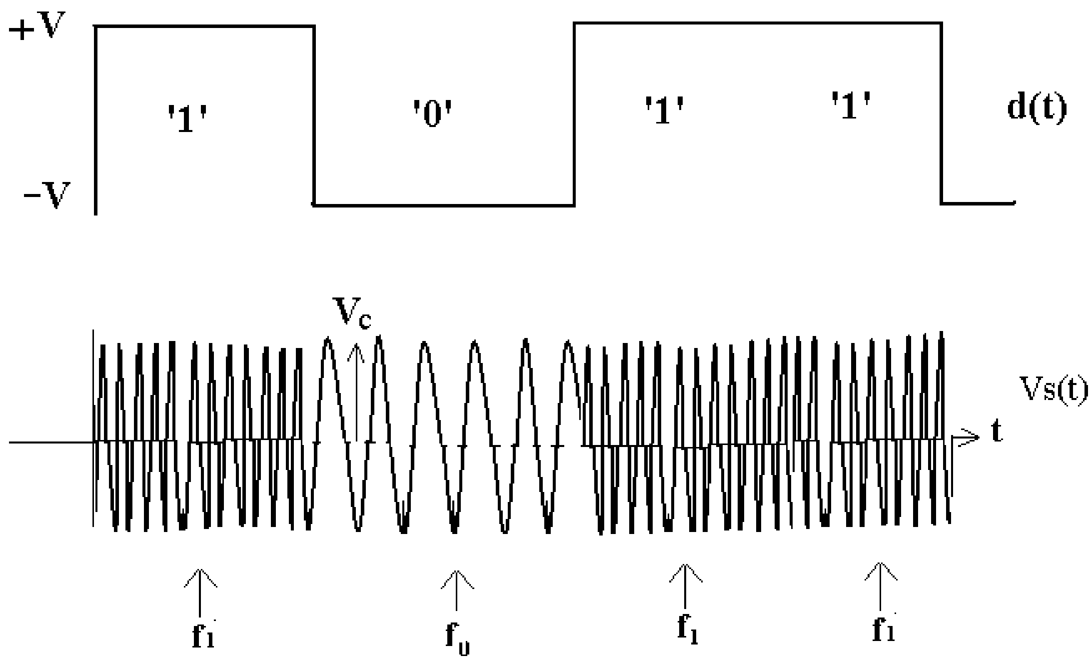
The diagrams below illustrate FM signal waveforms for various inputs



At this stage, an input digital data sequence, *d*(*t*), is introduced – the output in this case will be FSK, (Frequency Shift Keying).



FM Signal Waveforms.

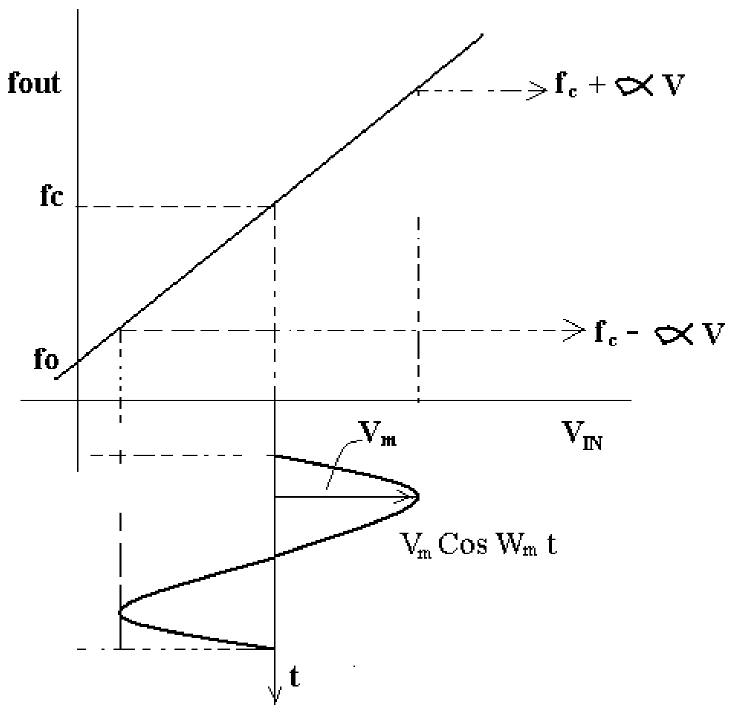


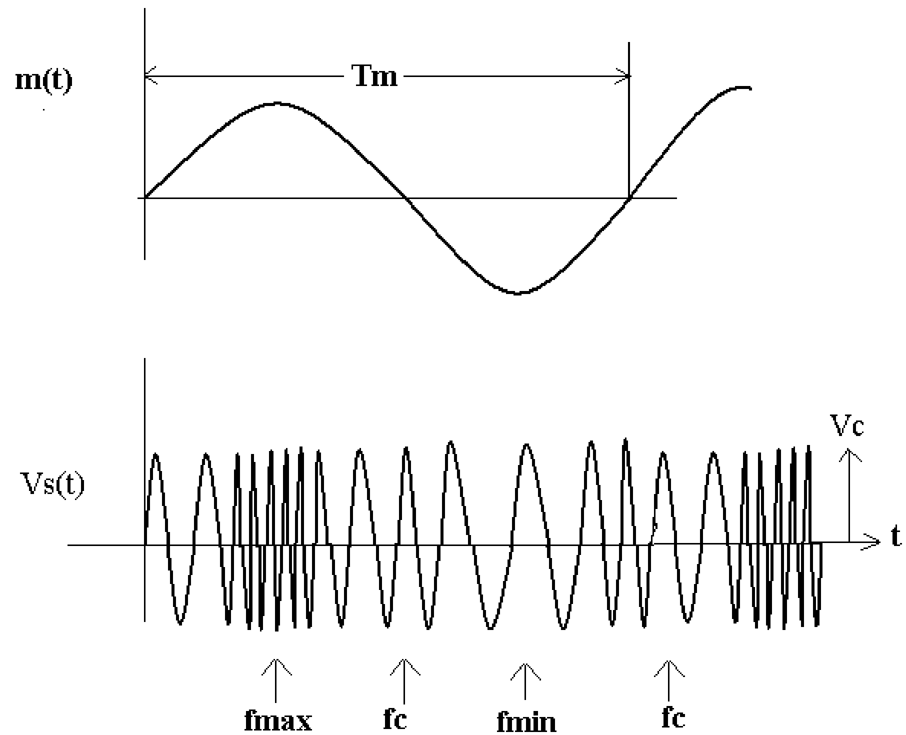
Assuming $d(t) = +V \text{ for 1's}$ $\quad f_{OUT} = f_1 = f_c + \alpha V \text{ for 1's}$

$= -V \text{ for 0's}$ $\quad f_{OUT} = f_0 = f_c - \alpha V \text{ for 0's}$

the output 'switches' between f_1 and f_0.

FM Signal Waveforms.

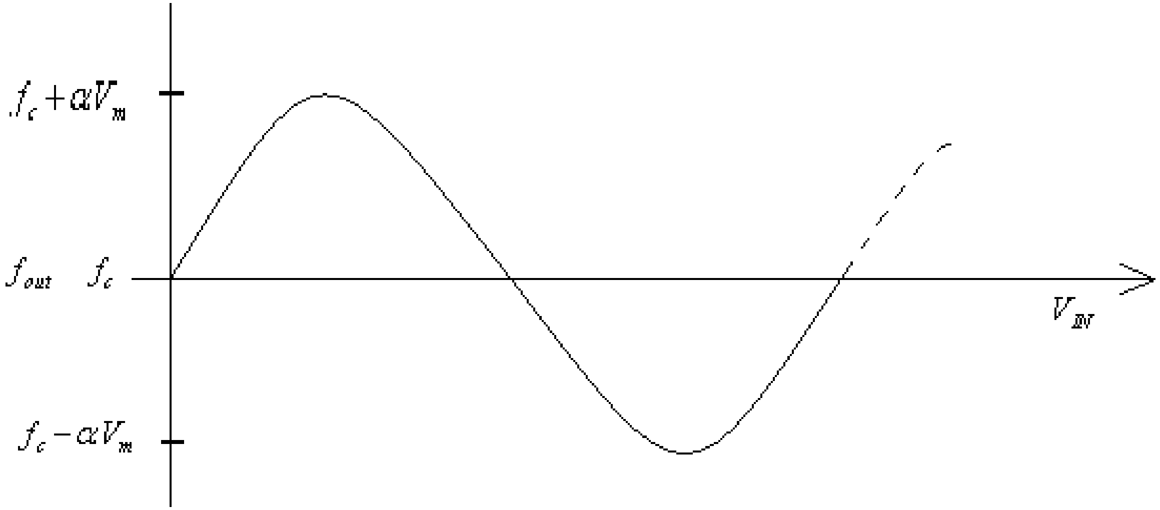




The output frequency varies 'gradually' from f_c to ($f_c + \alpha V_m$), through f_c to ($f_c - \alpha V_m$) *etc.*

FM Signal Waveforms.

If we plot f_{OUT} as a function of V_{IN}:



In general, *m*(*t*) will be a 'band of signals', *i.e.* it will contain amplitude and frequency variations. Both amplitude and frequency change in *m*(*t*) at the input are translated to (just) frequency changes in the FM output signal, *i.e.* the amplitude of the output FM signal is constant.

Amplitude changes at the input are translated to deviation from the carrier at the output. The larger the amplitude, the greater the deviation.