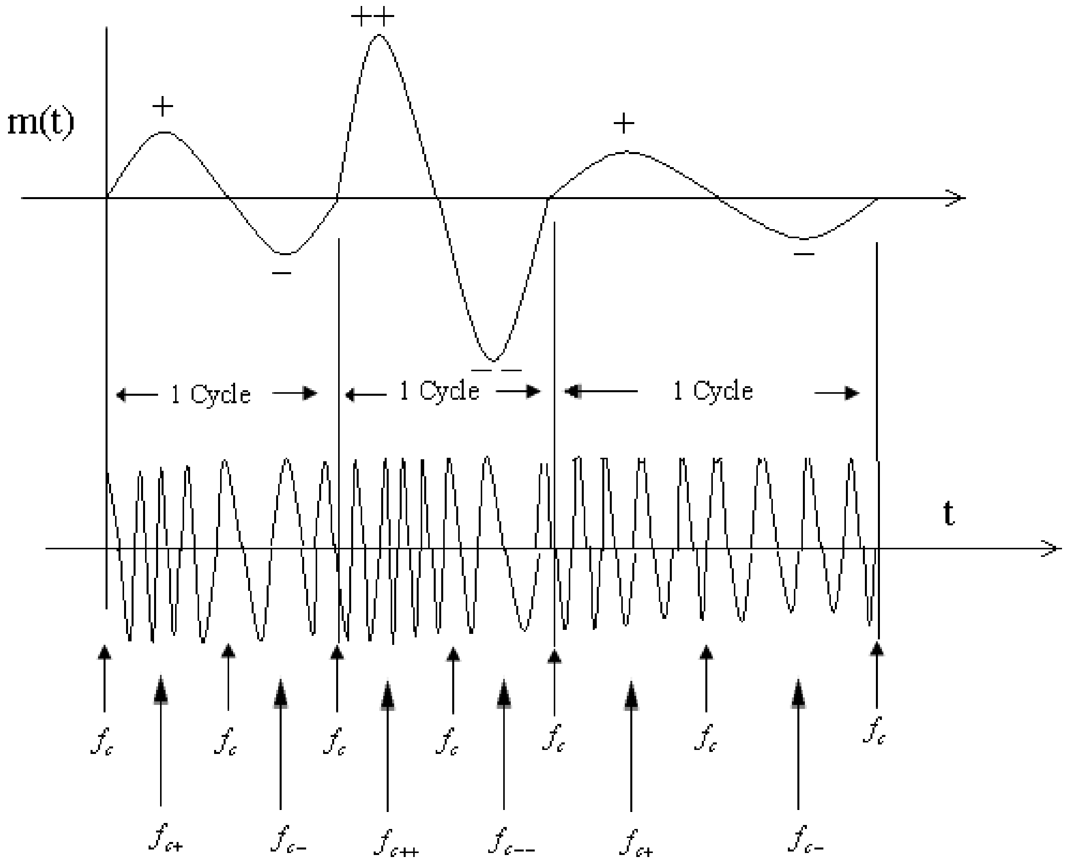
FM Signal Waveforms.

Frequency changes at the input are translated to rate of change of frequency at the output. An attempt to illustrate this is shown below:



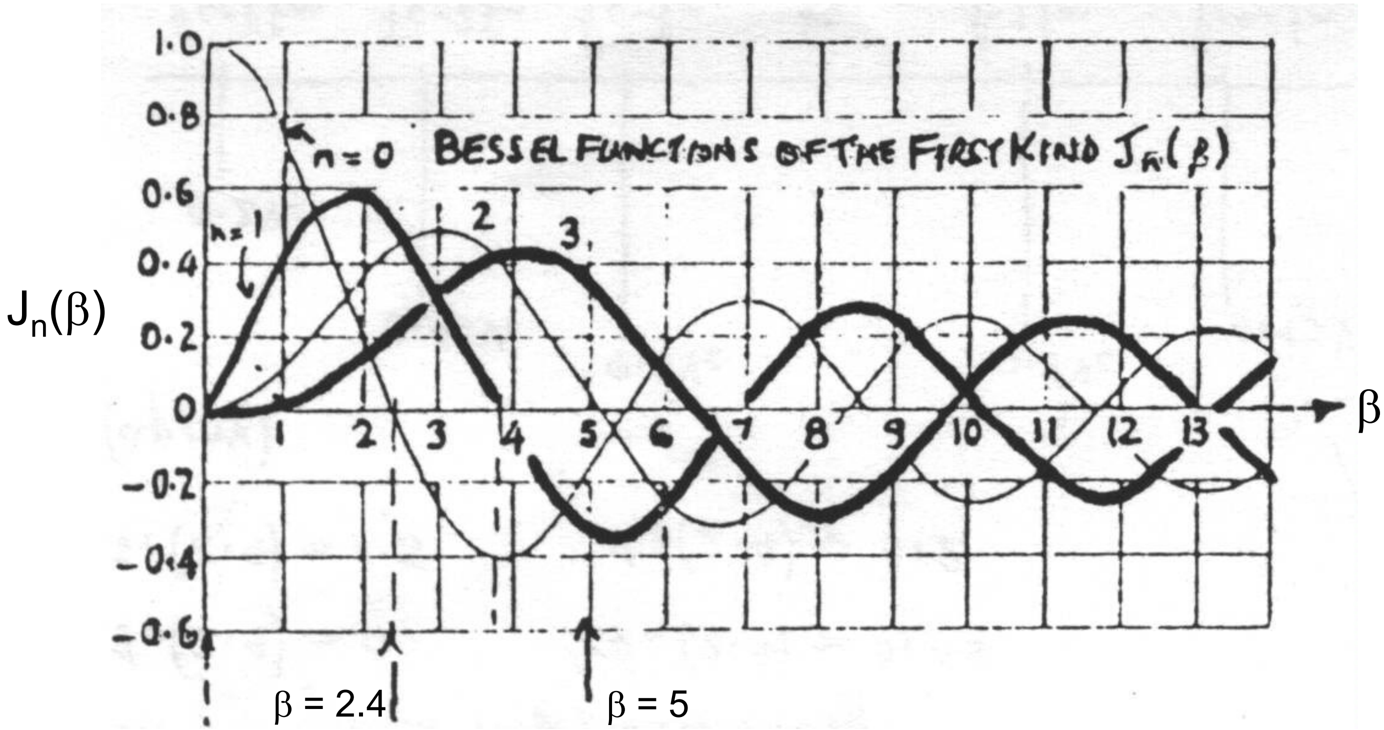
FM Spectrum – Bessel Coefficients.

The FM signal spectrum may be determined from

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

The values for the Bessel coefficients, $J_n(\beta)$ may be found from graphs or, preferably, tables of 'Bessel functions of the first kind'.

FM Spectrum – Bessel Coefficients.

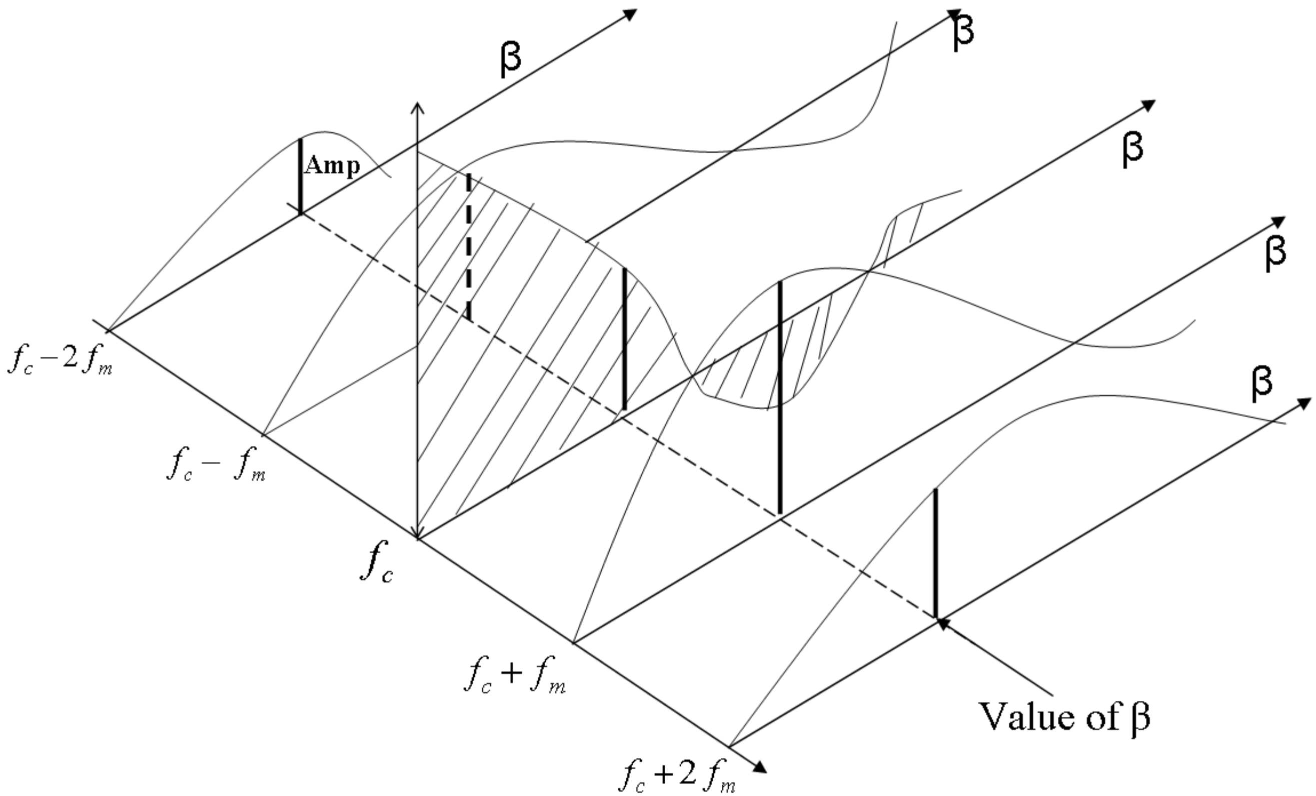


In the series for $v_s(t)$, n = 0 is the carrier component, *i.e.* $V_c J_0(\beta)\cos(\omega_c t)$, hence the n = 0 curve shows how the component at the carrier frequency, f_c, varies in amplitude, with modulation index β.

FM Spectrum – Bessel Coefficients.

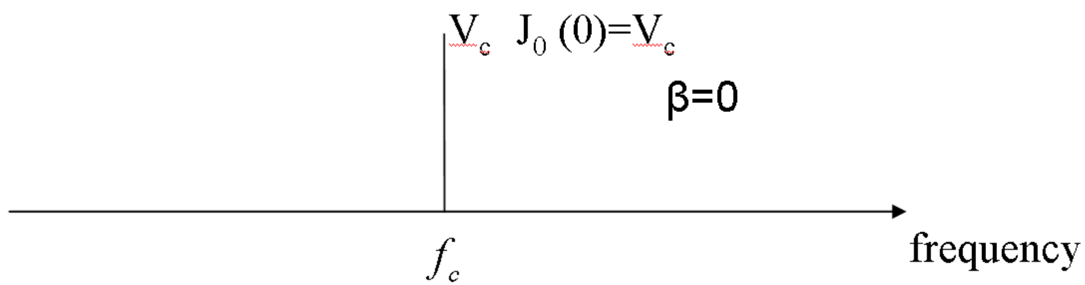
Hence for a given value of modulation index β, the values of $J_n(\beta)$ may be read off the graph and hence the component amplitudes ($V_c J_n(\beta)$) may be determined.

A further way to interpret these curves is to imagine them in 3 dimensions



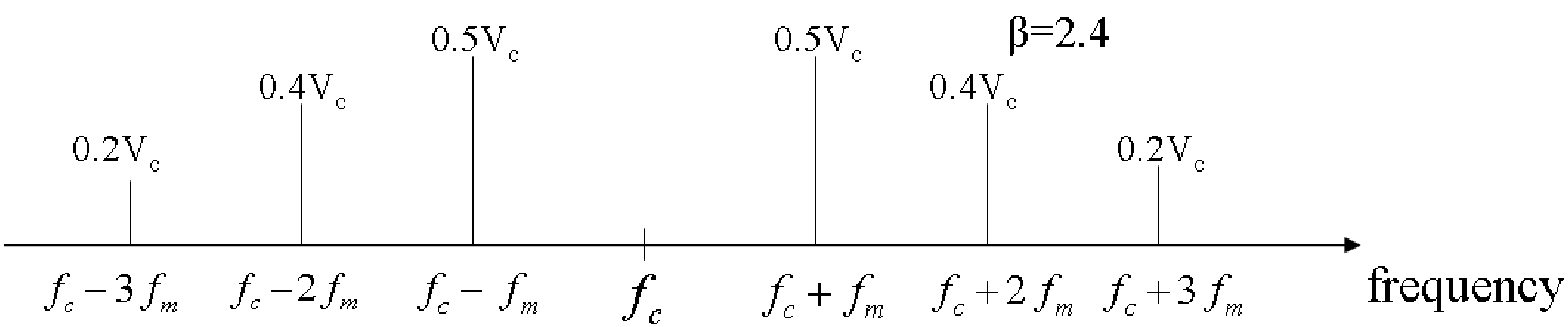
Examples from the graph

β = 0: When β **= 0 the carrier is unmodulated and $J_0(0) = 1$, all other $J_n(0) = 0$, *i.e.***



β = 2.4: From the graph (approximately)

$J_0(2.4) = 0$, $J_1(2.4) = 0.5$, $J_2(2.4) = 0.45$ and $J_3(2.4) = 0.2$



Significant Sidebands – Spectrum.

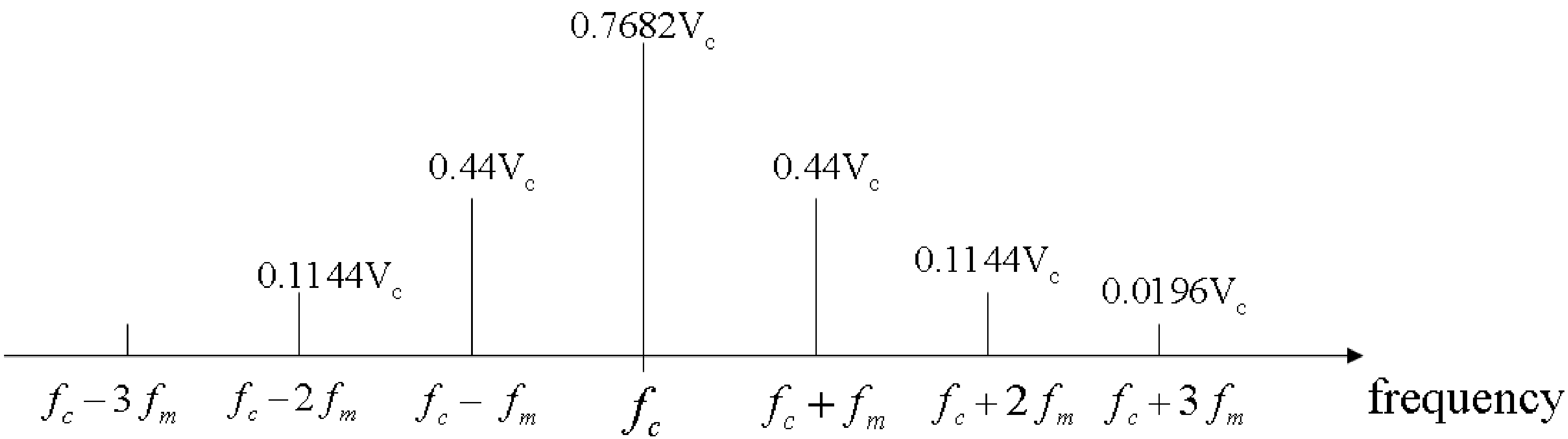
As may be seen from the table of Bessel functions, for values of *n* above a certain value, the values of $J_n(\beta)$ become progressively smaller. In FM the sidebands are considered to be significant if $J_n(\beta) \geq 0.01$ (1%).

Although the bandwidth of an FM signal is infinite, components with amplitudes $V_c J_n(\beta)$, for which $J_n(\beta) < 0.01$ are deemed to be insignificant and may be ignored.

Example: A message signal with a frequency f_m Hz modulates a carrier f_c to produce FM with a modulation index $\beta = 1$. Sketch the spectrum.

n	$J_n(1)$	Amplitude	Frequency
0	0.7652	$0.7652V_c$	f_c
1	0.4400	$0.44V_c$	f_c+f_m $f_c - f_m$
2	0.1149	$0.1149V_c$	f_c+2f_m $f_c - 2f_m$
3	0.0196	$0.0196V_c$	f_c+3f_m $f_c -3 f_m$
4	0.0025	*Insignificant*	
5	0.0002	*Insignificant*	

Significant Sidebands – Spectrum.



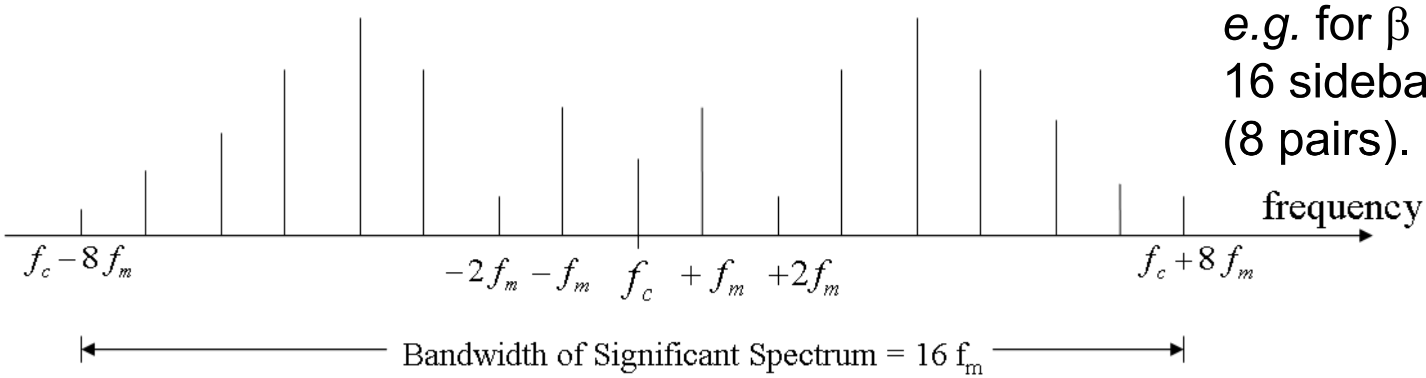
Bandwidth of Significant Spectrum = 6 f_m

As shown, the bandwidth of the spectrum containing significant components is $6f_m$, for $\beta = 1$.

Significant Sidebands – Spectrum.

The table below shows the number of significant sidebands for various modulation indices (β) and the associated spectral bandwidth.

β	No of sidebands ≥ 1% of unmodulated carrier	Bandwidth
0.1	2	$2f_m$
0.3	4	$4f_m$
0.5	4	$4f_m$
1.0	6	$6f_m$
2.0	8	$8f_m$
5.0	16	$16f_m$
10.0	28	$28f_m$



e.g. for β = 5, 16 sidebands (8 pairs).

Carson's Rule for FM Bandwidth.

An approximation for the bandwidth of an FM signal is given by BW = 2(Maximum frequency deviation + highest modulated frequency)

$$\text{Bandwidth} = 2(\Delta f_c + f_m)$$ **Carson's Rule**

Narrowband and Wideband FM

Narrowband FM NBFM

From the graph/table of Bessel functions it may be seen that for small β, ($\beta \leq 0.3$) there is only the carrier and 2 significant sidebands, *i.e.* BW = 2*fm*.

FM with $\beta \leq 0.3$ is referred to as **narrowband FM** (NBFM) (Note, the bandwidth is the same as DSBAM).

Wideband FM WBFM

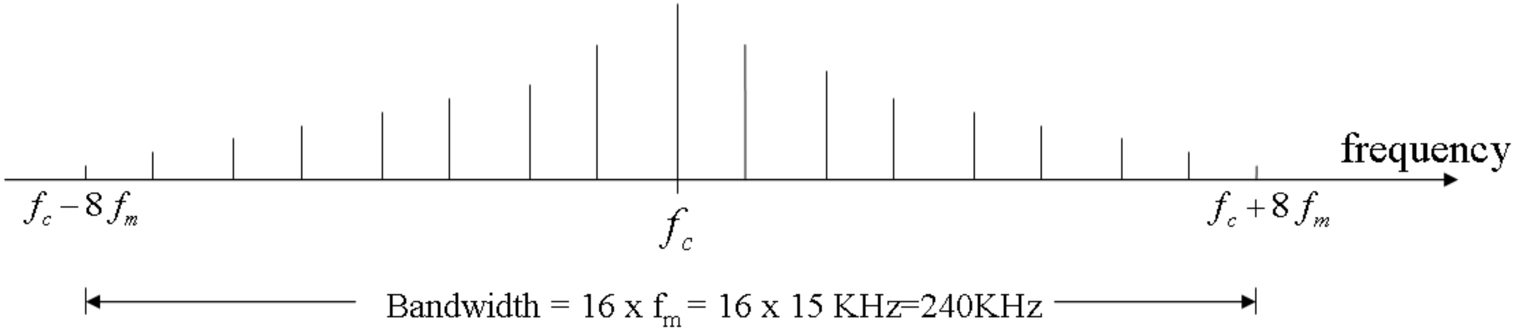
For $\beta > 0.3$ there are more than 2 significant sidebands. As β increases the number of sidebands increases. This is referred to as **wideband FM** (WBFM).

VHF/FM

VHF/FM (Very High Frequency band = 30MHz – 300MHz) radio transmissions, in the band 88MHz to 108MHz have the following parameters:

Max frequency input (*e.g.* music)	15kHz	f_m
Deviation	75kHz	$\Delta f_c = \alpha V_m$
Modulation Index β	5	$\beta = \frac{\Delta f_c}{f_m}$

For β = 5 there are 16 sidebands and the FM signal bandwidth is 16*fm* = 16 x 15kHz = 240kHz. Applying Carson's Rule BW = 2(75+15) = 180kHz.



Comments FM

- The FM spectrum contains a carrier component and an infinite number of sidebands at frequencies $f_c \pm nf_m$ (n = 0, 1, 2, …)

$$\text{FM signal, } v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

- In FM we refer to sideband pairs not upper and lower sidebands. Carrier or other components may not be suppressed in FM.

- The relative amplitudes of components in FM depend on the values $J_n(\beta)$, where $\beta = \dfrac{\alpha V_m}{f_m}$ thus the component at the carrier frequency depends on $m(t)$, as do all the other components and none may be suppressed.

Comments FM

- Components are significant if $J_n(\beta) \geq 0.01$. For β<<1 ($\beta \approx 0.3$ or less) only $J_0(\beta)$ and $J_1(\beta)$ are significant, *i.e.* only a carrier and 2 sidebands. Bandwidth is $2f_m$, similar to DSBAM in terms of bandwidth - called NBFM.

- Large modulation index $\beta = \frac{\Delta f_c}{f_m}$ means that a large bandwidth is required – called WBFM.

- The FM process is non-linear. The principle of superposition does not apply. When $m(t)$ is a band of signals, *e.g.* speech or music the analysis is very difficult (impossible?). Calculations usually assume a single tone frequency equal to the maximum input frequency. *E.g.* $m(t) \equiv$ band 20Hz $\rightarrow$ 15kHz, *fm* = 15kHz is used.

Power in FM Signals.

From the equation for FM $v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$

we see that the peak value of the components is $V_c J_n(\beta)$ for the n^{th} component.

Single normalised average power = $\left(\frac{V_{pk}}{\sqrt{2}}\right)^2 = (V_{RMS})^2$ then the n^{th} component is

$$\left(\frac{V_c J_n(\beta)}{\sqrt{2}}\right)^2 = \frac{(V_c J_n(\beta))^2}{2}$$

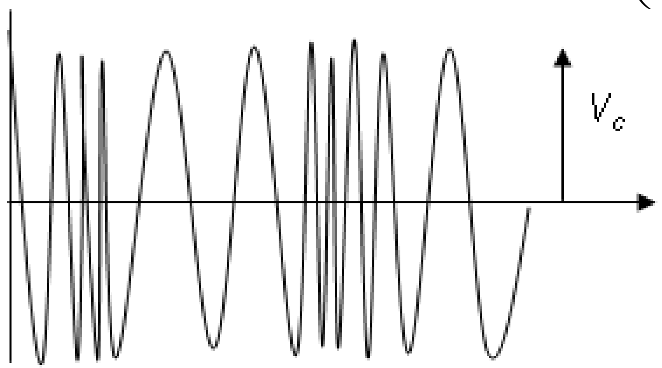
Hence, the total power in the infinite spectrum is

Total power $P_T = \sum_{n=-\infty}^{\infty} \frac{(V_c J_n(\beta))^2}{2}$

Power in FM Signals.

By this method we would need to carry out an infinite number of calculations to find P_T. But, considering the waveform, the peak value is V_{-c}, which is constant.

Since we know that the RMS value of a sine wave is $\left(\frac{V_{pk}}{\sqrt{2}}\right)^2 = \frac{V_c}{\sqrt{2}}$

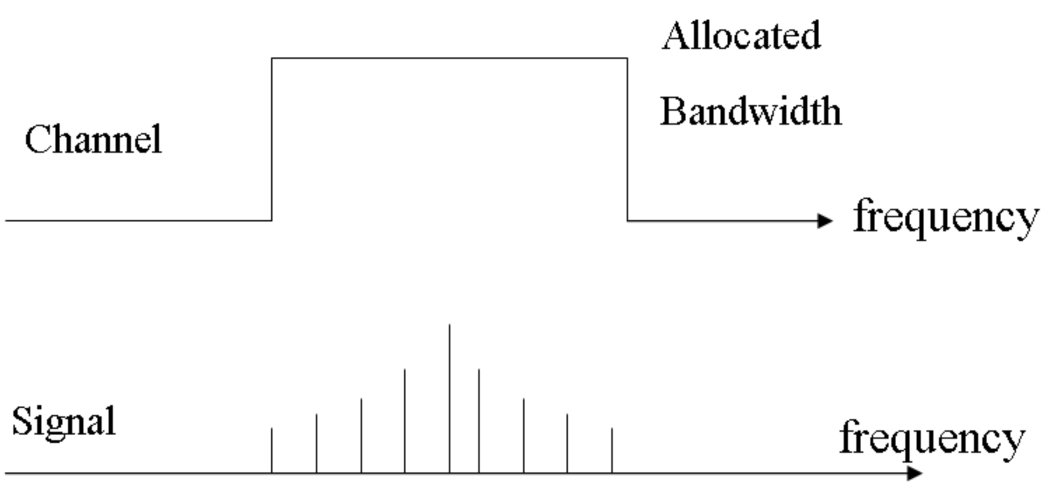


and power = $(V_{RMS})^2$ then we may deduce that $P_T = \left(\frac{V_c}{\sqrt{2}}\right)^2 = \frac{V_c^2}{2} = \sum_{n=-\infty}^{\infty} \frac{\left(V_c J_n(\beta)\right)^2}{2}$

Hence, if we know V_c for the FM signal, we can find the total power P_T for the infinite spectrum with a simple calculation.

Power in FM Signals.

Now consider – if we generate an FM signal, it will contain an infinite number of sidebands. However, if we wish to transfer this signal, *e.g.* over a radio or cable, this implies that we require an infinite bandwidth channel. Even if there was an infinite channel bandwidth it would not all be allocated to one user. Only a limited bandwidth is available for any particular signal. Thus we have to make the signal spectrum fit into the available channel bandwidth. We can think of the signal spectrum as a 'train' and the channel bandwidth as a tunnel – obviously we make the train slightly less wider than the tunnel if we can.



Phase Locked Loops PLL

- The input f_{IN} is applied to the multiplier and multiplied with the VCO frequency output f_O, to produce $\Sigma = (f_{IN} + f_O)$ and $\Delta = (f_{IN} - f_O)$.
- The low pass filter passes only $(f_{IN} - f_O)$ to give *VOUT* which is proportional to $(f_{IN} - f_O)$.
- If $f_{IN} \approx f_O$ but not equal, $V_{OUT} = V_{IN}$, $\alpha f_{IN} - f_O$ is a low frequency (beat frequency) signal to the VCO.
- This signal, V_{IN}, causes the VCO output frequency f_O to vary and move towards f_{IN}.
- When $f_{IN} = f_O$, V_{IN} $(f_{IN} - f_O)$ is approximately constant (DC) and f_O is held constant, *i.e* locked to f_{IN}.
- As f_{IN} changes, due to deviation in FM, f_O tracks or follows f_{IN}. $V_{OUT} = V_{IN}$ changes to drive f_O to track f_{IN}.
- V_{OUT} is therefore proportional to the deviation and contains the message signal $m(t)$.