Pulse Code Modulation (PCM)

- Pulse code modulation (PCM) is produced by analog-to-digital conversion process. Quantized PAM

- As in the case of other pulse modulation techniques, the rate at which samples are taken and encoded must conform to the Nyquist sampling rate.

- The sampling rate must be greater than, twice the highest frequency in the analog signal,

$$f_s > 2f_A(\max)$$

- Telegraph time-division multiplex (TDM) was conveyed as early as 1853, by the American inventor M.B. Farmer. The electrical engineer W.M. Miner, in 1903.
- PCM was invented by the British engineer Alec Reeves in 1937 in France.
- It was not until about the middle of 1943 that the Bell Labs people became aware of the use of PCM binary coding as already proposed by Alec Reeves.

Pulse Code Modulation

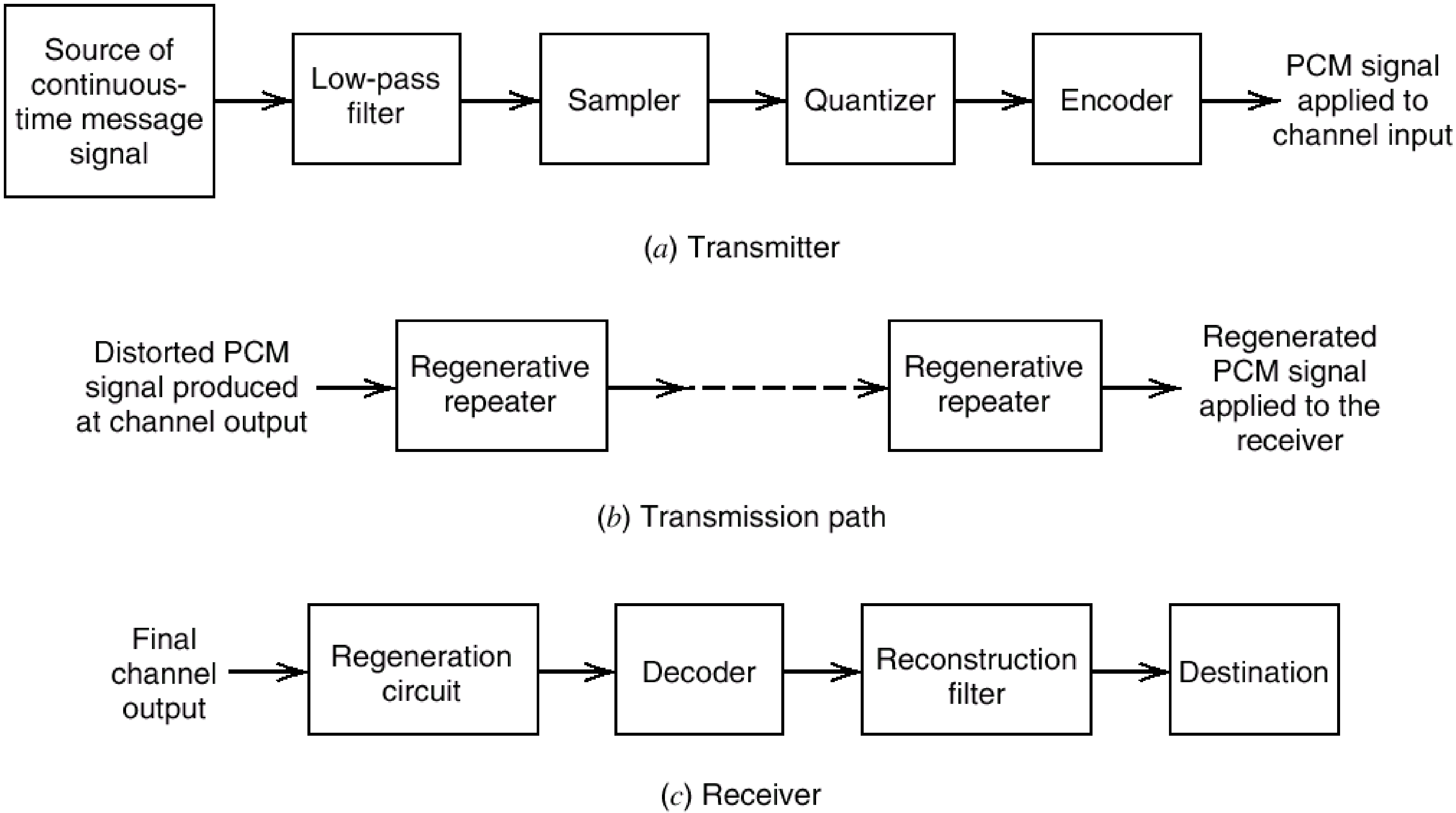
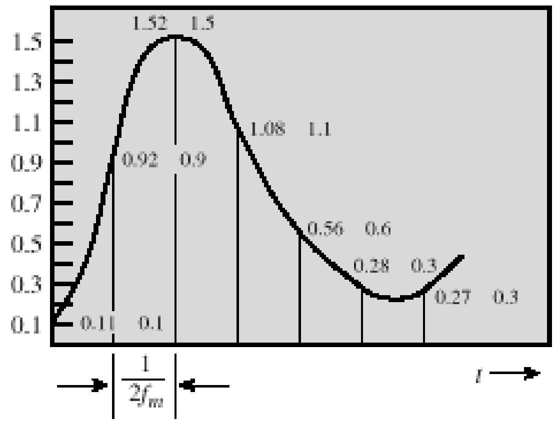


Figure ***The basic elements of a PCM system.***

Encoding



(a)

Digit	Binary Equivalent	PCM waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	

Digit	Binary Equivalent	PCM waveform
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

(b)

Virtues, Limitations and Modifications of PCM

Advantages of PCM

1. Robustness to noise and interference
2. Efficient regeneration
3. Efficient SNR and bandwidth trade-off
4. Uniform format
5. Ease add and drop
6. Secure

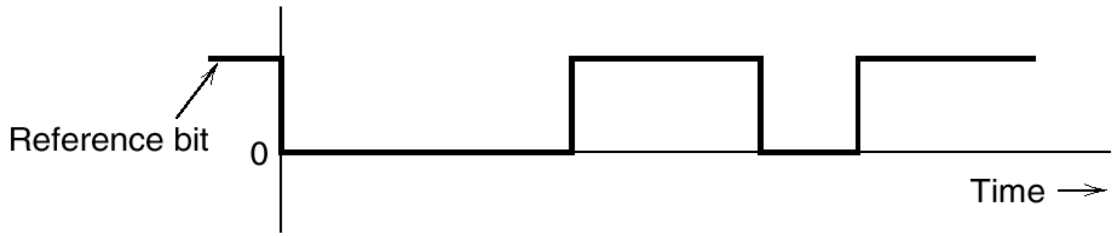
DS0: a basic digital signaling rate of 64 kbit/s. To carry a typical phone call, the audio sound is digitized at an 8 kHz sample rate using 8-bit pulse-code modulation. 4K baseband, 8*6+1.8 dB

Differential Encoding

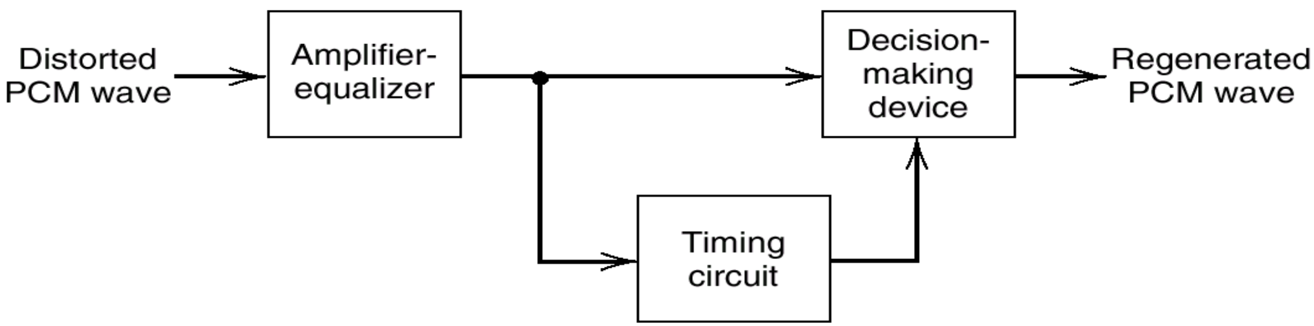
- Encode information in terms of signal transition; a transition is used to designate Symbol 0

(a) Original binary data		1	1	1	0	1	1	0	1
(b) Differentially encoded data	1	0	0	0	1	1	0	1	1

(c) Waveform



Regeneration (reamplification, retiming, reshaping)



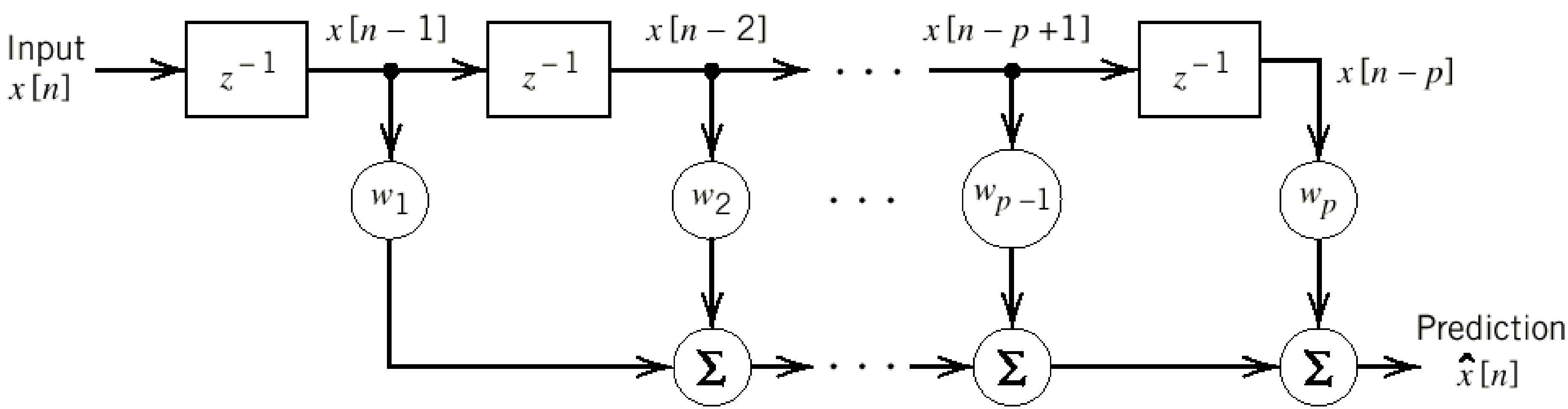
3dB performance loss, easier decoder

Linear Prediction Coding (LPC)

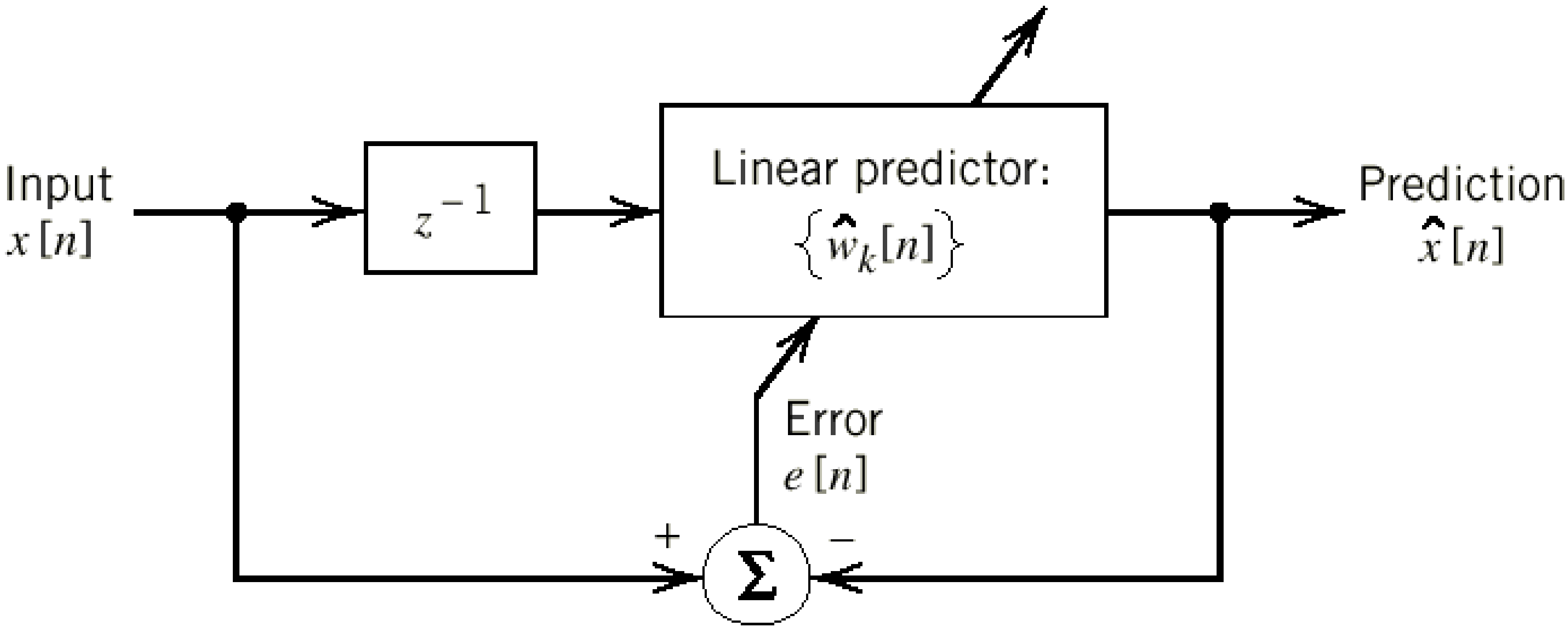
Consider a finite-duration impulse response (FIR)

discrete-time filter which consists of three blocks :

1. Set of p (***p*: prediction order**) unit-delay elements (z^{-1})
2. Set of multipliers with coefficients $w_1, w_2, \ldots w_p$
3. Set of adders ($\sum$)



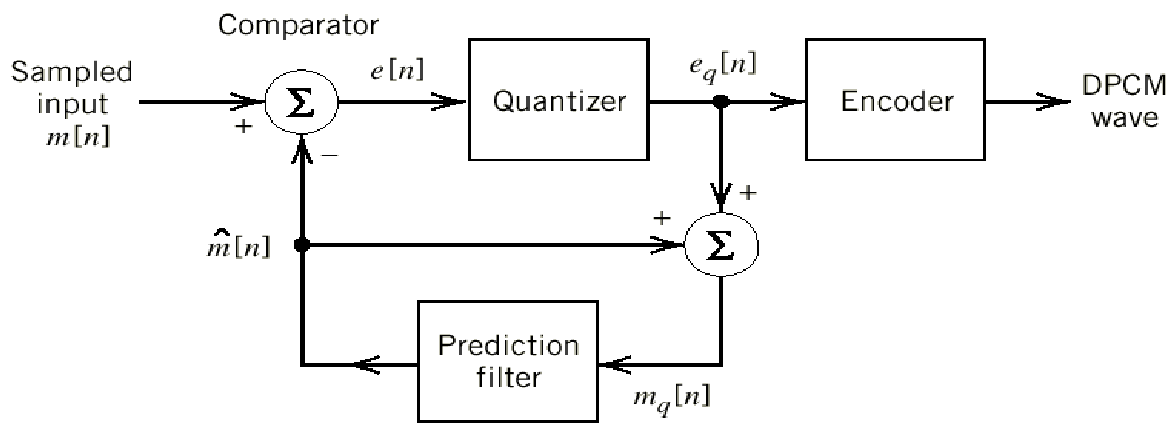
Reduce the sampling rate



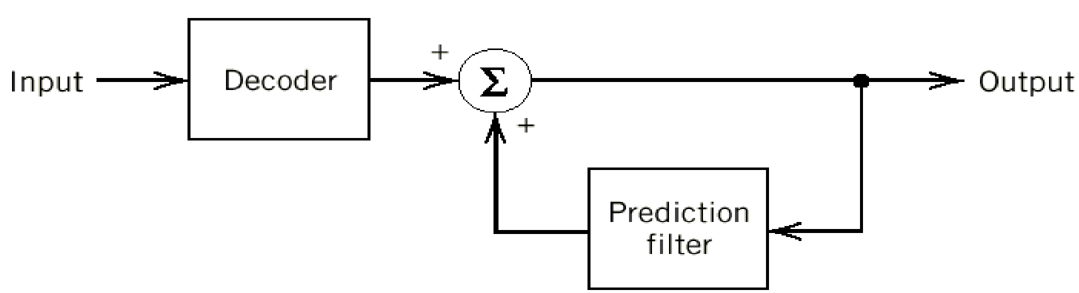
Block diagram illustrating the linear adaptive prediction process.

Differential Pulse-Code Modulation (DPCM)

Usually **PCM** has the sampling rate higher than the **Nyquist rate**.
The encode signal contains redundant information. **DPCM** can efficiently remove this redundancy. 32 Kbps for PCM Quality



(a)



(b)

Processing Gain

The $(\mathrm{SNR})_o$ of the DPCM system is

$$(\mathrm{SNR})_o = \frac{\sigma_M^2}{\sigma_Q^2}$$

where σ_M^2 and σ_Q^2 are variances of $m[n]$ $(E[m[n]]=0)$ and $q[n]$

$$(\mathrm{SNR})_o = \left(\frac{\sigma_M^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_Q^2}\right)$$

$$= G_p (\mathrm{SNR})_Q$$

where σ_E^2 is the variance of the predictions error

and the signal-to-quantization noise ratio is

$$(\mathrm{SNR})_Q = \frac{\sigma_E^2}{\sigma_Q^2}$$

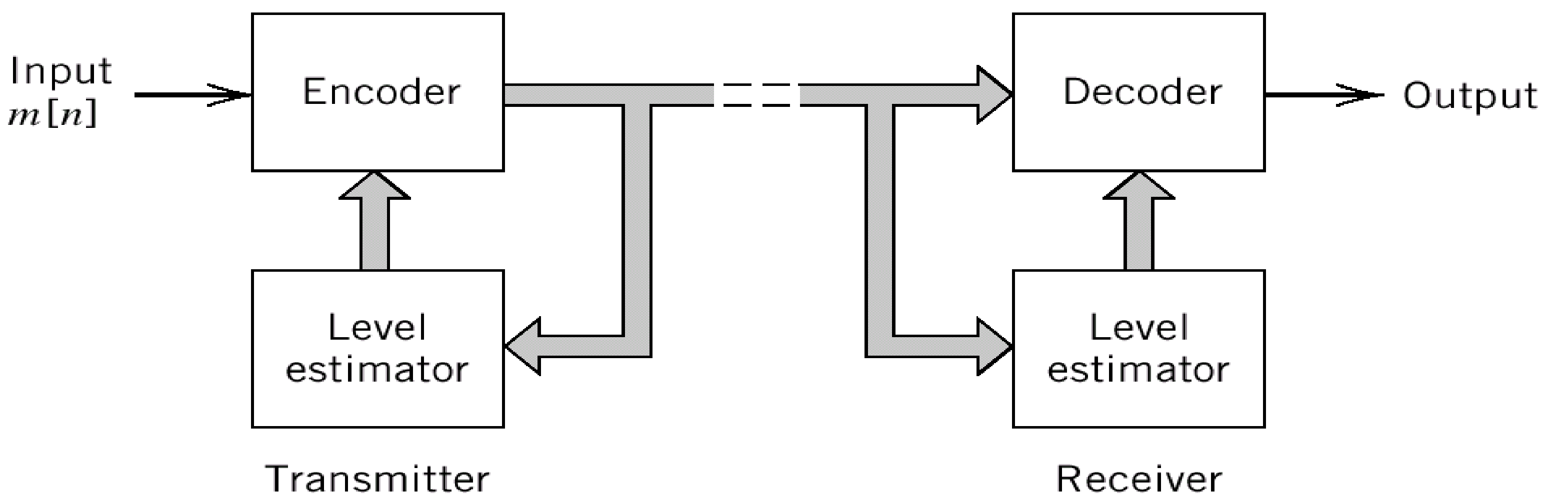
Processing Gain, $G_p = \frac{\sigma_M^2}{\sigma_E^2}$

Design a prediction filter to maximize G_p (minimize σ_E^2)

Adaptive Differential Pulse-Code Modulation (ADPCM)

Need for coding speech at low bit rates , we have two aims in mind:

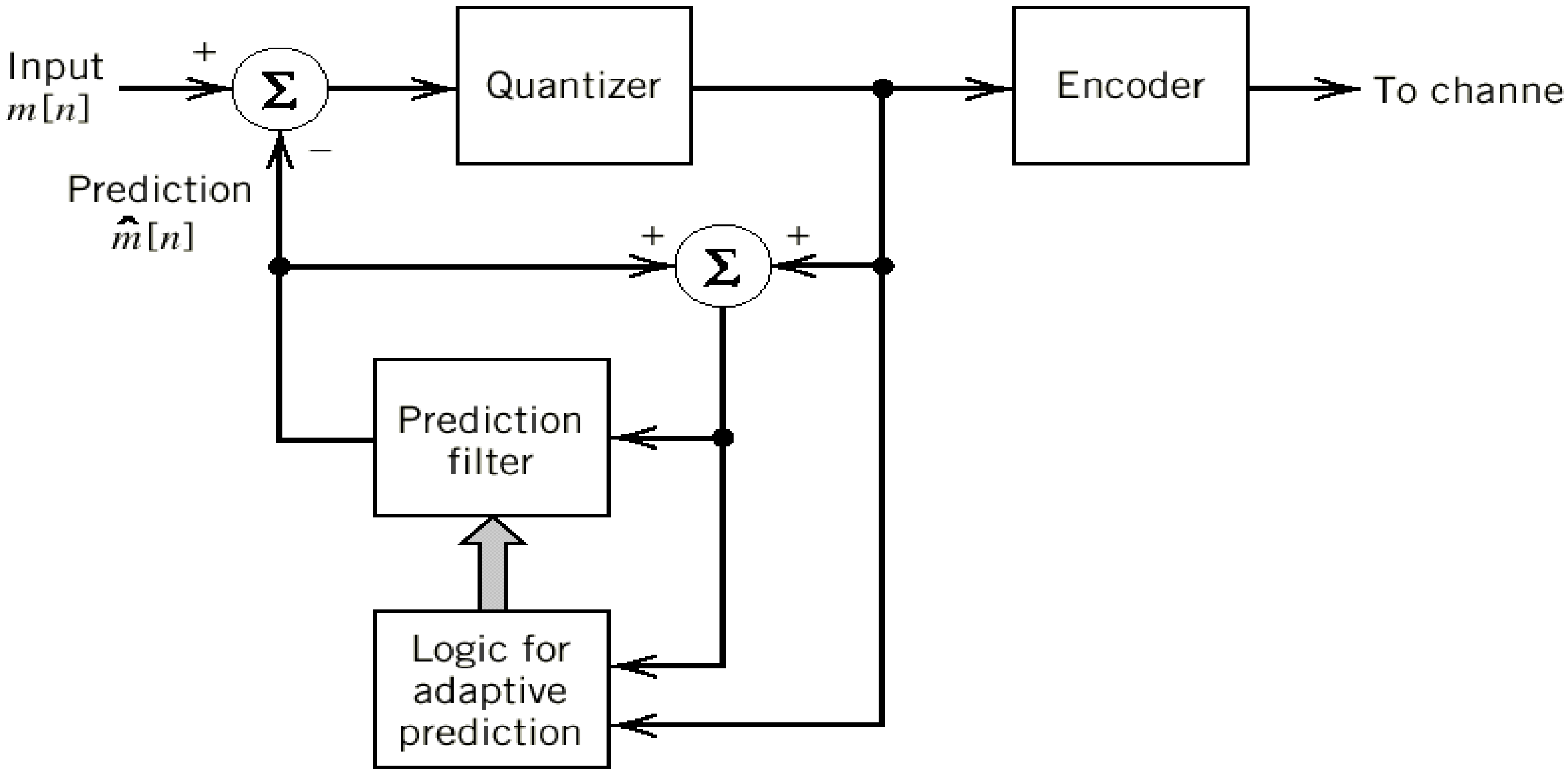
1. Remove redundancies from the speech signal as far as possible.
2. Assign the available bits in a perceptually efficient manner.



Adaptive quantization with backward estimation (AQB).

ADPCM

8-16 kbps with the same quality of PCM



Adaptive prediction with backward estimation (APB).

Coded Excited Linear Prediction (CELP)

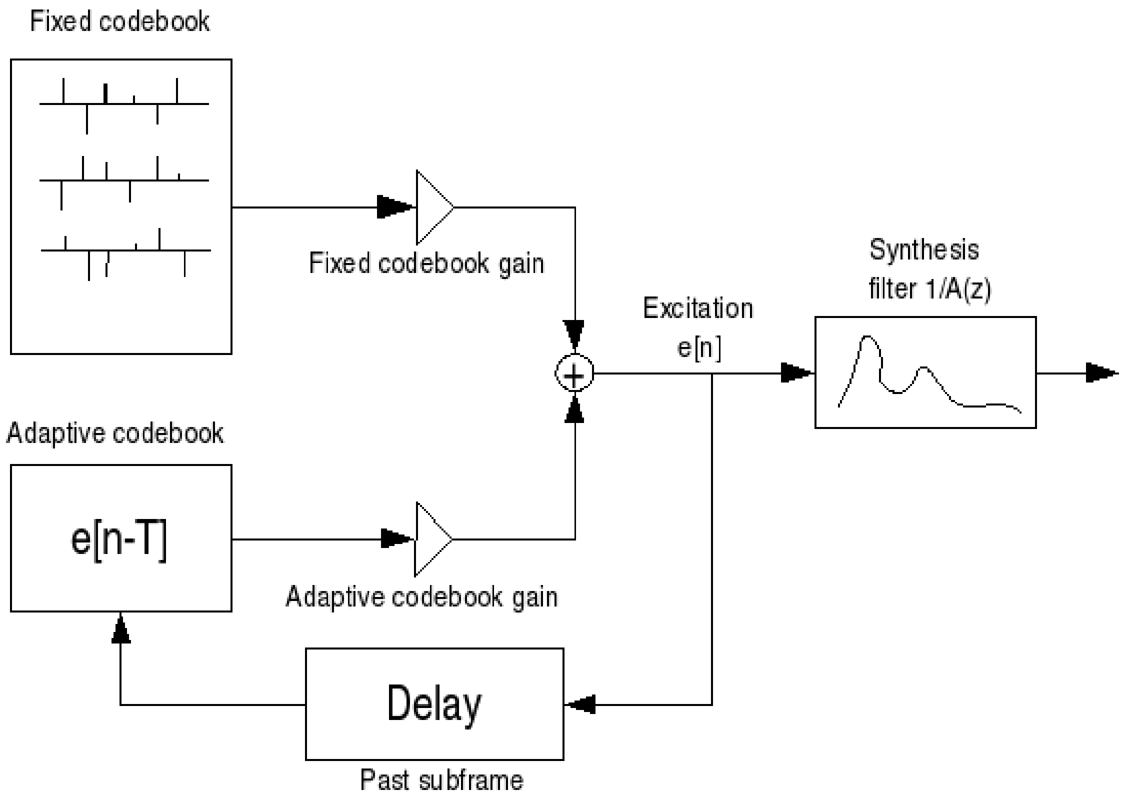
- Currently the most widely used speech coding algorithm
- Code books
- Vector Quantization
- <8kbps
- Compared to CD

44.1 k sampling

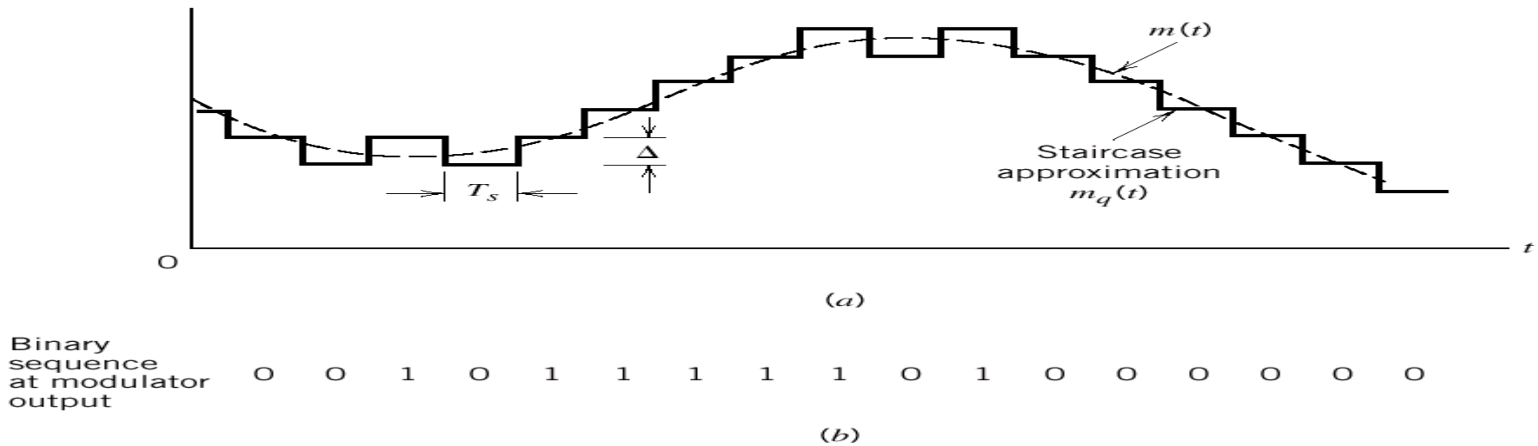
16 bits quantization

705.6 kbps

100 times difference



Delta Modulation (DM)



Let $m[n] = m(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$

where T_s is the sampling period and $m(nT_s)$ is a sample of $m(t)$.

The error signal is

$$e[n] = m[n] - m_q[n-1]$$

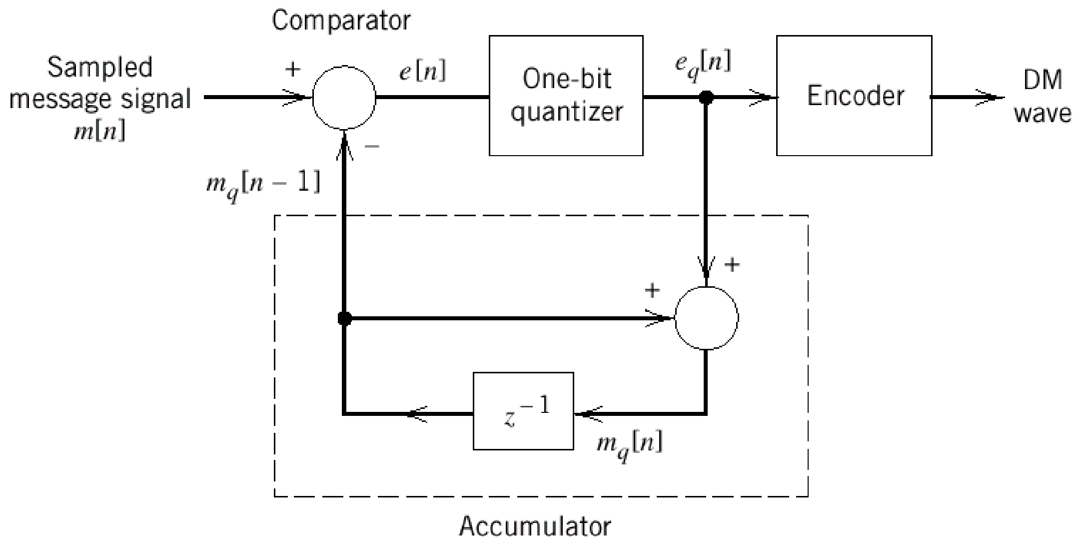
$$e_q[n] = \Delta \operatorname{sgn}(e[n])$$

$$m_q[n] = m_q[n-1] + e_q[n]$$

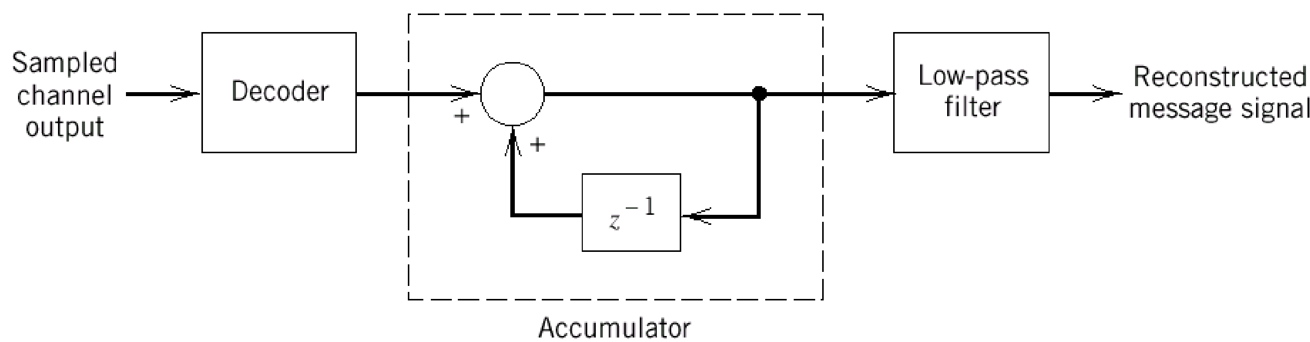
where $m_q[n]$ is the quantizer output, $e_q[n]$ is

the quantized version of $e[n]$, and Δ is the step size

DM System: Transmitter and Receiver.



(*a*)



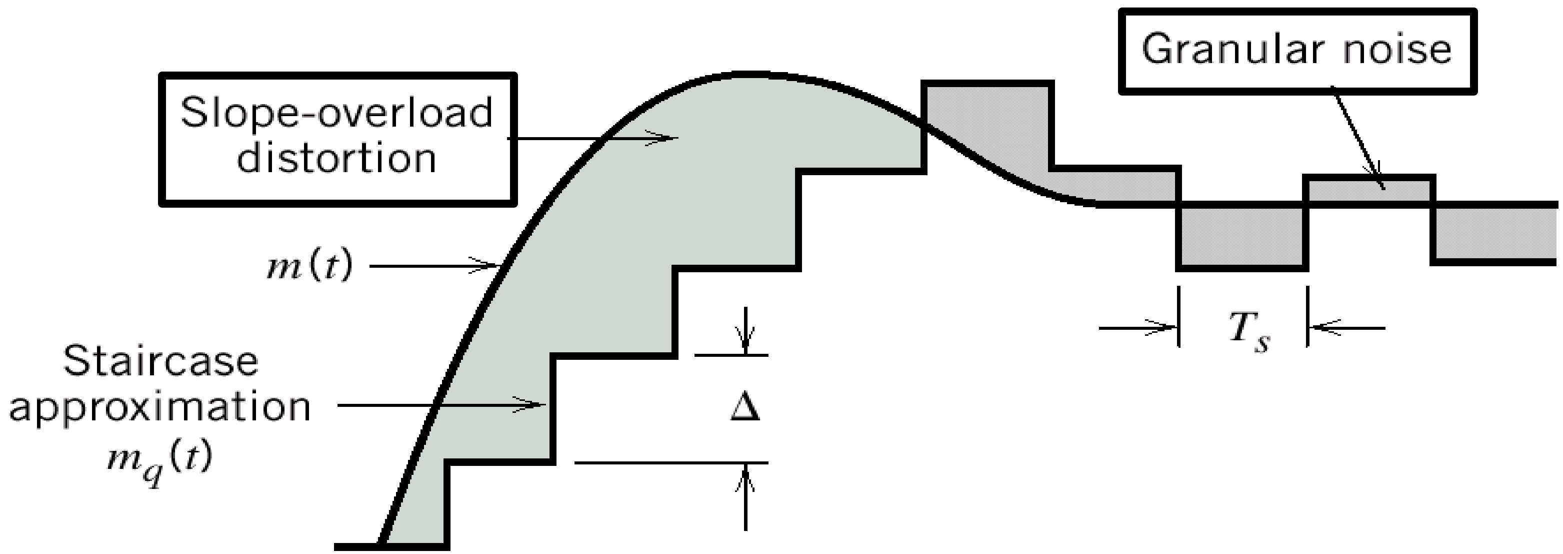
(*b*)

Slope overload distortion and **granular noise**

The modulator consists of a comparator, a quantizer, and an accumulator. The output of the accumulator is

$$m_q[n] = \Delta \sum_{i=1}^{n} \operatorname{sgn}(e[i])$$

$$= \sum_{i=1}^{n} e_q[i]$$



Slope Overload Distortion and Granular Noise

Denote the quantization error by $q[n]$,

$$m_q[n] = m[n] - q[n]$$

We have

$$e[n] = m[n] - m[n-1] - q[n-1]$$

Except for $q[n-1]$, the quantizer input is a first backward difference of the input signal **(differentiator)**

To avoid slope-overload distortion, we require

(slope) $$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

On the other hand, granular noise occurs when step size Δ is too large relative to the local slope of $m(t)$.

Delta-Sigma modulation (sigma-delta modulation)

The $\Delta - \Sigma$ modulation which has an **integrator** can relieve the draw back of delta modulation (**differentiator**)

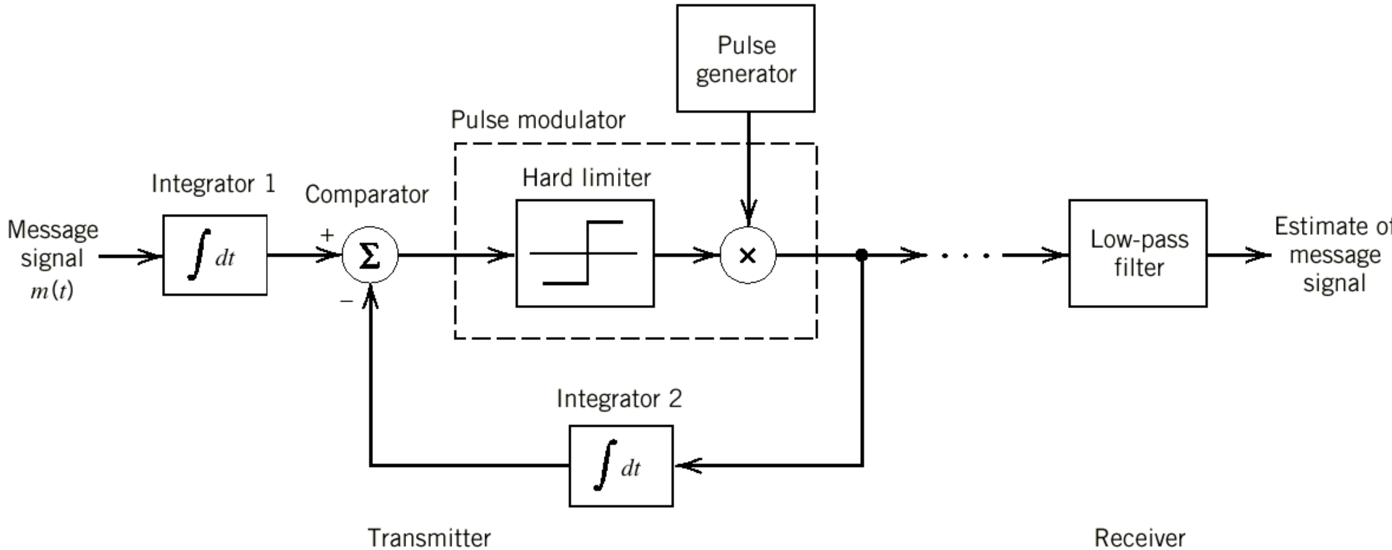
Beneficial effects of using integrator:

1. Pre-emphasize the low-frequency content
2. Increase correlation between adjacent samples
   (reduce the variance of the error signal at the quantizer input)
3. Simplify receiver design

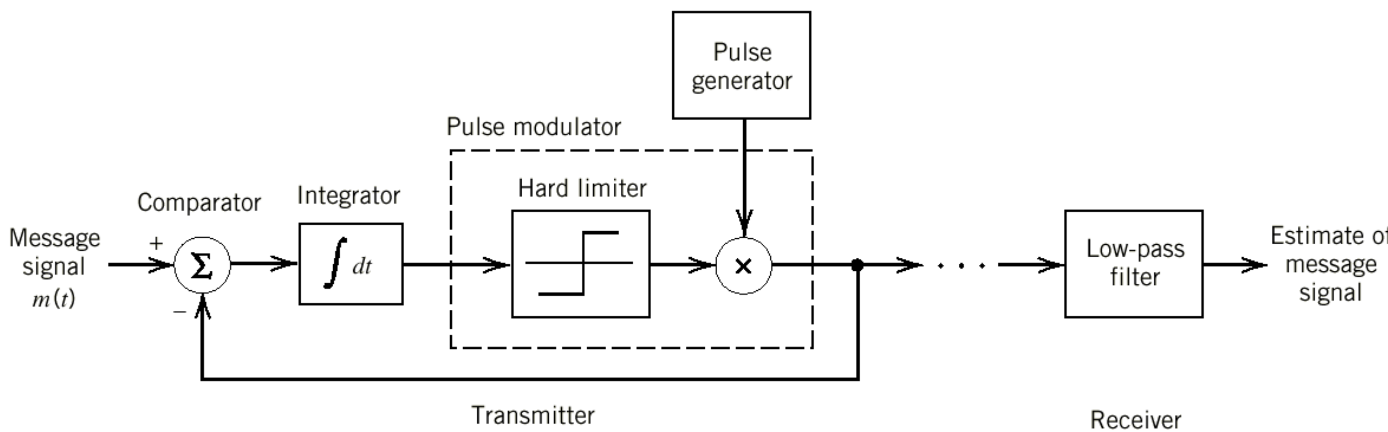
Because the transmitter has an integrator , the receiver consists simply of a low-pass filter.

(The differentiator in the conventional DM receiver is cancelled by the integrator)

delta-sigma modulation system.



(a)



(b)

Two Types of Errors

- Round off error
- Detection error
- Variance of sum of the independent random variables is equal to the sum of the variances of the independent random variables.
- The final error energy is equal to the sum of error energy for two types of errors
- Round off error in PCM

$$\sigma_q^2 = \frac{1}{3}\left(\frac{m_p}{L}\right)^2$$

Mean Square Error in PCM

- If transmit 1101 (13), but receive 0101 (5), error is 8
- Error in different location produces different MSE

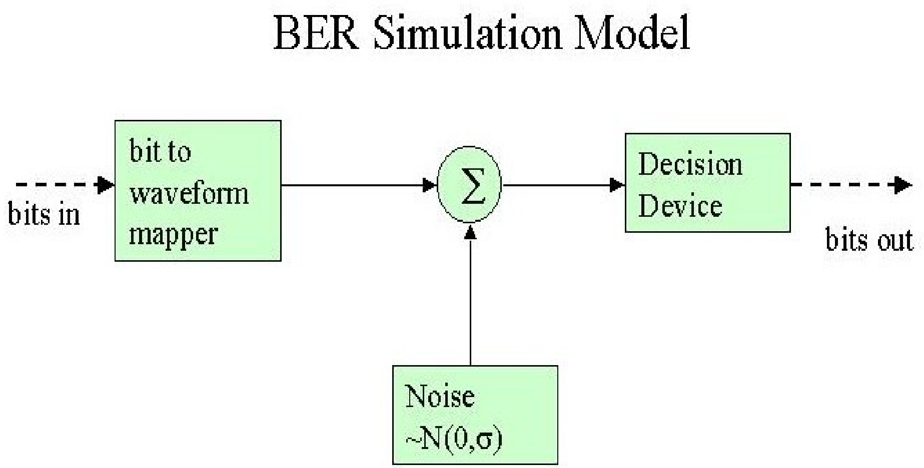
$$\varepsilon_i = (2^{-i})(2m_p)$$

- Overall error probability

$$MSE = \sum_{i=1}^{n} \varepsilon_i^2 P_e(\varepsilon_i) = P_e \sum_{i=1}^{n} \varepsilon_i^2 = \frac{4m_p^2 P_e (2^{2n} - 1)}{3(2^{2n})}$$

  - Gray coding: if one bit occur, the error is minimized.

Bit Errors in PCM Systems



Simplest case is Additive White Gaussian Noise for baseband PCM scheme -- see the analysis for this case. For signal levels of +A and -A we get

$p_e = Q(A/\sigma)$

Notes

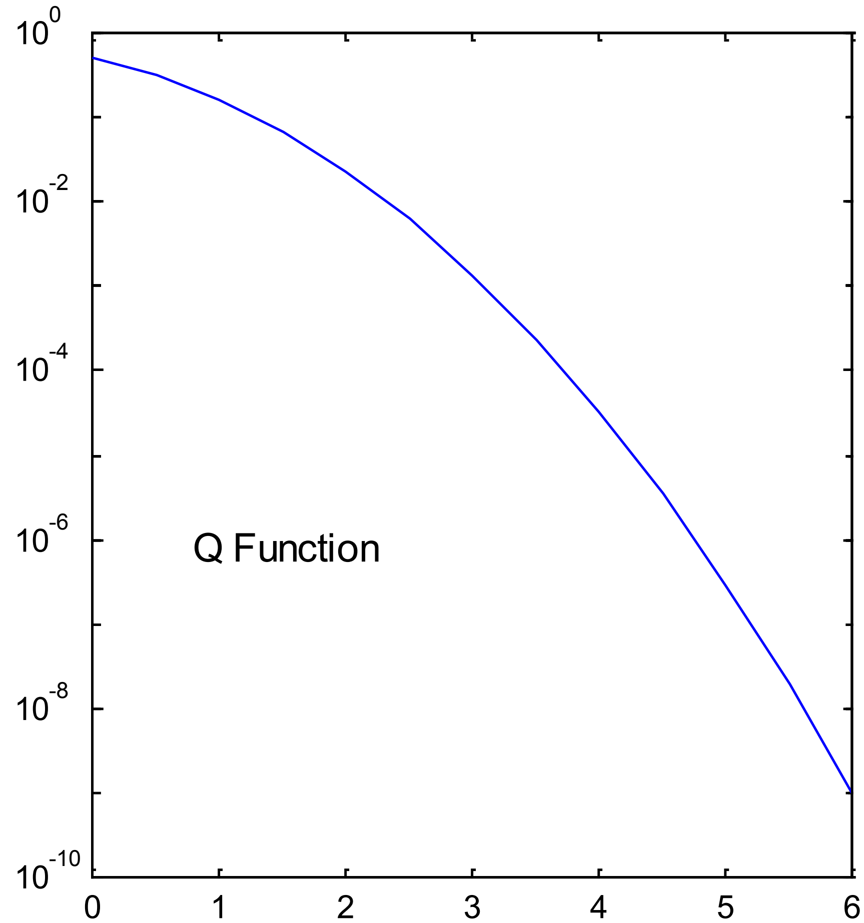
- $Q(A/\sigma)$ represents the area under one tail of the normal pdf
- $(A/\sigma)^2$ represents the Signal to Noise (SNR) ratio
- Our analysis has neglected the effects of transmit and receive filters - it can be shown that the same results apply when filters with the correct response are used.

Q Function

- For Q function:
  - The remain of cdf of Gaussian distribution
  - Physical meaning
  - Equation

$$P_e = Q\left(\sqrt{\gamma}\right)$$

- Matlab: erfc
  - y = **Q**(x)
  - y = 0.5***erfc**(x/sqrt(2));
- Note how rapidly Q(x) decreases as x increases - this leads to the threshold characteristic of digital communication systems

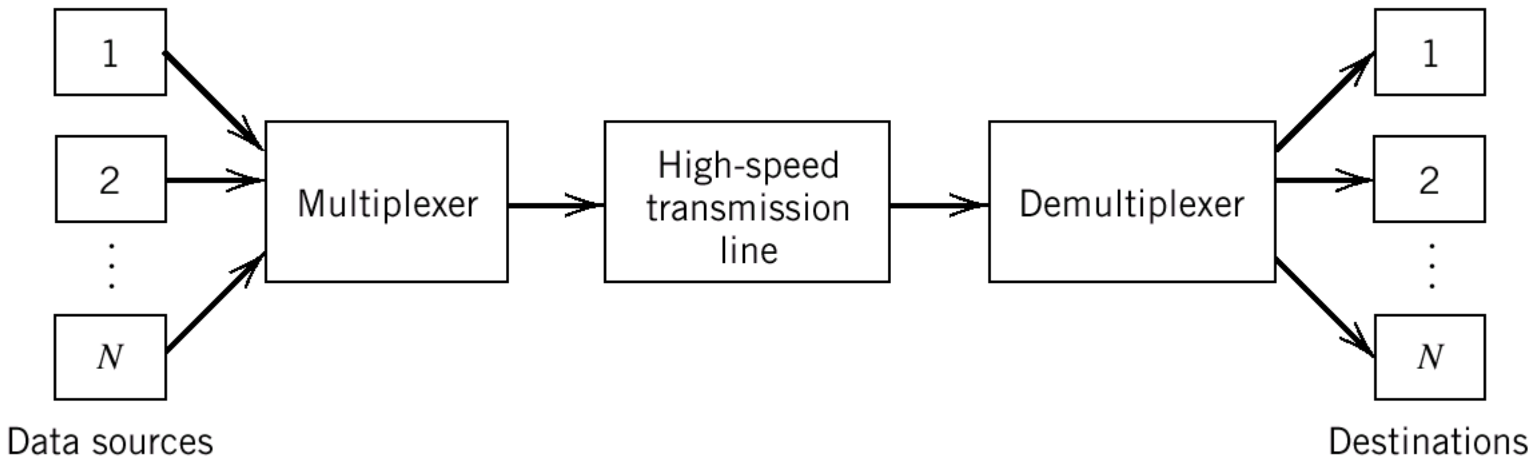


SNR vs. γ

$$\frac{S_0}{N_0} = \frac{3(2^{2n})}{1 + 4(2^{2n} - 1)Q(\sqrt{\gamma / n_0})} \left(\frac{\overline{m}^2}{m_p^2} \right)$$

- Threshold
- Saturation
  - slightly better than ADC
- Exchange of SNR for bandwidth is much more efficient than in angle modulation
- Repeaters

Time-Division Multiplexing



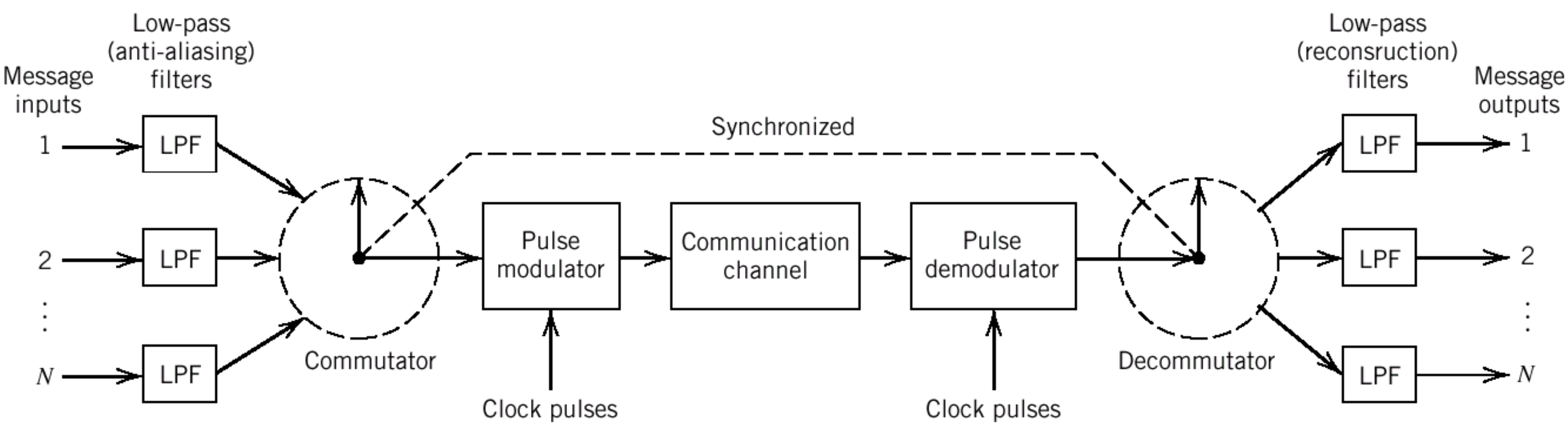


Figure Block diagram of TDM system.