Shannon’s Information Theory

OVERVIEW

- What is Information Theory
- What is Noise and Information
- General Model of Communication
- Examples (Information Theory)
- Definition of Entropy.

Overview Cont..

- Shannon’s Theorem
- Examples (Shannon’s Theorem)
- Shannon’s Law
- Conclusion
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What is Information Theory

Information theory deals with measurement and transmission of information through a channel.

A fundamental work in this area is the Shannon's Information Theory, which provides many useful tools that are based on measuring information in terms of bits or - more generally - in terms of (the minimal amount of) the complexity of structures needed to encode a given piece of information.

NOISE

Noise can be considered data without meaning; that is, data that is not being used to transmit a signal, but is simply produced as an unwanted by-product of other activities. Noise is still considered information, in the sense of Information Theory.

Information Theory Cont…

Shannon's ideas

- Form the basis for the field of Information Theory

- Provide the yardsticks for measuring the efficiency of communication system.

- Identified problems that had to be solved to get to what he described as ideal communications systems

Information

In defining information, Shannon identified the critical relationships among the elements of a communication system

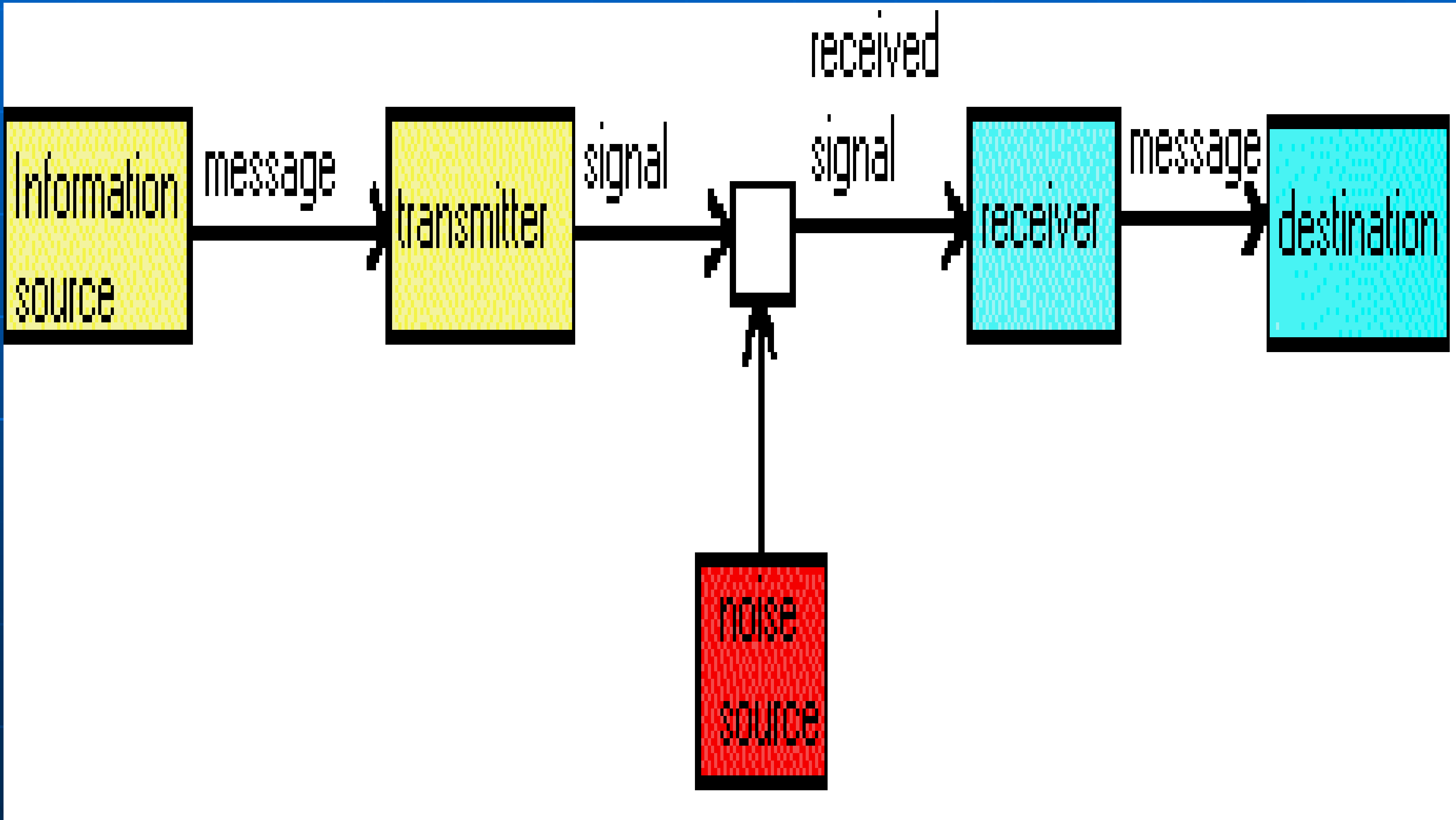
the power at the **source** of a signal

the **bandwidth** or frequency range of an information channel through which the signal travels

the **noise** of the channel, such as unpredictable static on a radio, which will alter the signal by the time it reaches the last element of the System

the **receiver**, which must decode the signal.

General Model Of Communication



Information Theory Cont.

To get a high-level understanding of his theory, a few basic points should be made.
First, words are symbols to carry information between people. If one says to an American, “Let’s go!”, the command is immediately understood. But if we give the commands in Russian, “Pustim v xod!”, we only get a quizzical look.
Russian is the wrong code for an American.

Information Theory Cont.

Second, all communication involves three steps

- Coding a message at its source
- Transmitting the message through a communications channel
- Decoding the message at its destination.

Information Theory Cont….

In the first step, the message has to be put into some kind of symbolic
representation – words, musical notes, icons, mathematical equations, or bits.
When we write “Hello,” we encode a greeting.
When we write a musical score,
it’s the same thing – only we’re encoding sounds.

Information Theory Cont.

For any code to be useful it has to be transmitted to someone or, in a computer's case, to something.

Transmission can be by voice, a letter, a billboard, a telephone conversation, a radio or television broadcast.

At the destination, someone or something has to receive the symbols, and then decode them by matching them against his or her own body of information to extract the data.

Information Theory Cont….

Fourth, there is a distinction between a communications channel's designed symbol rate of so many bits per second and its actual information capacity. Shannon defines channel capacity as how many kilobits per second of user information can be transmitted over a noisy channel with as small an error rate as possible, which can be less than the channel's "raw" symbol rate.

EXAMPLE

Suppose we are watching cars going past on a highway. For simplicity, suppose 50% of the cars are black, 25% are white,
12.5% are red, and 12.5% are blue. Consider the flow of cars as an information
source with four words: black, white, red, and blue. A simple way of encoding
this source into binary symbols would be to associate each color with two bits,
that is:
black = 00, white = 01, red = 10, and blue = 11, an average of 2.00 bits
per color.

A Better Code Using Information Theory

A better encoding can be constructed by allowing for the frequency of certain symbols, or words:
black = 0, white = 10, red = 110, blue = 111.

How is this encoding better?

0.50 black x 1 bit = .500
0.25 white x 2 bits = .500
0.125 red x 3 bits = .375
0.125 blue x 3 bits = .375
Average-- 1.750 bits per car

ENTROPY

A quantitative measure of the disorder of a system and inversely related to the amount of energy available to do work in an isolated system. The more energy has become dispersed, the less work it can perform and the greater the entropy.

Information Theory Cont..

Furthermore Information Theory tells us that the entropy of this information source is 1.75 bits per car and thus no encoding scheme will do better than the scheme we just described.

In general, an efficient code for a source will not represent single letters, as in our example before, but will represent strings of letters or words. If we see three black cars, followed by a white car, a red car, and a blue car, the sequence would be encoded as 00010110111, and the original sequence of cars can readily be recovered from the encoded sequence.

Shannon’s Theorem

Shannon's theorem, proved by Claude Shannon in 1948, describes the maximum possible efficiency of error correcting methods versus levels of noise interference and data corruption.

Shannon’s theorem

The theory doesn't describe how to construct the error-correcting method, it only tells us how good the best possible method can be. Shannon's theorem has wide-ranging applications in both communications and data storage applications.

$$C \leq W \log_2(1 + S/N)$$

where

C is the post-correction effective channel capacity in bits per second;

W is the raw channel capacity in hertz (the bandwidth); and

S/N is the signal-to-noise ratio of the communication signal to the Gaussian noise interference expressed as a straight power ratio (not as decibels)

Shannon’s Theorem Cont..

Channel capacity, shown often as "C" in communication formulas, is the amount of discrete information bits that a defined area or segment in a communications medium can hold.

Shannon Theorem Cont..

The phrase signal-to-noise ratio, often abbreviated SNR or S/N, is an engineering term for the ratio between the magnitude of a signal (meaningful information) and the magnitude of background noise. Because many signals have a very wide dynamic range, SNRs are often expressed in terms of the logarithmic decibel scale.

Example

If the SNR is 20 dB, and the bandwidth available is 4 kHz, which is appropriate for telephone communications, then C = 4 log2(1 + 100) = 4 log2 (101) = 26.63 kbit/s. Note that the value of 100 is appropriate for an SNR of 20 dB.

Example

If it is required to transmit at 50 kbit/s, and a bandwidth of 1 MHz is used, then the minimum SNR required is given by 50 = 1000 log2(1+S/N) so S/N = 2C/W -1 = 0.035 corresponding to an SNR of -14.5 dB. This shows that it is possible to transmit using signals which are actually much weaker than the background noise level.

SHANNON'S LAW

Shannon's law is any statement defining the theoretical maximum rate at which error free digits can be transmitted over a bandwidth limited channel in the presence of noise