Multiple FSK (MFSK)

- More than two frequencies (M frequencies) are used
- More bandwidth efficient compared to BFSK
- More susceptible to noise compared to BFSK
- MFSK signal:

$$s_i(t) = A\cos(2\pi f_i t), \qquad 1 \le i \le M$$

where

$$f_i = f_c + (2i - 1 - M) f_d$$

$$f_c = \textit{the carrier frequency}$$

$$f_d = \textit{the difference frequency}$$

$$M = \textit{number of different signal elements} = 2^L$$

$$L = \textit{number of bits per signal element}$$

Multiple FSK (MFSK)

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$$M = number\ of\ different\ signal\ elements = 2^L$$

$$L = number\ of\ bits\ per\ signal\ element$$

- Period of signal element

$$T_s = LT_b, \qquad T_s : signal\ element\ period \quad T_b : bit\ period$$

- Minimum frequency separation

$$1/T_s = 2f_d \quad \Rightarrow \quad 1/(LT_b) = 2f_d \;\Rightarrow\; 1/T_b = 2Lf_d\ (bit\ rate)$$

- MFSK signal bandwidth:

$$W_d = M(2f_d) = 2Mf_d$$

Example

- With f_c=250KHz, f_d=25KHz, and M=8 (L=3 bits), we have the following frequency assignment for each of the 8 possible 3-bit data combinations:

$$f_i = f_c + (2i - 1 - M) f_d$$

$$\left.\begin{array}{lll} 000 & \rightarrow & f_1 = 75KHz \\ 001 & \rightarrow & f_2 = 125KHz \\ 010 & \rightarrow & f_3 = 175KHz \\ 011 & \rightarrow & f_4 = 225KHz \\ 100 & \rightarrow & f_5 = 275KHz \\ 101 & \rightarrow & f_6 = 325KHz \\ 110 & \rightarrow & f_7 = 375KHz \\ 111 & \rightarrow & f_8 = 425KHz \end{array}\right\} \quad bandwidth = W_s = 2Mf_d = 400KHz$$

- This scheme can support a data rate of:

$$1/T_b = 2Lf_d = 2(3bits)(25Hz) = 150Kbps$$

Example

- The following figure shows an example of MFSK with M=4. An input bit stream of 20 bits is encoded 2bits at a time, with each of the possible 2-bit combinations transmitted as a different frequency.

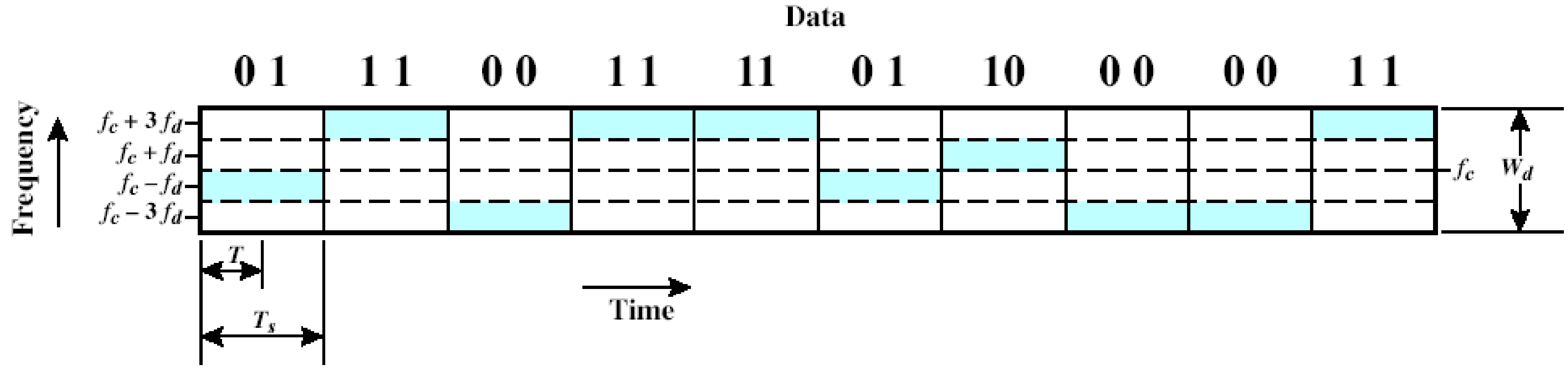
$$f_i = f_c + (2i - 1 - M) f_d$$

$$00 \rightarrow i = 1 \rightarrow f_1 = f_c - 3f_d$$

$$01 \rightarrow i = 2 \rightarrow f_2 = f_c - f_d$$

$$10 \rightarrow i = 3 \rightarrow f_3 = f_c + f_d$$

$$11 \rightarrow i = 4 \rightarrow f_4 = f_c + 3f_d$$



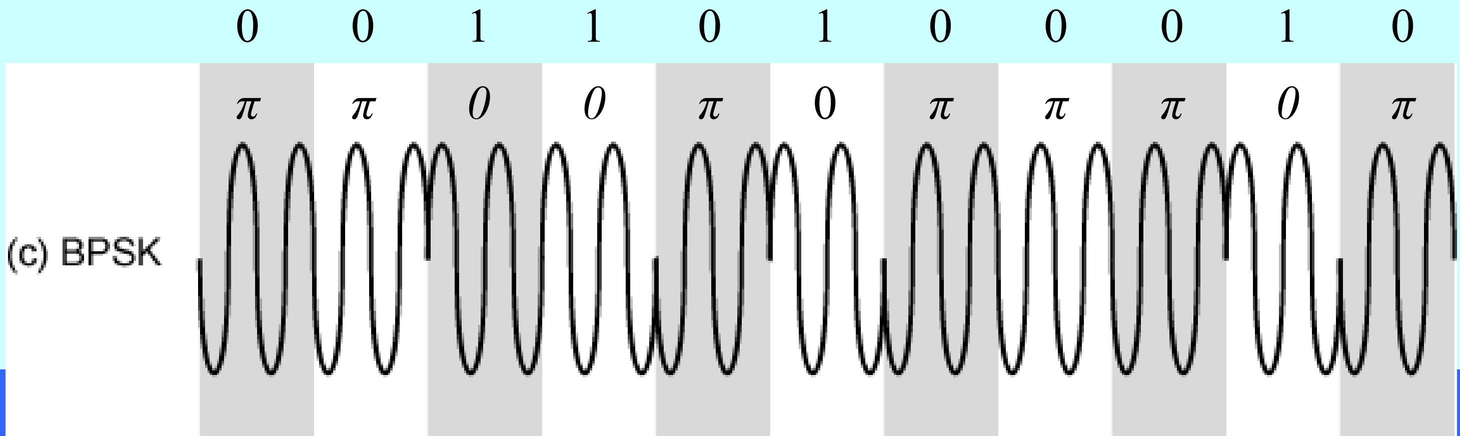
Phase Shift Keying (PSK)

- Phase of carrier signal is shifted to represent data
- Binary PSK (BPSK): two phases represent two binary digits

$$s(t) = \begin{cases} A\cos(2\pi f_c t), & binary\ 1 \\ A\cos(2\pi f_c t + \pi), & binary\ 0 \end{cases}$$

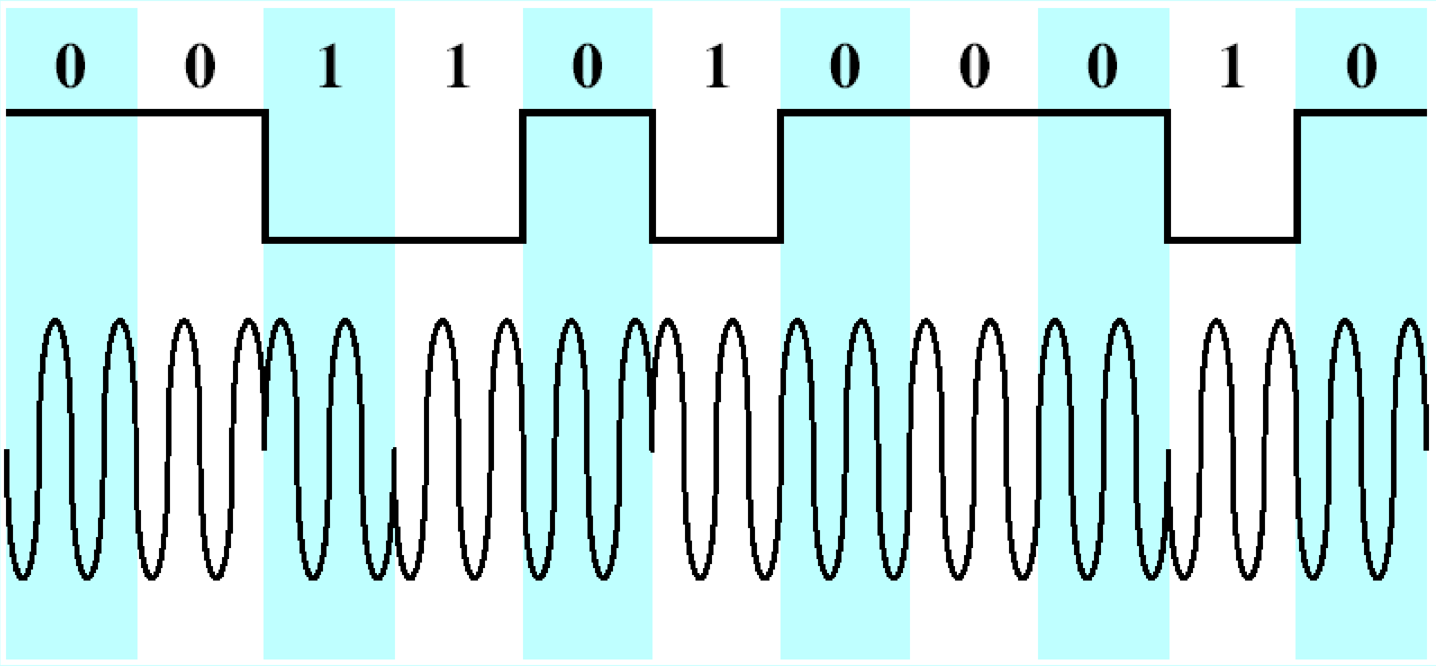
$$= \begin{cases} A\cos(2\pi f_c t), & binary\ 1 \\ -A\cos(2\pi f_c t), & binary\ 0 \end{cases}$$

$$= Ad(t)\cos(2\pi f_c t), \quad d(t) = \pm 1$$



Differential PSK (DPSK)

- In DPSK, the phase shift is with reference to the previous bit transmitted rather than to some constant reference signal
- Binary 0:signal burst with the same phase as the previous one
- Binary 1:signal burst of opposite phase to the preceding one



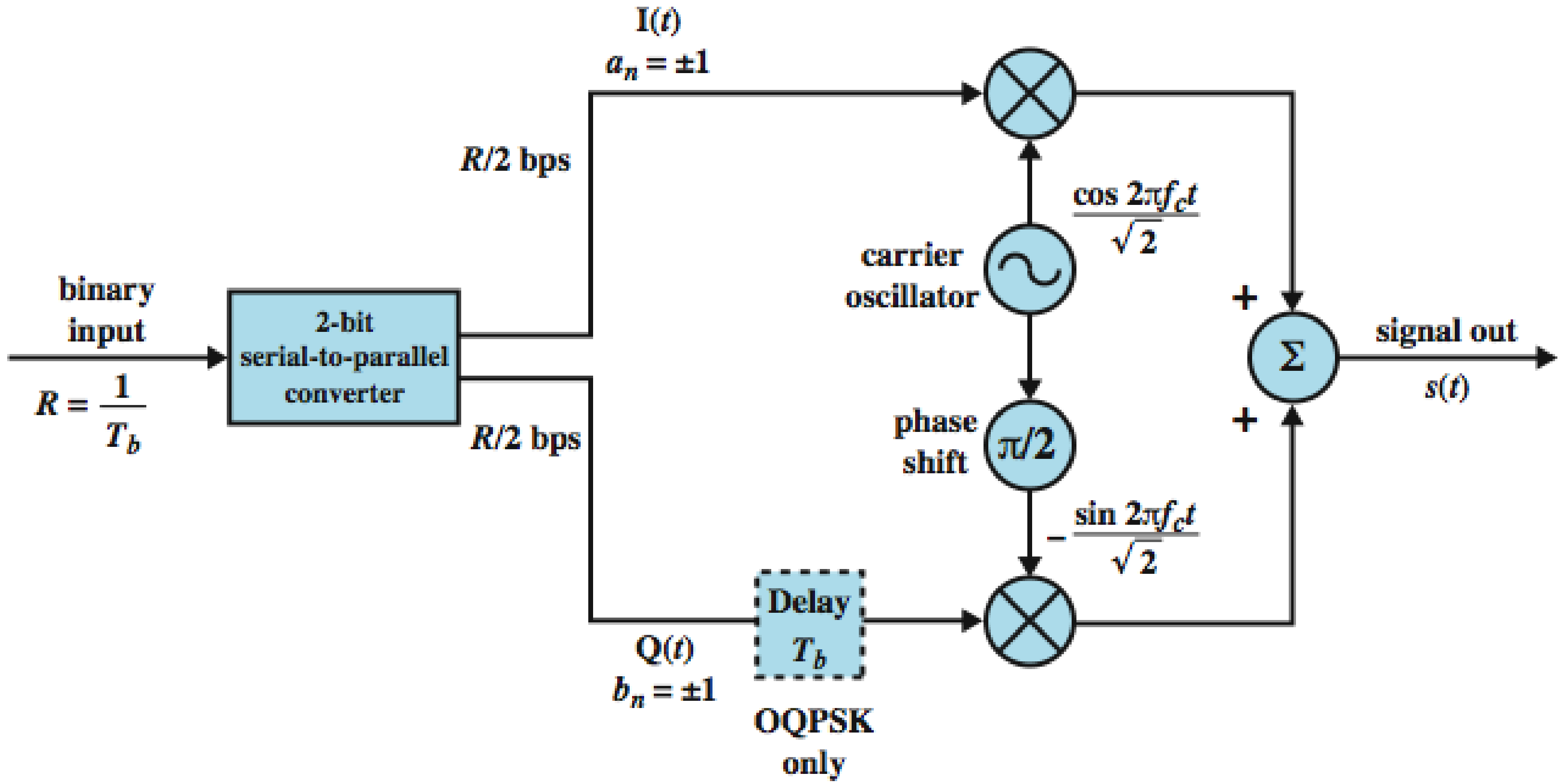
Four-level PSK: Quadrature PSK (QPSK)

- More efficient use of bandwidth if each signal element represents more than one bit
  - eg. shifts of $\pi/2$ (90°)
  - each signal element represents two bits
  - split input data stream in two & modulate onto the phase of the carrier

$$s(t) = \begin{cases} A\cos(2\pi f_c t + \frac{\pi}{4}) & \leftrightarrow \quad 11 \\ A\cos(2\pi f_c t + \frac{3\pi}{4}) & \leftrightarrow \quad 01 \\ A\cos(2\pi f_c t + \frac{3\pi}{4}) & \leftrightarrow \quad 00 \\ A\cos(2\pi f_c t - \frac{\pi}{4}) & \leftrightarrow \quad 10 \end{cases}$$

- can use 8 phase angles & more than one amplitude
  - 9600bps modem uses 12 phase angles, four of which have two amplitudes: this gives a total of 16 different signal elements

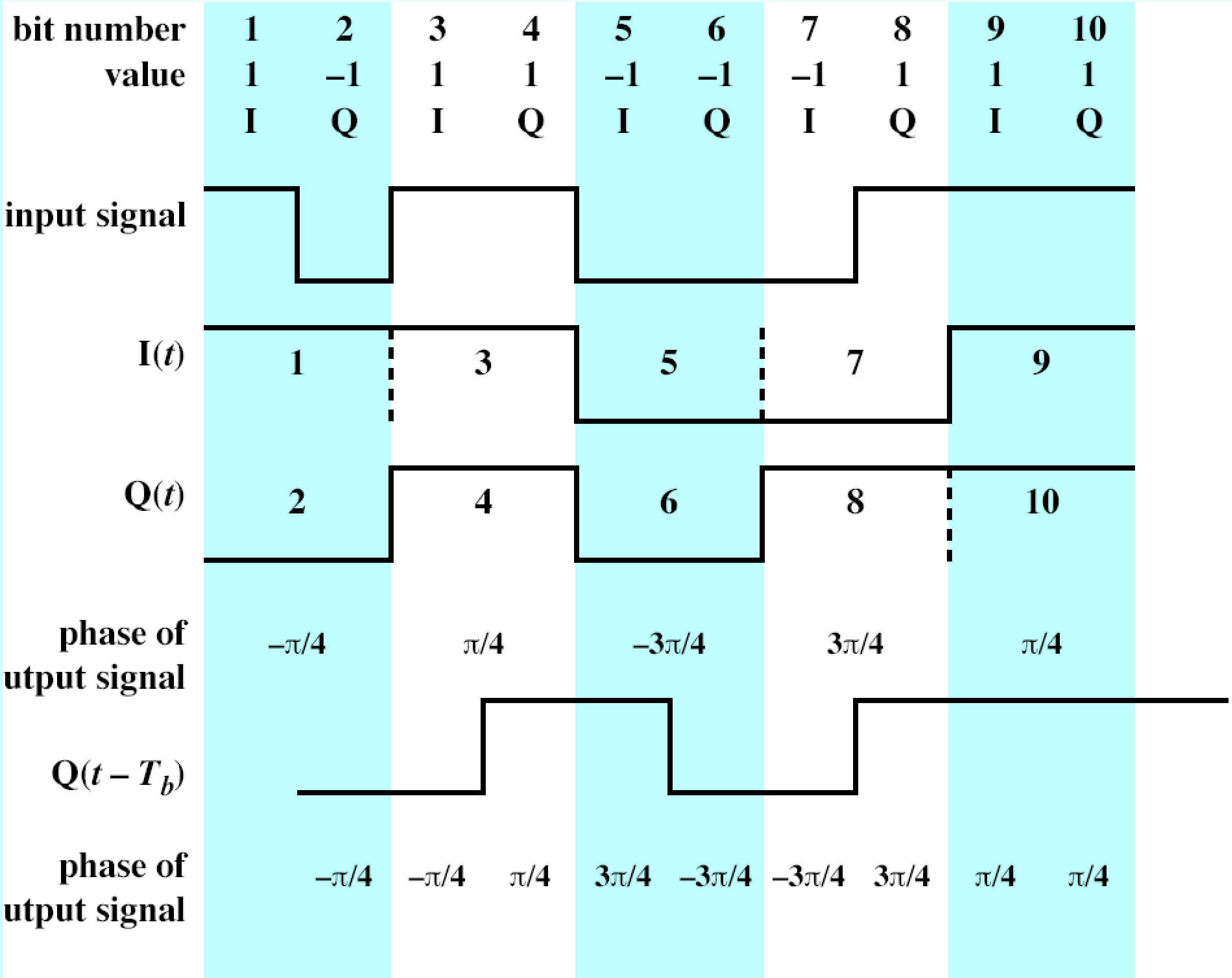
QPSK and Offset QPSK (OQPSK) Modulators



$$QPSK: \quad s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t)\sin(2\pi f_c t)$$

$$OQPSK: \quad s(t) = \frac{1}{\sqrt{2}} I(t)\cos(2\pi f_c t) - \frac{1}{\sqrt{2}} Q(t - T_b)\sin(2\pi f_c t)$$

Example of QPSK and OQPSK Waveforms



$$for \quad QPSK:$$

$$1 \;\; 1 \rightarrow 1 \;\; 1 \rightarrow \frac{\pi}{4}$$

$$0 \;\; 1 \rightarrow -1 \;\; 1 \rightarrow \frac{3\pi}{4}$$

$$0 \;\; 0 \rightarrow -1 \; -1 \rightarrow \frac{-3\pi}{4}$$

$$1 \;\; 0 \rightarrow 1 \; -1 \rightarrow \frac{-\pi}{4}$$

Performance of ASK, FSK, MFSK, PSK and MPSK

Bandwidth Efficiency

- ASK/PSK: $$\frac{data\ rate}{transmission\ bandwidth} = \frac{R}{B_T} = \frac{1}{1+r}, \qquad 0 < r < 1$$
- MPSK: $$\frac{R}{B_T} = \frac{\log_2 M}{1+r}, \qquad M: number\ of\ different\ signal\ elements$$
- MFSK: $$\frac{R}{B_T} = \frac{\log_2 M}{(1+r)M}$$

Bit Error Rate (BER)

- bit error rate of PSK and QPSK are about 3dB superior to ASK and FSK (see Fig. 5.4)
- for MFSK & MPSK have tradeoff between bandwidth efficiency and error performance

Performance of MFSK and MPSK

- MFSK: increasing M decreases BER and decreases bandwidth Efficiency
- MPSK: Increasing M increases BER and increases bandwidth efficiency

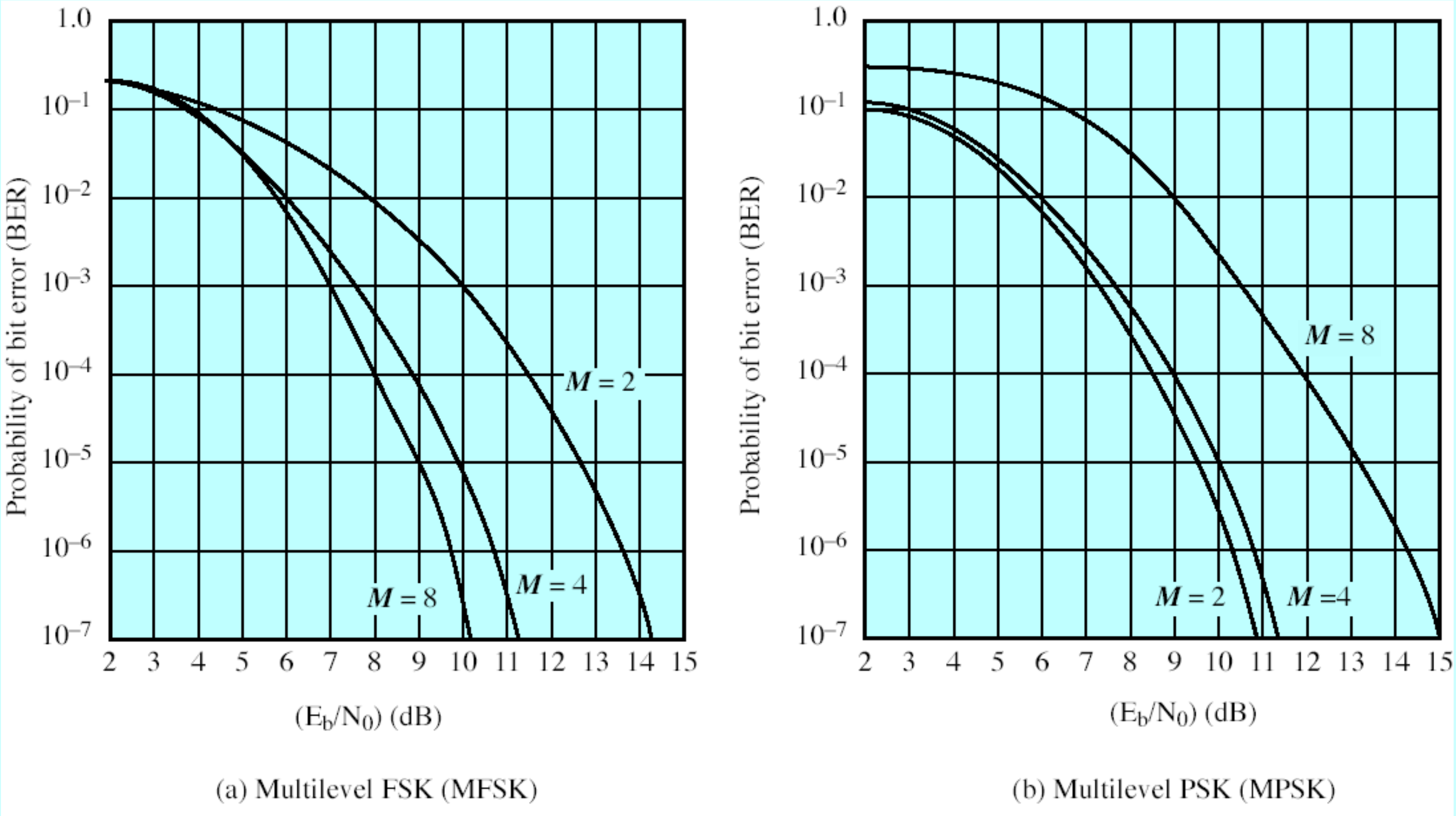
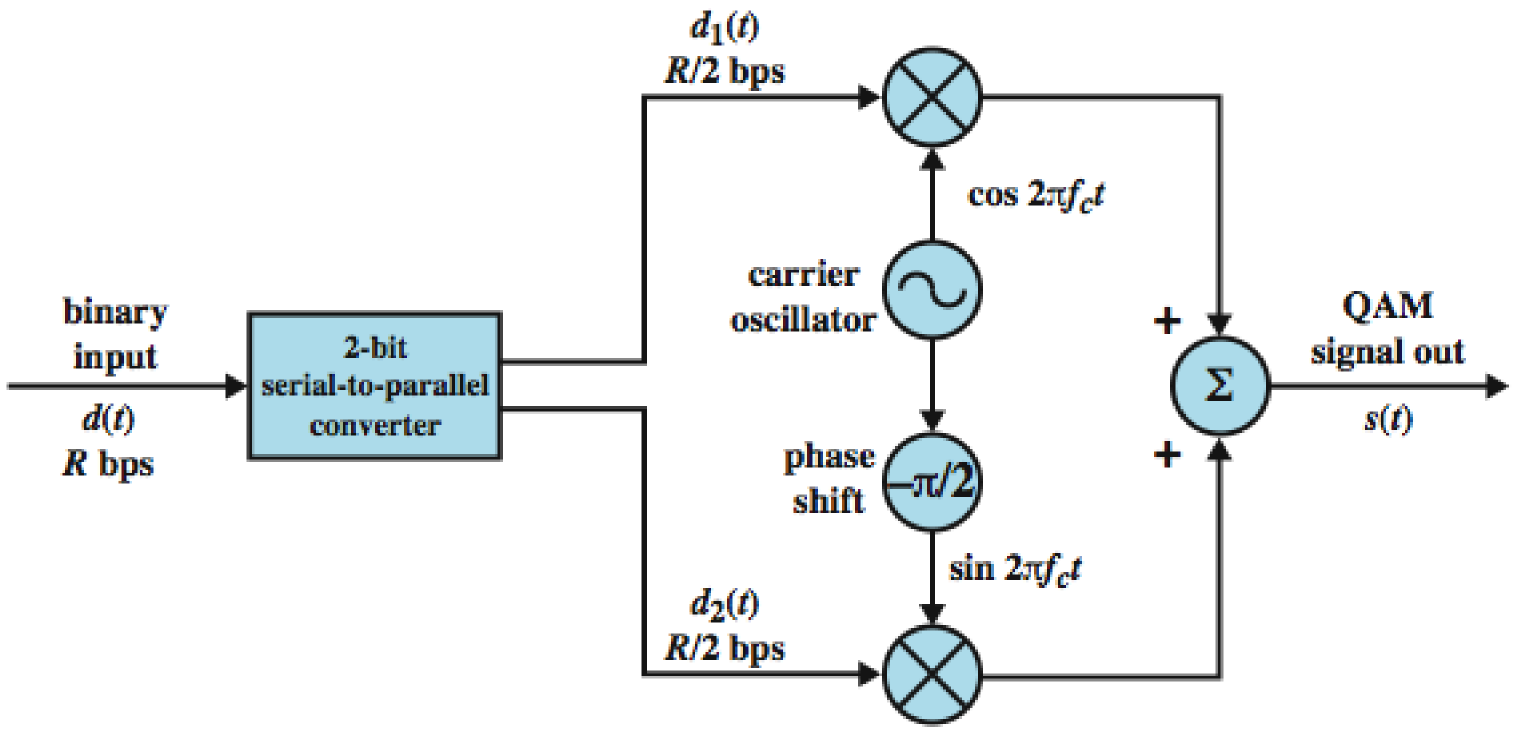


Figure 5.13 Theoretical Bit Error Rate for Multilevel FSK and PSK

Quadrature Amplitude Modulation (QAM)

- QAM used on asymmetric digital subscriber line (ADSL) and some wireless standards
- combination of ASK and PSK
- logical extension of QPSK
- send two different signals simultaneously on same carrier frequency
  - use two copies of carrier, one shifted by 90°
  - each carrier is ASK modulated

QAM modulator



$$QAM: \qquad s(t) = \underbrace{d_1(t)\cos(2\pi f_c t)}_{ASK} + \underbrace{d_2(t)\sin(2\pi f_c t)}_{ASK}$$

QAM Variants

- Two level ASK (two different amplitude levels)
  - each of two streams in one of two states
  - four state system
  - essentially QPSK
- Four level ASK (four different amplitude levels)
  - combined stream in one of 16 states
- Have 64 and 256 state systems
- Improved data rate for given bandwidth
  - but increased potential error rate