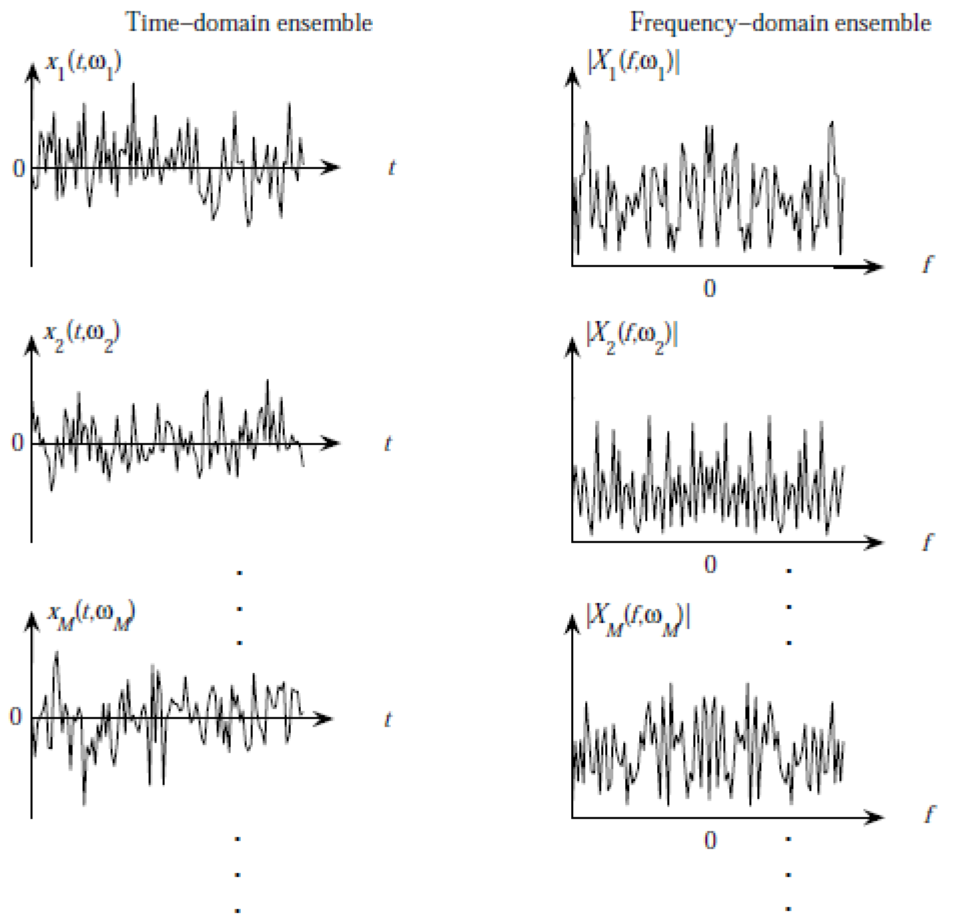
Spectral Density

SPECTRAL DENSITY

- The *spectral density* of a signal characterizes the distribution of the signal's energy or power, in the frequency domain
- This concept is particularly important when considering filtering in communication systems while evaluating the signal and noise at the filter output.
- The energy spectral density (ESD) or the power spectral density (PSD) is used in the evaluation.
- Need to determine how the average power or energy of the process is distributed in frequency.

SPECTRAL DENSITY

- Taking the Fourier transform of the random process does not work



ENERGY SPECTRAL DENSITY

- Energy spectral density describes the energy per unit bandwidth measured in joules/hertz
- Represented as $\varphi_x(t)$, the squared magnitude ***spectrum***

$$\varphi_x(t) = |x(f)|^2$$

- According to Parseval's Relation $E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$

- Therefore $E_x = \int_{-\infty}^{\infty} \psi_x(f)\,df$

- The Energy spectral density is symmetrical in frequency about origin and total energy of the signal x(t) can be expressed as

$$E_x = 2\int_{0}^{\infty} \psi_x(f)\,df$$

POWER SPECTRAL DENSITY

- The power spectral density (PSD) function $G_x(f)$ of the periodic signal x(t) is a real, even ad nonnegative function of frequency that gives the distribution of the power of x(t) in the frequency domain.
- PSD is represented as (Fourier Series):

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$$

- PSD of non-periodic signals:

$$G_x(f) = \lim_{T\to\infty} \frac{1}{T} |X_T(f)|^2$$

- Whereas the average power of a periodic signal x(t) is represented as:

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Noise

Noise in the Communication System

- The term *noise* refers to *unwanted* electrical signals that are always present in electrical systems: e.g. spark-plug ignition noise, switching transients and other electro-magnetic signals or atmosphere: the sun and other galactic sources
- Can describe thermal noise as zero-mean Gaussian random process
- A Gaussian process $n(t)$ is a random function whose value n at any arbitrary time t is statistically characterized by the Gaussian probability density function

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

WHITE NOISE

- The primary spectral characteristic of thermal noise is that its power spectral density is *the same* for all frequencies of interest in most communication systems
- A thermal noise source emanates an equal amount of noise power per unit bandwidth at all frequencies—from dc to about 10^{12} Hz.
- Power spectral density G(f)

$$G_n(f) = \frac{N_0}{2} \quad \text{watts/hertz}$$

- Autocorrelation function of white noise is

$$R_n(\tau) = \mathcal{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\delta(\tau)$$

- The average power P of white noise if infinite

$$P_n = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df = \infty$$

White Noise

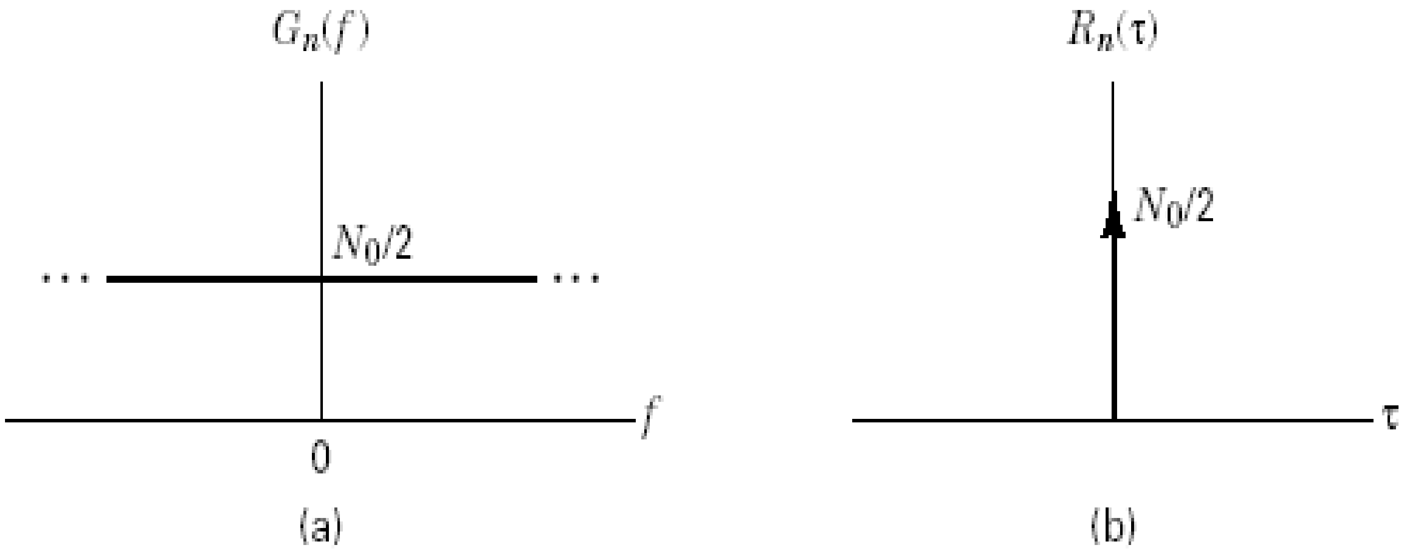


Figure 1.8 (a) Power spectral density of white noise. (b) Autocorrelation function of white noise.

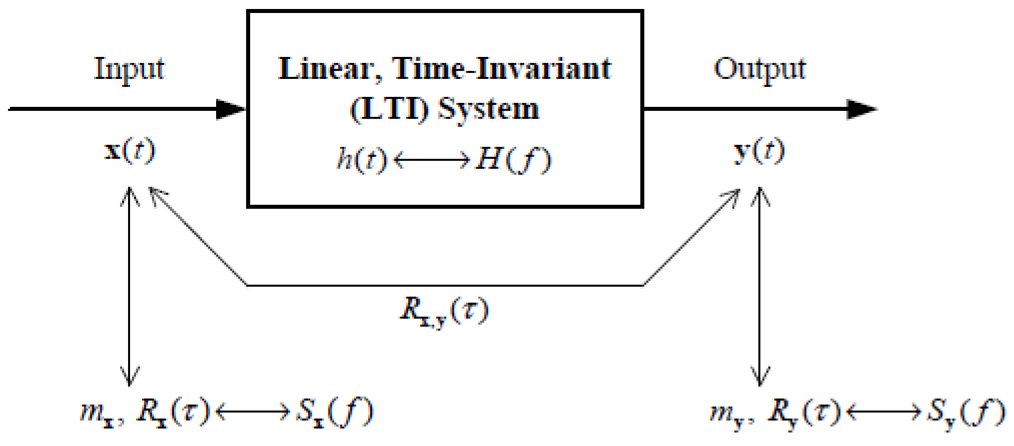
White Noise

- Since Rw(*T*) = 0 for *T* = 0, any two different samples of white noise, no matter how close in time they are taken, are uncorrelated.
- Since the noise samples of white noise are uncorrelated, if the noise is both white and Gaussian (for example, thermal noise) then the noise samples are also independent.

Additive White Gaussian Noise (AWGN)

- The effect on the detection process of a channel with *Additive White Gaussian Noise* (AWGN) is that the noise affects each transmitted symbol *independently*
- Such a channel is called a *memoryless channel*
- The term “additive” means that the noise is simply superimposed or added to the signal—that there are no multiplicative mechanisms at work

Random Processes and Linear Systems



- If a random process forms the input to a time-invariant linear system, the output will also be a random process

$$m_{\mathbf{y}} = E\{\mathbf{y}(t)\} = E\left\{\int_{-\infty}^{\infty} h(\lambda)\mathbf{x}(t-\lambda)\mathrm{d}\lambda\right\}$$

$$\begin{aligned} S_{\mathbf{y}}(f) &= |H(f)|^2 S_{\mathbf{x}}(f) \\ R_{\mathbf{y}}(\tau) &= h(\tau) * h(-\tau) * R_{\mathbf{x}}(\tau). \end{aligned}$$

Distortion less Transmission

Remember linear and non-linear group delays in DSP

DISTORTION LESS TRANSMISSION

What is required of a network for it to behave like an *ideal* transmission line?

- The output signal from an ideal transmission line may have some time delay and different amplitude as compared with the input
- It must have no distortion—it must have the same shape as the input
- For idea distortion less transmission

$$y(t) = Kx(t - t_0) \qquad Y(f) = KX(f)e^{-j2\pi f t_0}$$

$$H(f) = Ke^{-j2\pi f t_0}$$

Ideal Distortion Less Transmission

- The overall system response must have a constant magnitude response
- The phase shift must be linear with frequency
- All of the signal's frequency components must also arrive with identical time delay in order to add up correctly
- The time delay *t*0 is related to the phase shift and the radian frequency $\omega = 2\pi f$ by

$$t_0\,(\text{seconds}) = \frac{\theta\,(\text{radians})}{2\pi f\,(\text{radians/second})}$$

- A characteristic often used to measure delay distortion of a signal is called *envelope delay* or *group delay,* which is defined as

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$$