Receiver Structure

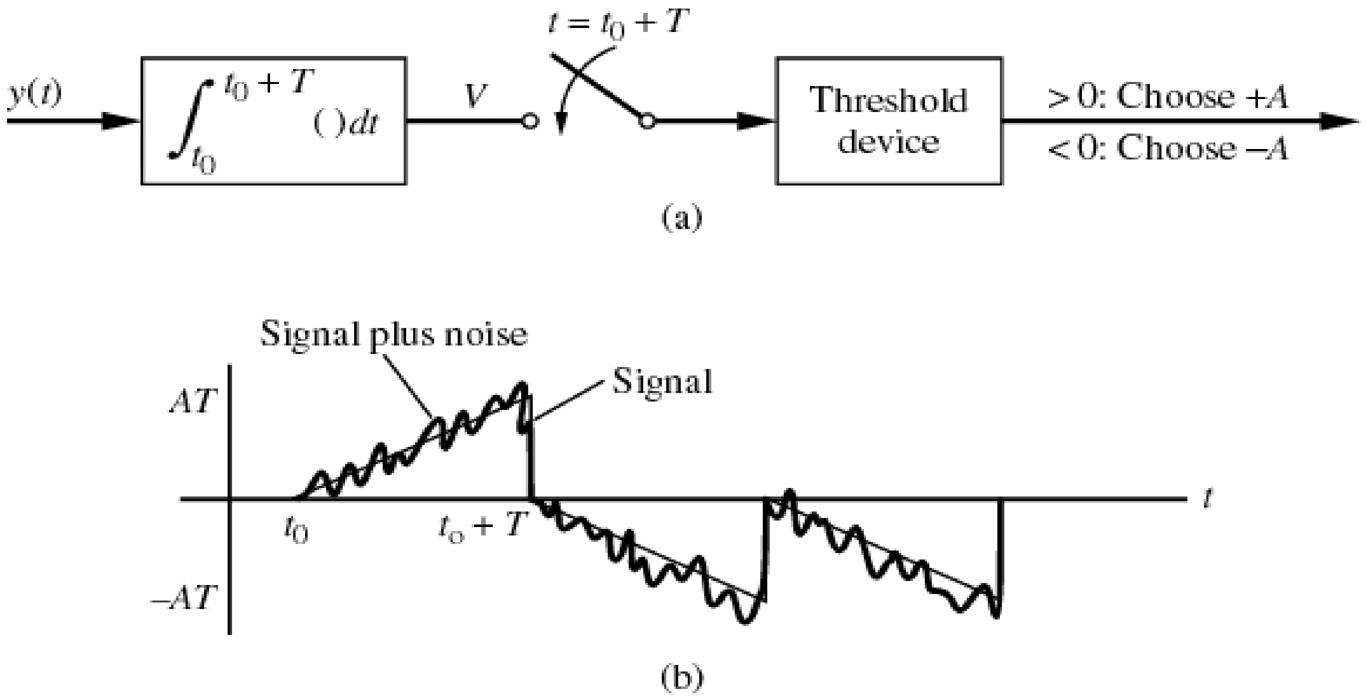


Figure 7-3 Receiver structure and integrator output. (a) Integrate-and-dump receiver. (b) Output from the integrator.

Receiver Preformance

- The output of the integrator:

$$V = \int_{t_0}^{t_0+T} [s(t) + n(t)]dt$$

$$= \begin{cases} AT + N & A \quad is \quad sent \\ -AT + N & -A \quad is \quad sent \end{cases}$$

- $N = \int_{t_0}^{t_0+T} n(t)dt$ is a random variable.
- N is Gaussian. Why?

Analysis

$$E[N] = E[\int_{t_0}^{t_0+T} n(t)dt] = \int_{t_0}^{t_0+T} E[n(t)]dt = 0$$

$$Var[N] = E[N^2] - E^2[N]$$

$$= E[N^2] \qquad Why?$$

$$= E\left\{\left[\int_{t_0}^{t_0+T} n(t)dt\right]^2\right\}$$

$$= \int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T} E[n(t)n(s)]dtds$$

$$= \int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T} \frac{N_0}{2}\delta(t-s)dtds \qquad Why?(White \quad noise \quad is \quad uncorrelated!)$$

$$= \frac{N_0 T}{2}$$

- Key Point
  - White noise is uncorrelated

Error Analysis

- Therefore, the pdf of N is:

$$f_N(n) = \frac{e^{-n^2/(N_0 T)}}{\sqrt{\pi N_0 T}}$$

- In how many different ways, can an error occur?

Error Analysis

- Two ways in which errors occur:
  - A is transmitted, AT+N<0 (0 received,1 sent)
  - -A is transmitted, -AT+N>0 (1 received,0 sent)

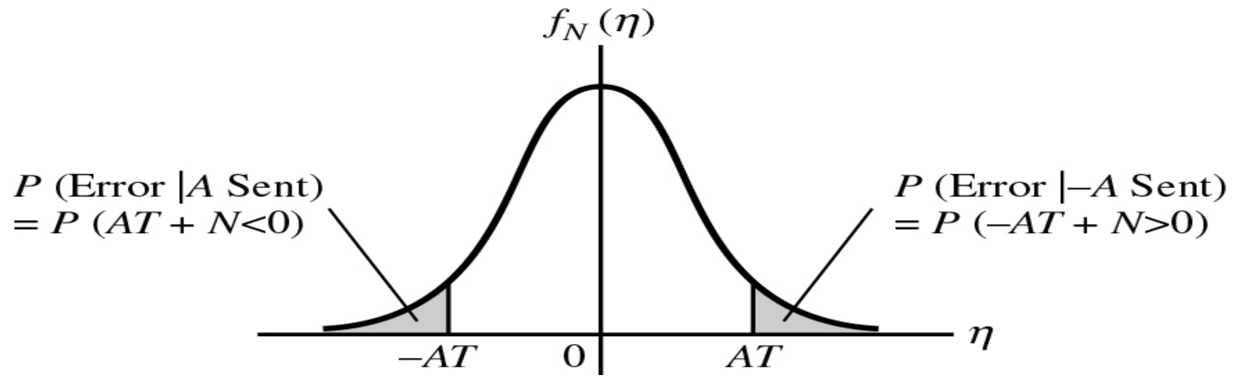


Figure 7-4 Illustration of error probabilities for binary signaling.

- $$P(Error \mid A) = \int_{-\infty}^{-AT} \frac{e^{-n^2/N_0T}}{\sqrt{\pi N_0 T}} dn = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- Similarly,

$$P(Error \mid -A) = \int_{AT}^{\infty} \frac{e^{-n^2/N_0T}}{\sqrt{\pi N_0 T}} dn = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- The average probability of error:

$$P_E = P(E \mid A)P(A) + P(E \mid -A)P(-A)$$

$$= Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- Energy per bit:

$$E_b = \int_{t_0}^{t_0+T} A^2 dt = A^2 T$$

- Therefore, the error can be written in terms of the energy.
- Define

$$z = \frac{A^2 T}{N_0} = \frac{E_b}{N_0}$$

- Recall: Rectangular pulse of duration T seconds has magnitude spectrum

$$ATsinc(Tf)$$

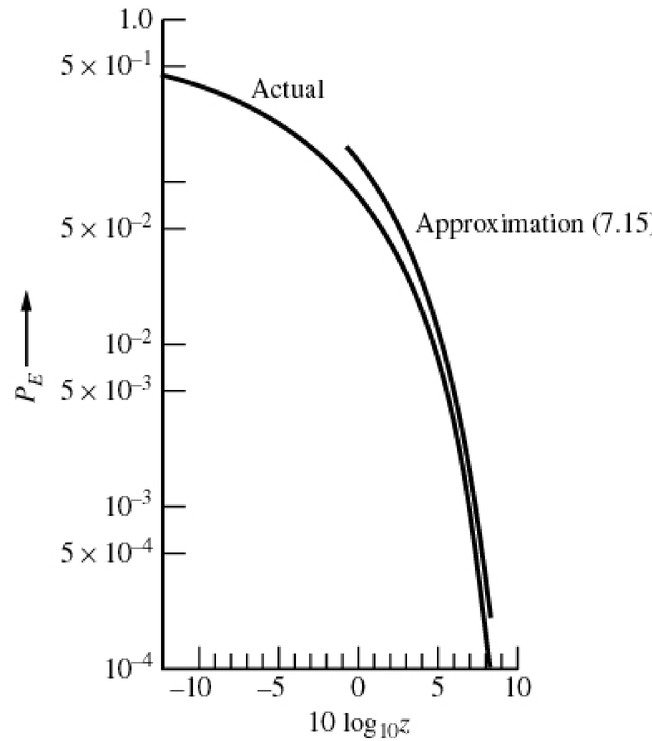
- Effective Bandwidth: $B_p = 1/T$
- Therefore,

$$z = \frac{A^2}{N_0 B_p}$$

- What's the physical meaning of this quantity?

Probability of Error vs. SNR

Figure 7-5
P_E for antipodal baseband digital signaling.



Error Approximation

- Use the approximation

$$Q(u) \cong \frac{e^{-u^2/2}}{u\sqrt{2\pi}}, u >> 1$$

$$P_E = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) \cong \frac{e^{-z}}{2\sqrt{\pi z}}, z >> 1$$

Example

- Digital data is transmitted through a baseband system with $N_0 = 10^{-7} W/Hz$, the received pulse amplitude A=20mV.

a)If 1 kbps is the transmission rate, what is probability of error?

$$B_p = \frac{1}{T} = \frac{1}{10^{-3}} = 10^3$$

$$SNR = z = \frac{A^2}{N_0 B_p} = \frac{400 \times 10^{-6}}{10^{-7} \times 10^3} = 400 \times 10^{-2} = 4$$

$$P_E \cong \frac{e^{-z}}{2\sqrt{\pi z}} = 2.58 \times 10^{-3}$$

b) If 10 kbps are transmitted, what must be the value of A to attain the same probability of error?

$$z = \frac{A^2}{N_0 B_p} = \frac{A^2}{10^{-7} \times 10^4} = 4 \Rightarrow A^2 = 4 \times 10^{-3} \Rightarrow A = 63.2mV$$

- Conclusion:

Transmission power vs. Bit rate

Binary Signaling Techniques

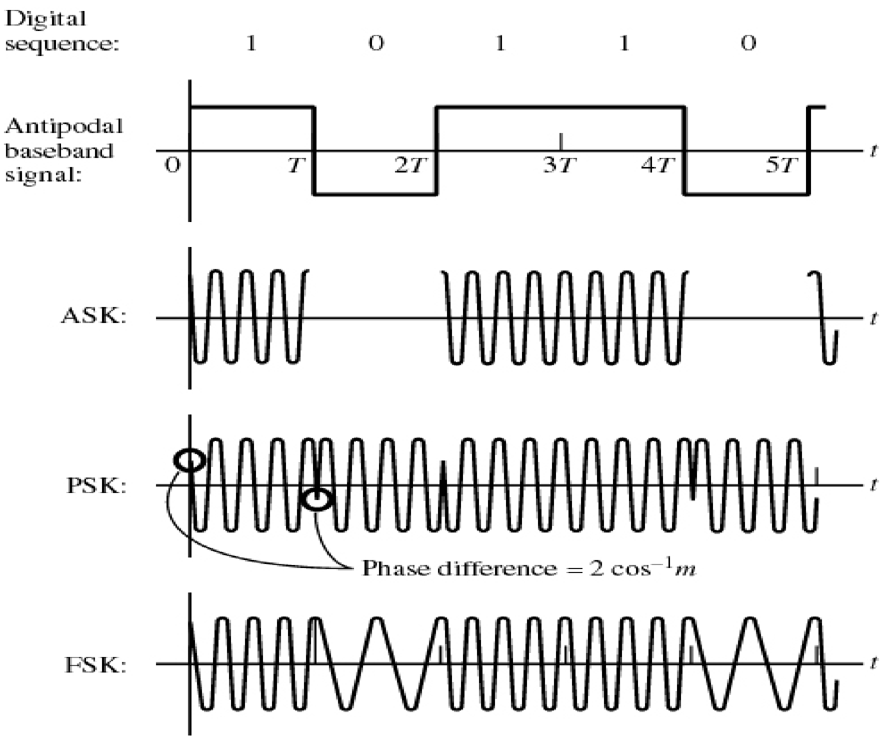


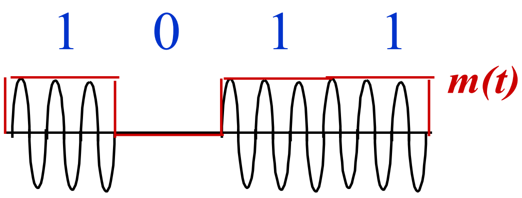
Figure 7-13
Waveforms for ASK, PSK, and FSK modulation.

Principles of Communications, 5/E by Rodger Ziemer and William Tranter

ASK, PSK, and FSK

- **Amplitude Shift Keying (ASK)**

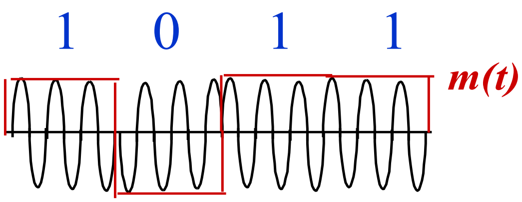
$$s(t) = m(t) A_c \cos(2\pi f_c t) = \begin{cases} A_c \cos(2\pi f_c t) & m(nT_b) = 1 \\ 0 & m(nT_b) = 0 \end{cases}$$



AM Modulation

- **Phase Shift Keying (PSK)**

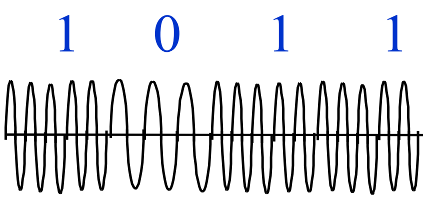
$$s(t) = A_c m(t) \cos(2\pi f_c t) = \begin{cases} A_c \cos(2\pi f_c t) & m(nT_b) = 1 \\ A_c \cos(2\pi f_c t + \pi) & m(nT_b) = -1 \end{cases}$$



PM Modulation

- **Frequency Shift Keying**

$$s(t) = \begin{cases} A_c \cos(2\pi f_1 t) & m(nT_b) = 1 \\ A_c \cos(2\pi f_2 t) & m(nT_b) = -1 \end{cases}$$



FM Modulation

Amplitude Shift Keying (ASK)

- 0→0
- 1→Acos(wct)
- What is the structure of the optimum receiver?

Receiver for binary signals in noise

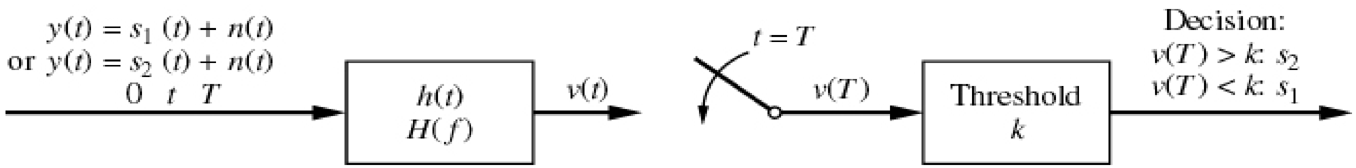


Figure 7-6 A possible receiver structure for detecting binary signals in white Gaussian noise.

Error Analysis

- 0→s1(t), 1→s2(t) in general.
- The received signal:

$$y(t) = s_1(t) + n(t), t_0 \le t \le t_0 + T$$
$$OR$$
$$y(t) = s_2(t) + n(t), t_0 \le t \le t_0 + T$$

- Noise is white and Gaussian.
- Find P_E
- In how many different ways can an error occur?