SINGLE PHASE INDUCTION MOTOR
UNIT- V

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Single Phase Induction Motor

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Lecture No. 2 & 3

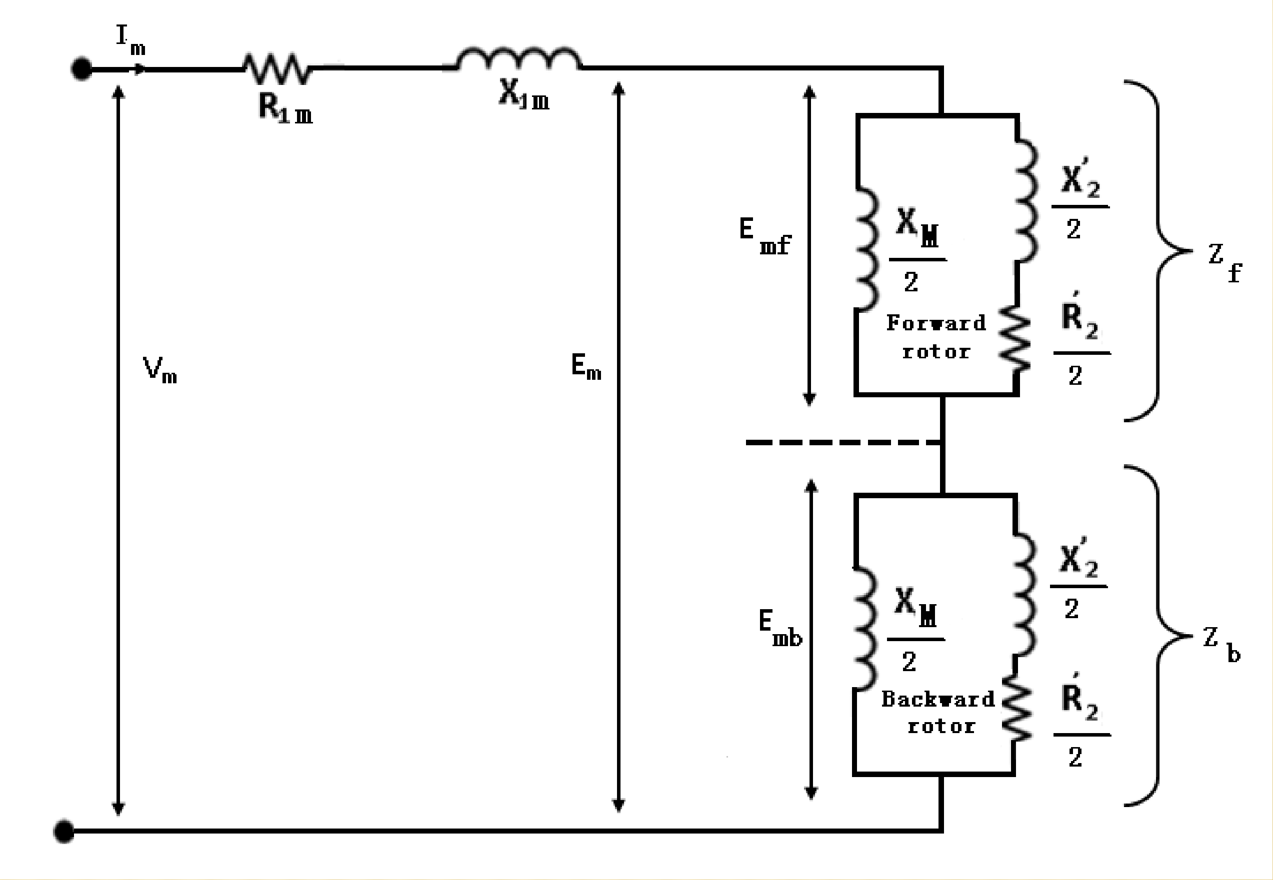
EQUIVALENT CIRCUIT

When the stator of single phase induction motor is connected to single – phase supply, the stator current produces a pulsating flux. According to the double – revolving field theory, the pulsating air – gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor.

Since the magnitude of each rotating flux is one – half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single – phase induction motor may be considered as consisting of two motors having a common stator winding and two imaginary rotors, which rotate in opposite directions.

CONT…

The equivalent circuit of single – phase induction motor at standstill is shown in the given fig.



CONT…

In this diagram, the portion of the equivalent circuit representing the effects of air gap flux is split into two portions. The first portion shows the effect of forward rotating flux, and the second portion shows the effect of backward rotating flux.

The forward flux induces a voltage Emf in the main stator winding. The backward rotating flux induces a voltage Emb in the main stator winding. The resultant induced voltage in the main stator winding is Em , where

Em = Emf + Emb

At standstill, Emf = Emb

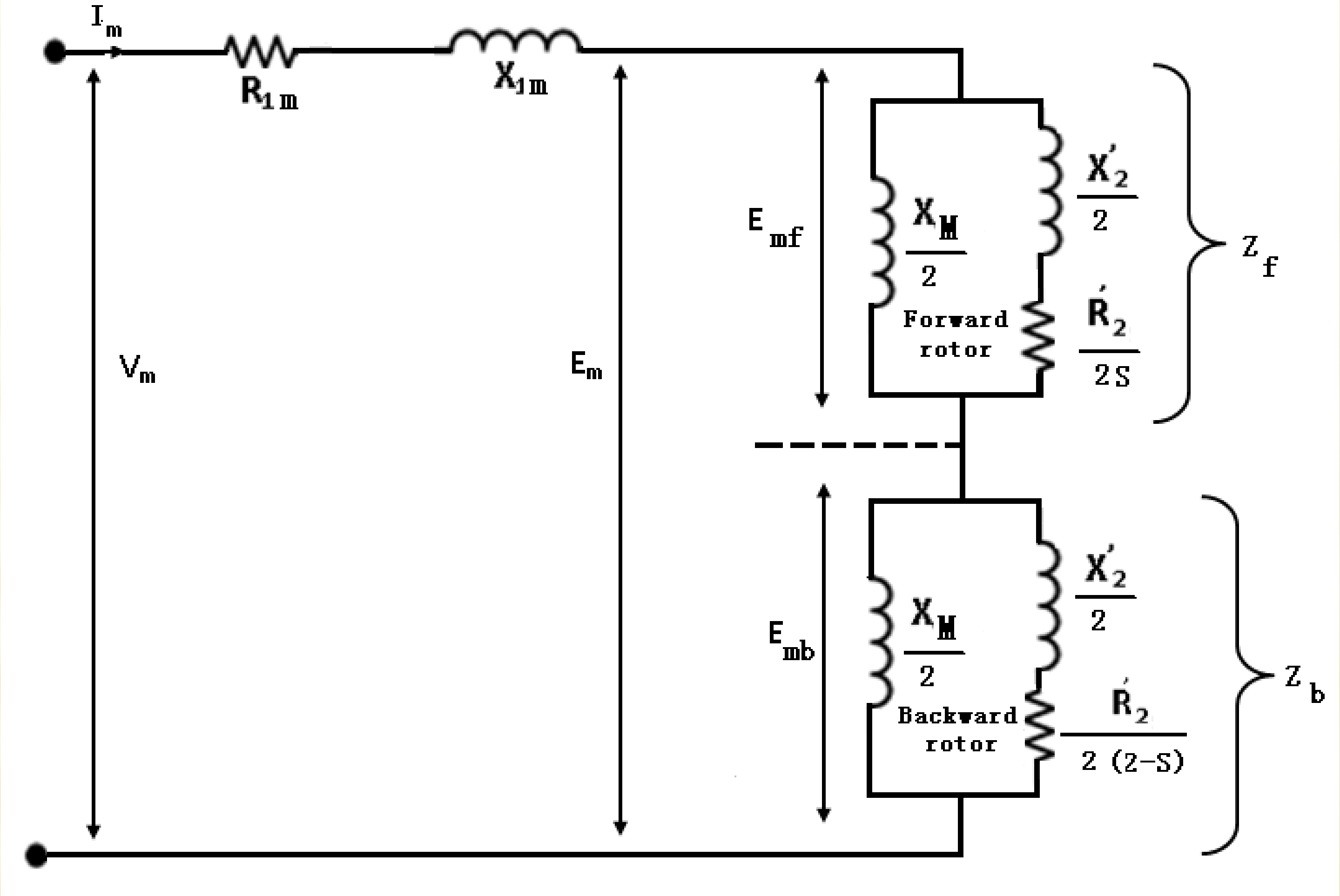
CONT…

Now suppose that the motor is started with the help of an auxiliary winding. The auxiliary winding is switched out after the motor gains it normal speed.

The effective rotor resistance of an induction motor depends on the slip of the rotor. The slip of the rotor with respect to the forward rotating flux is S. The slip the rotor with respect to the backward rotating flux is (2-S).

When the forward and backward slips are taken in account, the result is the equivalent circuit shown in fig. which represent the motor running on the main winding alone.

CONT…



Im
R1m
X1m
Vm
Em
Emf
Emb
XM/2
X'2/2
R'2/2S
Forward rotor
Backward rotor
R'2/2(2-S)
Zf
Zb

CONT…

The rotor impedance representing the effect of forward field referred to the stator winding m is given by an impedance

The rotor impedance representing the effect of forward field referred to the stator winding m is given by an impedance $(\frac{\acute{R}_2}{2S} + j\frac{\acute{X}_2}{2})$ in parallel with $j\frac{X_M}{2}$.

$$\therefore \qquad Z_f = R_f + jX_f = (\frac{\acute{R}_2}{2S} + j\frac{\acute{X}_2}{2}) \parallel (j\frac{X_M}{2})$$

$$Z_f = \frac{(\frac{\acute{R}_2}{2S} + j\frac{\acute{X}_2}{2})(j\frac{X_M}{2})}{\frac{\acute{R}_2}{2S} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

CONT…

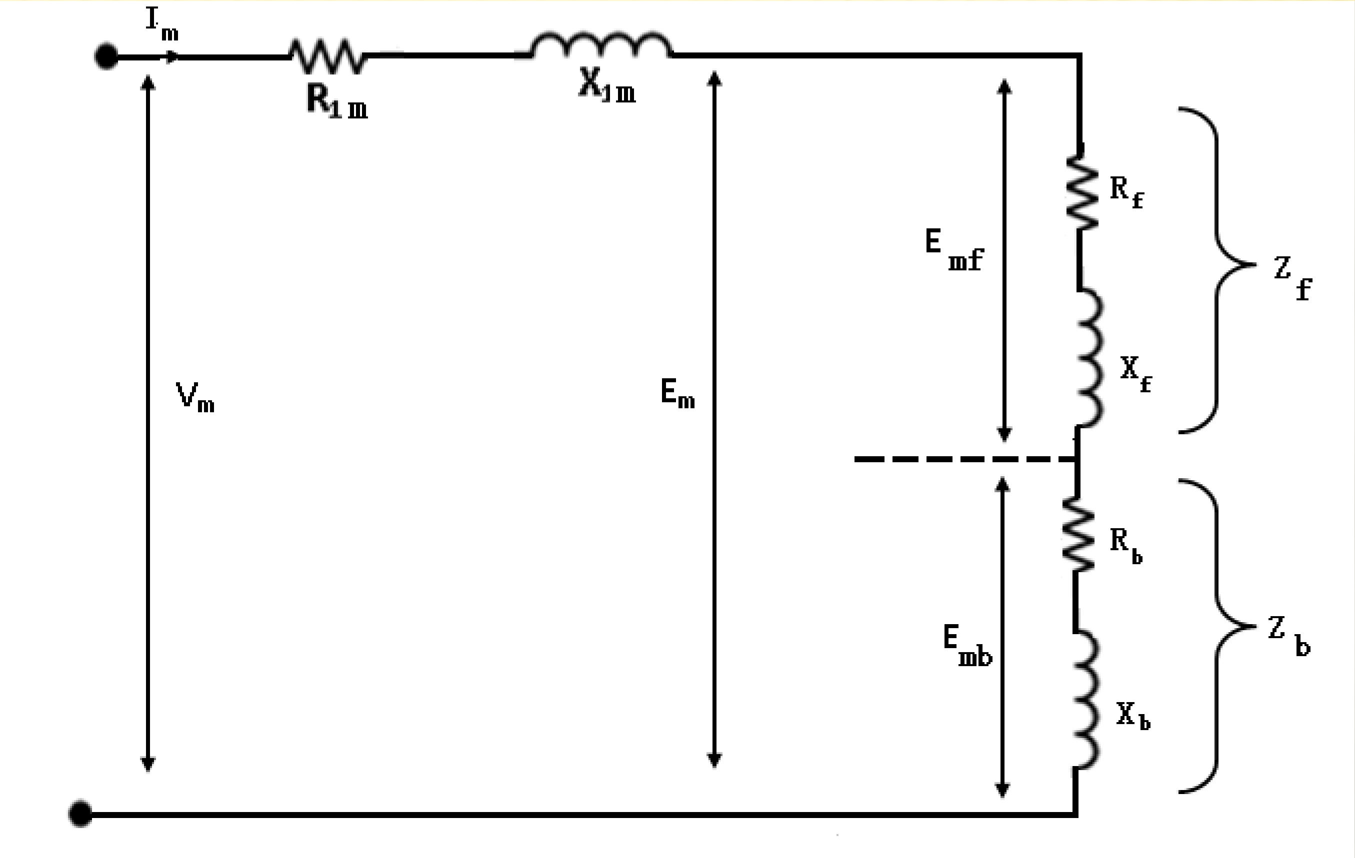
Similarly, the rotor impedance representing the effect of backward field referred to the stator winding m is given by an impedance $(\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2})$ in parallel with $j\frac{X_M}{2}$.

$$\therefore \qquad Z_b = R_b + jX_b = (\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2}) \parallel (j\frac{X_M}{2})$$

$$Z_b = \frac{(\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2})(j\frac{X_M}{2})}{\frac{\acute{R}_2}{2(2-S)} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

CONT…

The simplified equivalent circuit of single – phase induction motor with only main winding energized is shown in the given fig.



CONT…

The current in the stator winding is:

$$I_m = \frac{V_m}{Z_{1m} + Z_f + Z_b}$$

The torque of the backward field is in opposite direction to that of the forward field, and therefore the total air – gap power in a single phase induction motor is:

$$P_g = P_{gf} - P_{gb}$$

Where P_{gf} = air – gap power for forward field

$$P_{gf} = I_m^2 R_f$$

Where P_{gb} = air – gap power for backward field

$$P_{gb} = I_m^2 R_b$$

CONT…

$\therefore$

$$P_g = I_m^2 R_f - I_m^2 R_b = I_m^2 (R_f - R_b)$$

The torque produced by the forward field

$$T_f = \frac{1}{\omega_s} P_{gf} = \frac{P_{gf}}{2\pi n_s}$$

The torque produced by the backward field

$$T_b = \frac{1}{\omega_s} P_{gb} = \frac{P_{gb}}{2\pi n_s}$$

The resultant electromagnetic or induced torque T_{int} is the difference between the torque T_f and T_b :

$$T_{int} = T_f - T_b$$

CONT…

As in the case of the 3 - phase I.M., the induced torque is equal to the air gap power divided by synchronous angular velocity.

$$T_{int} = \frac{P_g}{\omega_s} = \frac{1}{\omega_s}(P_{gf} - P_{gb}) = \frac{I_m^2}{\omega_s}(R_f - R_b)$$

The total copper loss is the sum of rotor copper loss due to the forward field and the rotor copper loss due to the backward field.

$$P_{cr} = P_{crf} + P_{crb}$$

And rotor copper loss in a 3 – phase induction motor

$$P_{cr} = \text{slip} * \text{air gap power}$$

$$P_{cr} = sP_{gf} + (2 - s)P_{gb}$$

CONT…

The power converted from electrical to mechanical form in a single phase induction motor is given by

$$P_{mech} = P_{conv} = \omega T_{ind}$$

$$P_{mech} = (1-s)\omega_s T_{ind}$$

$$= (1-s)P_g = (1-s)(P_{gf} - P_{gb})$$

Or

$$P_{mech} = I_m^2\left(R_f - R_b\right)(1-s)$$

Shaft output power

$$P_{out} = P_{mech} - core\ loss - mechanical\ losses - stray\ losses$$

$$P_{out} = P_{mech} - P_{rot}$$

Where $P_{rot} = rotational\ losses$

CONT…

Example

A 230 V, 50 Hz, 4 – pole single phase induction motor has the following equivalent circuit impedances:

$$R_{1m} = 2.2\Omega, \qquad \acute{R}_2 = 4.5\Omega$$

$$X_{1m} = 3.1\Omega, \quad \acute{X}_2 = 2.6\Omega, \quad X_M = 80\Omega.$$

Friction, windage and core loss = 40 W

For a slip of 0.03pu, calculation (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency.

Solution. Form the given data

$$\frac{\grave{R}_2}{2s} = \frac{4.5}{2 \times 0.03} = 75\Omega$$

$$\frac{\grave{R}_2}{2(2-s)} = \frac{4.5}{2(2-0.03)} = 1.142\Omega$$

$$\frac{1}{2}\grave{X}_2 = \frac{1}{2} \times 2.6 = 1.3\Omega$$

$$\frac{1}{2}X_M = \frac{1}{2} \times 80 = 40\Omega$$

For the forward field circuit

$$Z_f = R_f + jX_f = \frac{\left(\frac{\acute{R}_2}{2s} + j\frac{\acute{X}_2}{2}\right)\left(j\frac{X_M}{2}\right)}{\frac{\acute{R}_2}{2s} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

$$= \frac{(75 + j1.3)(j40)}{75 + j1.3 + j40} = \frac{(75.011\angle 0.993°)(40\angle 90°)}{85.619\angle 28.84°}$$

$$= 35.04\angle 62.15°\Omega = 16.37 + j30.98\Omega$$

For the backward field

$$Z_b = R_b + jX_b = \frac{\left(\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2}\right)\left(j\frac{X_M}{2}\right)}{\frac{\acute{R}_2}{2(2-s)} + j\frac{\acute{X}_2}{2} + j\frac{X_M}{2}}$$

$$= \frac{(1.142 + j1.3)(j40)}{1.142 + j1.3 + j40} = \frac{(1.73\angle 48.7°)(40\angle 90°)}{41.316\angle 88.4°}$$

$$= 1.675\angle 50.3° = 1.07 + j1.29\Omega$$

$$Z_{1m} = R_{1m} + jX_{1m} = 2.2 + j3.1$$

The total series impedance

$$Z_e = Z_{1m} + Z_f + Z_b$$

$$= 2.2 + j3.1 + 16.37 + j30.98 + 1.07 + j1.29$$

CONT…

$$= 19.64 + j35.37 = 40.457\angle 60.96°\Omega$$

(a) Input current

$$I_m = \frac{V_m}{Z_e} = \frac{230\angle 0°}{40.457\angle 60.96°} = 5.685\angle -60.96° A.$$

(b) Power factor = $\cos(-60.95°) = 0.4856\ lagging.$

(c) Developed power

$$P_{conv} = P_d = I_m^2\left(R_f - R_b\right)(1 - s)$$

$$= (5.685)^2(16.37 - 1.07)(1 - 0.03) = 479.65\ W$$

(d) Output power = $\mathrm{P_d} - \mathrm{P_{rot}} = 479.65 - 40 = 439.65\ \mathrm{W}$

Input power = $VI_m \cos\emptyset = 230 \times 5.685 \times 0.4856 = 634.9\ W$

(e) Efficiency = $\frac{output}{input} = \frac{439.65}{634.9} = 0.692\ pu.$