Schrödinger's Equation

Class Objectives

- How do we get at the information in the wave function?
- Introduce Schrödinger's equation.
- Develop the time independent Schrödinger's Equation (TISE).

Schrödinger's Equation

- The fundamental problem in QM is: given the wave function at some instant t=0, find the wave function at a subsequent time.
- The wave function $\Psi(x,0)$ gives the initial information.
- $\Psi(x,t)$ is determined from the Schrödinger equation.

Schrödinger's Equation

- In developing his theory, Schrödinger adopted de Broglie's equations: $\lambda = h/p$, $v = E/h$
- As well he defined the total energy E as $E = p^2/2m + V$

Schrödinger’s Equation

- For a particle acted on by a force F, $\Psi(x,t)$ must be found from Schrödinger's wave equation.

Schrödinger’s Equation

- Schrödinger's wave equation for 1D is written as:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+U(x)\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$

- U(x) is the potential energy function for the force F. ie. $F=-dU/dx$

Schrödinger's Equation

- How do we obtain an equation for $\Psi(x,t)$?
- Schrödinger's equation is a partial differential equation for Ψ in terms of two variables. A standard technique is to look for solutions having separable form. Ie. $\Psi(x,t) = \psi(x)\phi(t)$
- Where $\psi(x)$ is a function of x only and $\phi(t)$ a function of t only.

Schrödinger's Equation

- Substituting into Schrödinger's equation we get:

$$-\frac{\hbar^2}{2m}\psi''(x)\phi(t)+U(x)\psi(x)\phi(t)=i\hbar\psi(x)\phi'(t)$$

- Dividing by $\psi(x)\phi(t)$ gives

$$-\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)}+U(x)=i\hbar\frac{\phi'(t)}{\phi(t)}$$

Schrödinger's Equation

- LHS is a function of x only.
- RHS is a function of t only.
- Since changing t cannot effect LHS (changing x does not affect RHS), the differential can be separated into two ODEs.

Schrödinger's Equation

- Both sides must equal to the same separation constant. So that,

$$i\hbar\frac{\phi'(t)}{\phi(t)} = i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C \qquad ....S1$$

$$-\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)} + U(x) = C \qquad ....S2$$

Schrödinger's Equation

- S1 is a 1st order ODE (Φ as a function of t). These have the solution $\phi(t) = e^{-iCt/\hbar}$
- NB: You should verify this!
- It is easy to show that C = E, the total energy.
- Thus $\phi(t) = e^{-iEt/\hbar}$
- And $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$

Schrödinger's Equation

- $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$ is the time independent Schrödinger equation.
- We can write the solution to Schrödinger equation as $\Psi(x,t)=\psi(x)e^{-iEt/h}$
- The expression gives a relationship between the time independent and dependent wave functions.
- The solutions for $\psi(x)$ are that of planes. ie $\psi(x)=e^{ikx}$

Schrödinger's Equation

- The functions of $\psi(x)$ are called *eigenfunctions*.
- Solutions of Schrödinger's equation are stationary states.
- This because they are time independent and the probability distributions are time independent.

$$\left|\Psi(x,t)\right| = \left|\psi(x)\right|^2$$

Schrödinger's Equation

- Because the probabilities are static they can be calculated from the time independent wave form.