Einstein Relations - A and B Coefficients

The distribution of atoms in the two energy levels will change by absorption or emission of radiation. Einstein introduced three empirical coefficients to quantify the change of population of the two levels.

- **Absorption -** If B_{12} is the probability (per unit time) of absorption of radiation, the population of the upper level increases. The rate is clearly proportional to the population of atoms in the lower level and to the energy density $u(\nu)$ of radiation in the system. Thus the rate of increase of population of the excited state is given by

  $$\frac{\partial N_2}{\partial t} = B_{12} u(\nu) N_1$$

  where B_{12} is a constant of proportionality with dimensions m $_3$/s $_2$-J.

- **Spontaneous Emission -** The population of the upper level will decrease due to spontaneous transition to the lower level with emission of radiation. The rate of emission will depend on the population of the upper level. If A_{12} is the probability that an atom in the excited state will spontaneously decay to the ground state,

  $$\frac{\partial N_2}{\partial t} = -A_{12} N_2$$

The equation above has the solution

$$N_2(t) = N_2(0) e^{-t/\tau}$$

where $\tau = 1/A_{12}$ gives the average *lifetime* of an atom in the excited level before the

atom returns to the ground state. Thus the spontaneous emission depends on the lefetime of the atom in the excited state. The process is statistical and the emitted quanta bear no phase relationship with one another, i.e. the emission is **incoherent** .

- **Stimulated Emission -** Stimulated or induced emission depends on the number of atoms in the excited level as well as on the energy density of the incident radiation. If

  be the transition be the transition probability per unit time per unit energy density

B_{21}

of radiation, the rate of decrease of the population of the excited state is $B_{21}u(\nu)N_2$.

The rate equation for the population of the upper level is

$$\frac{dN_2}{dt} = B_{12}u(\nu)N_1 - [A_{21} + B_{21}u(\nu)]N_2$$

Since $N_1 + N_2 =$constant,

$$\frac{\partial N_2}{\partial t} = -\frac{\partial N_1}{\partial t}$$

The emitted quanta under stimulated emission are coherent with the impressed field. The spontaneous emission, being incoherent, is a source of noise in lasers. When equilibrium is reached, the population of the levels remains constant, so that $dN_2/dt = 0$and the rate of emission equals rate of absorption, so that

$$B_{12}u(\nu)N_1 = [A_{21} + B_{21}u(\nu)]N_2$$

Using the Boltzmann facor $N_2/N_1 = (g_2/g_1)\exp(-h\nu/kT)$, and simplifying, we get

$$u(\nu) = \frac{A_{21}\frac{g_2}{g_1}}{B_{12}e^{h\nu/kT} - B_{21}\frac{g_2}{g_1}} = \frac{A_{21}/B_{21}}{\frac{g_1}{g_2}e^{h\nu/kT}\frac{B_{12}}{B_{21}} - 1}$$

If we regard the matter to be a blackbody and compare the above expression for the energy density with the corresponding energy density expression derived for the blackbody radiation, viz.,

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

and

$$\frac{B_{21}}{B_{12}} = \frac{g_1}{g_2}$$

The last equation shows that n the absence of degeneracy, the probability of stimulated emission is equal to that of absorption. In view of this we replace the two coefficients by a single coefficient B and term them as B- coefficient. The spontaneous emission coefficient will be called the A- coefficient.

The ratio of spontaneous emission probability to the stimulated emission probability is

$$\frac{A}{Bu(\nu)} = e^{h\nu/kT} - 1$$

so that for low temperatures, when $h\nu/kT \gg 1$, spontaneous emission is much more

probable than induced emission and the latter may be neglected. For high enough temperatures, stimulated emission probability can be significant though for optical frequencies, this requires very high temperature. For microwave frequencies the stimulated emission processes may be significant even at room temperatures.