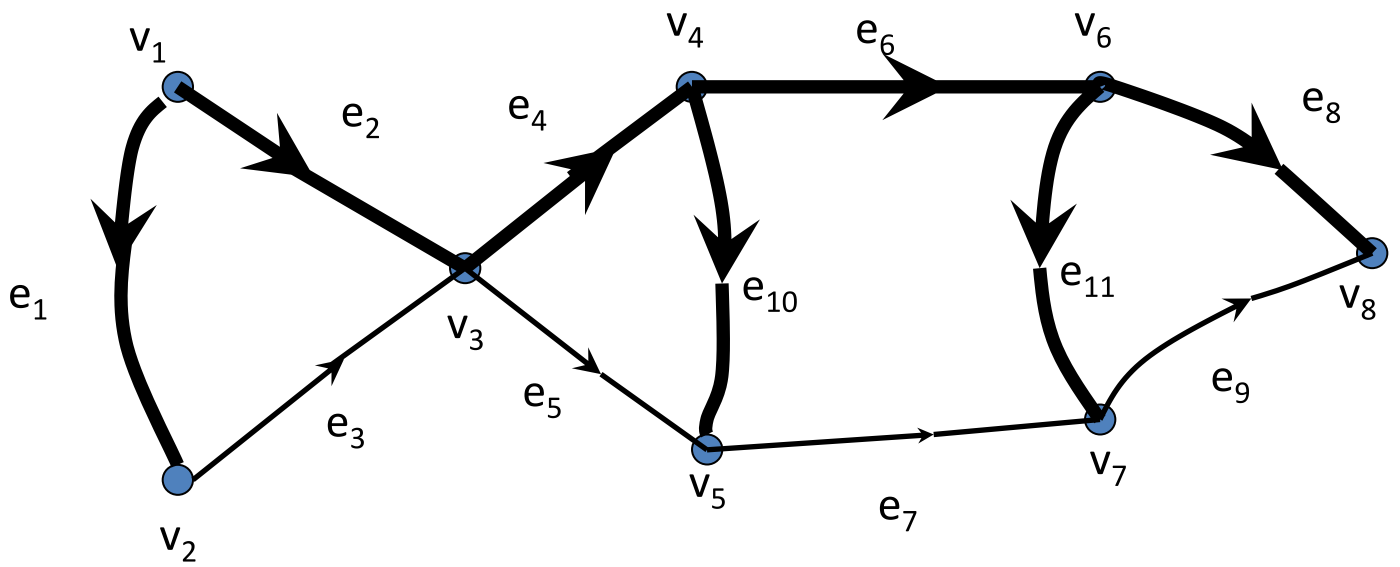
NETWORK ANALYSIS AND SYNTHESIS

Unit 1

Graph Theory

Matrices of Oriented Graphs



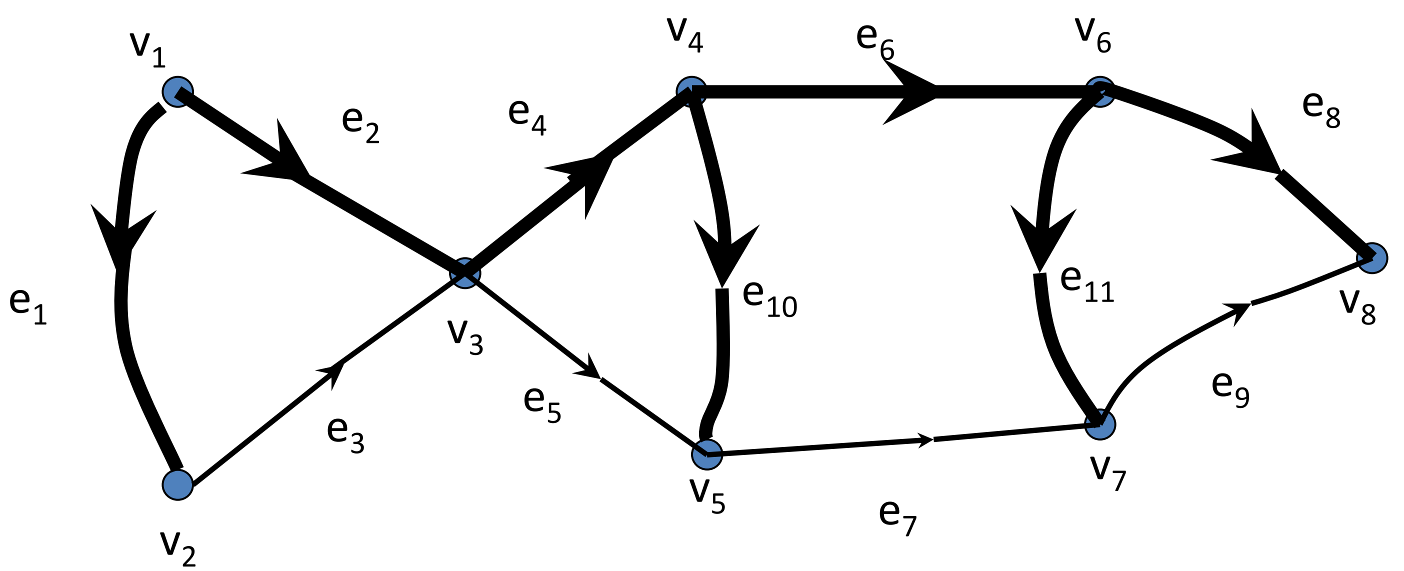
- The edge e_1 which has an orientation from vertex v_1 to vertex v_2 simply indicates that any transmission from v_1 to v_2 along e_1 is assumed to be positive.
- Any transmission from v_2 to v_1 along e_1 is assumed to be negative.

Matrices of Oriented Graphs

- DEFINITION: Let e and v represent respectively the number of edges and vertices of a graph G. The incident matrix $\Pi = \left[\pi_{ij} \right]_{v,e}$ having v rows and e columns is defined as

$$\pi_{ij} = \begin{cases} 1 & \text{if edge } e_j \text{ is incident at vertex } v_i \text{ and is oriented away from } v_i \\ -1 & \text{if edge } e_j \text{ is incident at vertex } v_i \text{ and is oriented toward } v_i \\ 0 & \text{if edge } e_j \text{ is not incident on vertex } v_i \end{cases}$$

Matrices of Oriented Graphs



Incident Matrix:

$$\Pi = \left[\begin{array}{ccccccccccc|c}
(e_1) & (e_2) & (e_3) & (e_4) & (e_5) & (e_6) & (e_7) & (e_8) & (e_9) & (e_{10}) & (e_{11}) & \\
\hline
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (v_1) \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (v_2) \\
0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & (v_3) \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & (v_4) \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 0 & (v_5) \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & (v_6) \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & (v_7) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & (v_8)
\end{array}\right]$$

Property:

Any column of Π contains exactly two nonzero entries of opposite sign.

Matrices of Oriented Graphs

- Property: The determinant of any square submatrix of order q ($1 \leq q \leq v$) of Π is either one of the following values: 1, -1, 0.
- Now, consider a graph G of p connected parts:

$$\Pi = \left[\begin{array}{ccccc|c} E_1 & E_2 & \cdots & E_p & \\ \hline \Pi_1 & 0 & \cdots & 0 & V_1 \\ 0 & \Pi_2 & \cdots & 0 & V_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \Pi_p & V_p \end{array}\right]$$

THANKS….

Queries Please…