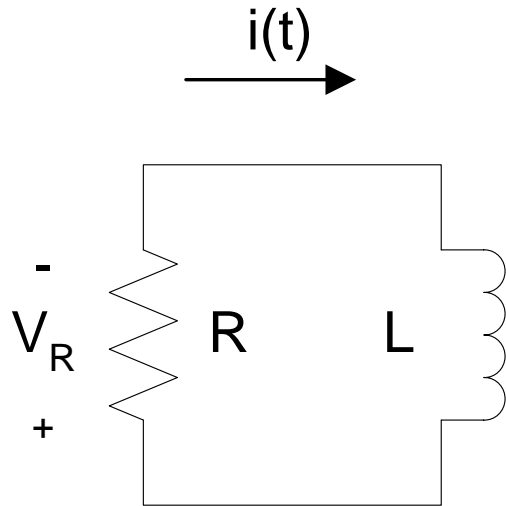


# **NETWORK ANALYSIS AND SYNTHESIS**

# RL CIRCUITS

Initial condition

$$i(t = 0) = I_0$$

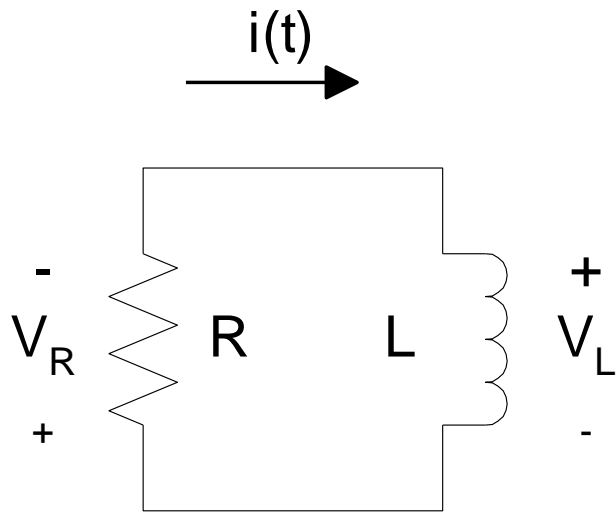


$$v_R + v_L = 0 = Ri + L \frac{di}{dt}$$

$$\frac{L}{R} \frac{di}{dt} + i = 0$$

*Solving the differential equation*

# RL CIRCUITS



Initial condition  
 $i(t = 0) = I_o$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\frac{di}{i} = -\frac{R}{L}dt, \quad \int_{I_o}^{i(t)} \frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\ln i \Big|_{I_o}^i = -\frac{R}{L}t \Big|_0^t$$

$$\ln i - \ln I_o = -\frac{R}{L}t$$

$$i(t) = I_o e^{-Rt/L}$$

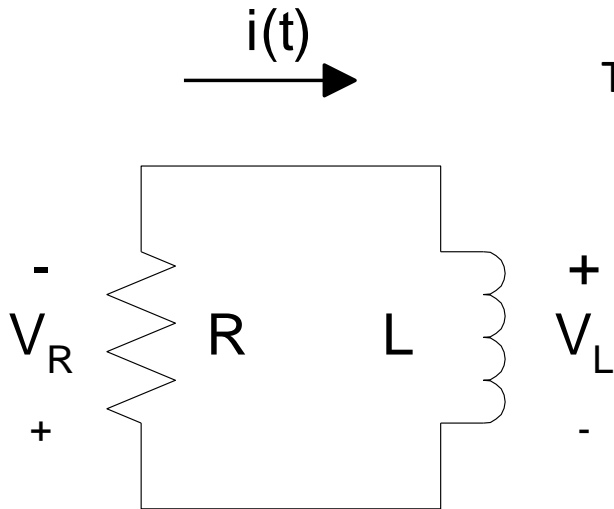
# RL CIRCUIT

Power dissipation in the resistor is:

$$p_R = i^2 R = I_o^2 e^{-2Rt/L} R$$

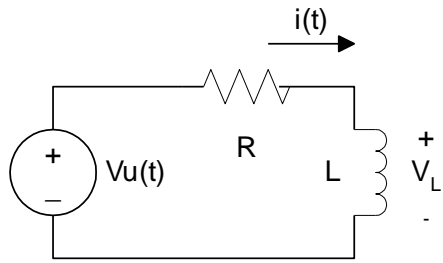
Total energy turned into heat in the resistor

$$\begin{aligned} W_R &= \int_0^{\infty} p_R dt = I_o^2 R \int_0^{\infty} e^{-2Rt/L} dt \\ &= I_o^2 R \left( -\frac{L}{2R} \right) e^{-2Rt/L} \Big|_0^{\infty} \\ &= \frac{1}{2} L I_o^2 \end{aligned}$$

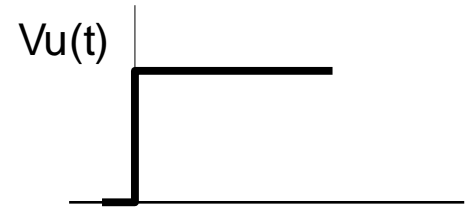


It is expected as the energy stored in the inductor is

$$\frac{1}{2} L I_o^2$$



# RL CIRCUIT



$$Ri + L \frac{di}{dt} = V$$

$$\frac{Ldi}{V - Ri} = dt$$

*Integrating both sides,*

$$-\frac{L}{R} \ln(V - Ri) = t + k$$

$$i(0^+) = 0, \text{ thus } k = -\frac{L}{R} \ln V$$

$$-\frac{L}{R} [\ln(V - Ri) - \ln V] = t$$

$$\frac{V - Ri}{V} = e^{-Rt/L} \quad \text{or}$$

$$i = \frac{V}{R} - \frac{V}{R} e^{-Rt/L}, \text{ for } t > 0$$

where  $L/R$  is the time constant

# DC STEADY STATE

The steps in determining the forced response for *RL* or *RC* circuits with dc sources are:

1. Replace capacitances with open circuits.
2. Replace inductances with short circuits.
3. Solve the remaining circuit.

**THANKS....**

Queries Please...