Performance Parameters
Of
Phase Controlled Rectifiers

Output dc power (avg. or dc o/p power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \;;\; i.e.,\; P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ avg./ dc value of o/p voltage.

$I_{O(dc)} = I_{dc} =$ avg./dc value of o/p current

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}}$; % Efficiency $\eta = \dfrac{P_{O(dc)}}{P_{O(ac)}} \times 100$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

$$\therefore \qquad V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC load output (load) voltage}}$$

The Ripple Factor (RF) w.r.t. o/p voltage w/f

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

$$\therefore \quad r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor $r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

$V_{r(pp)} =$ peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(\max)} - V_{O(\min)}$$

$I_{r(pp)} =$ peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

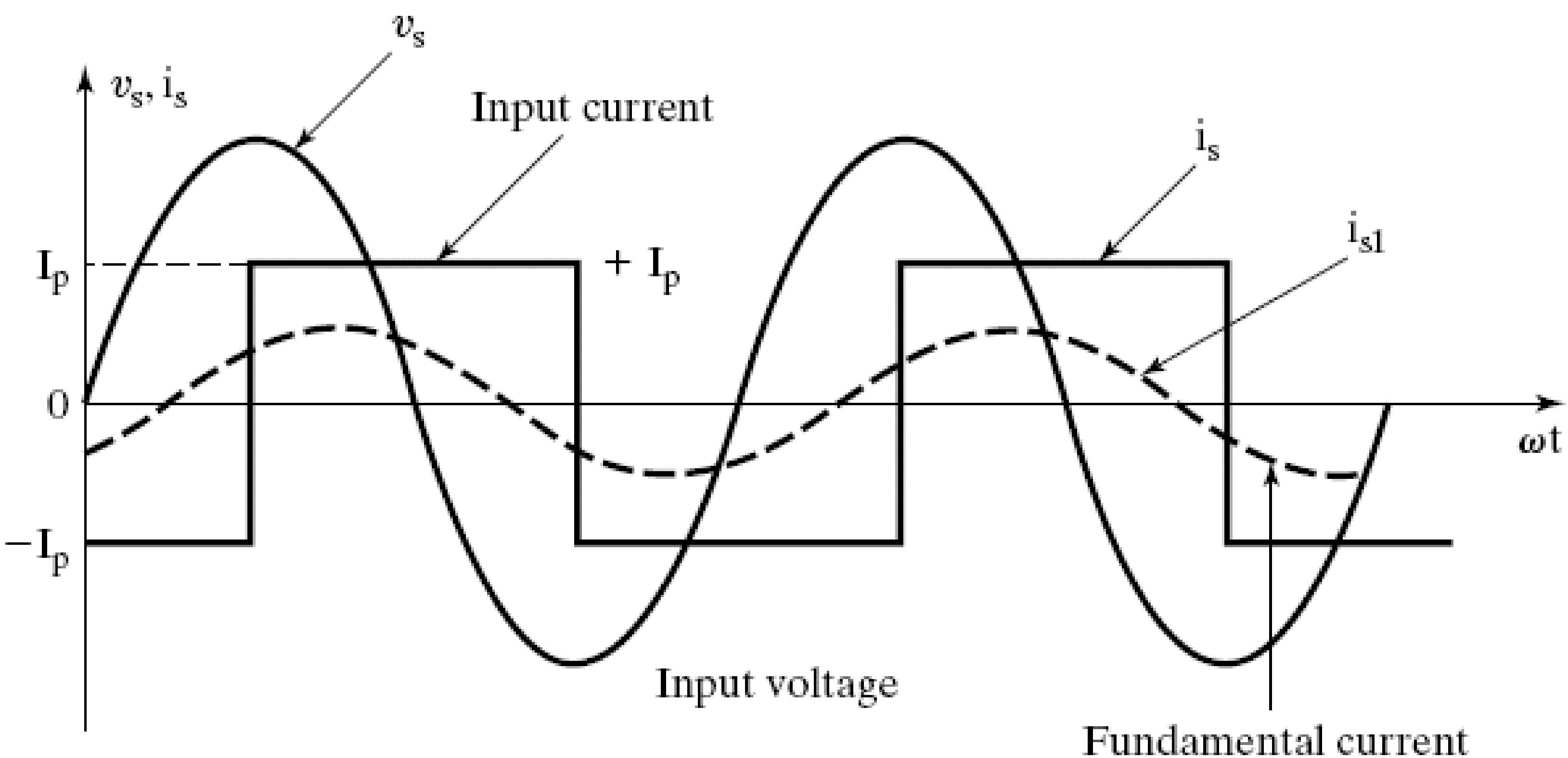
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

V_S = RMS supply (secondary) voltage

I_S = RMS supply (secondary) current


v_s
v_s, i_s
Input current
i_s
i_{s1}
I_p
$+ I_p$
0
ωt
$-I_p$
Input voltage
Fundamental current

Where

$v_S =$ Supply voltage at the transformer secondary side

$i_S =$ i/p supply current

(transformer secondary winding current)

$i_{S1} =$ Fundamental component of the i/p supply current

$I_P =$ Peak value of the input supply current

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of i/p supply current & the input supply voltage.

$\phi =$ Displacement angle (phase angle)

For an RL load

$\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or

Fundamental Power Factor

$$DF = Cos\phi$$

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2} \right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}} \right)^2 - 1 \right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the i/p supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

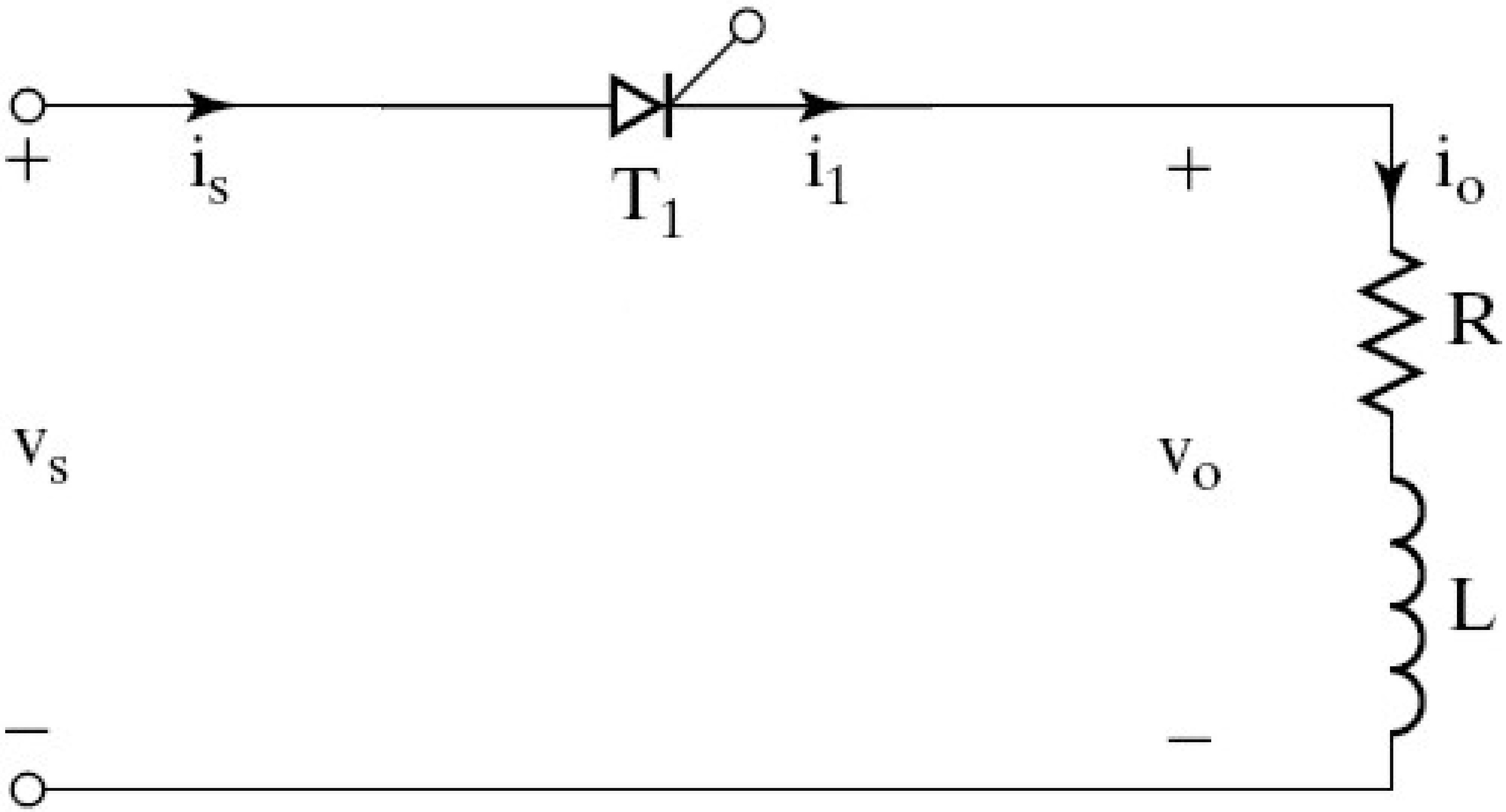
$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

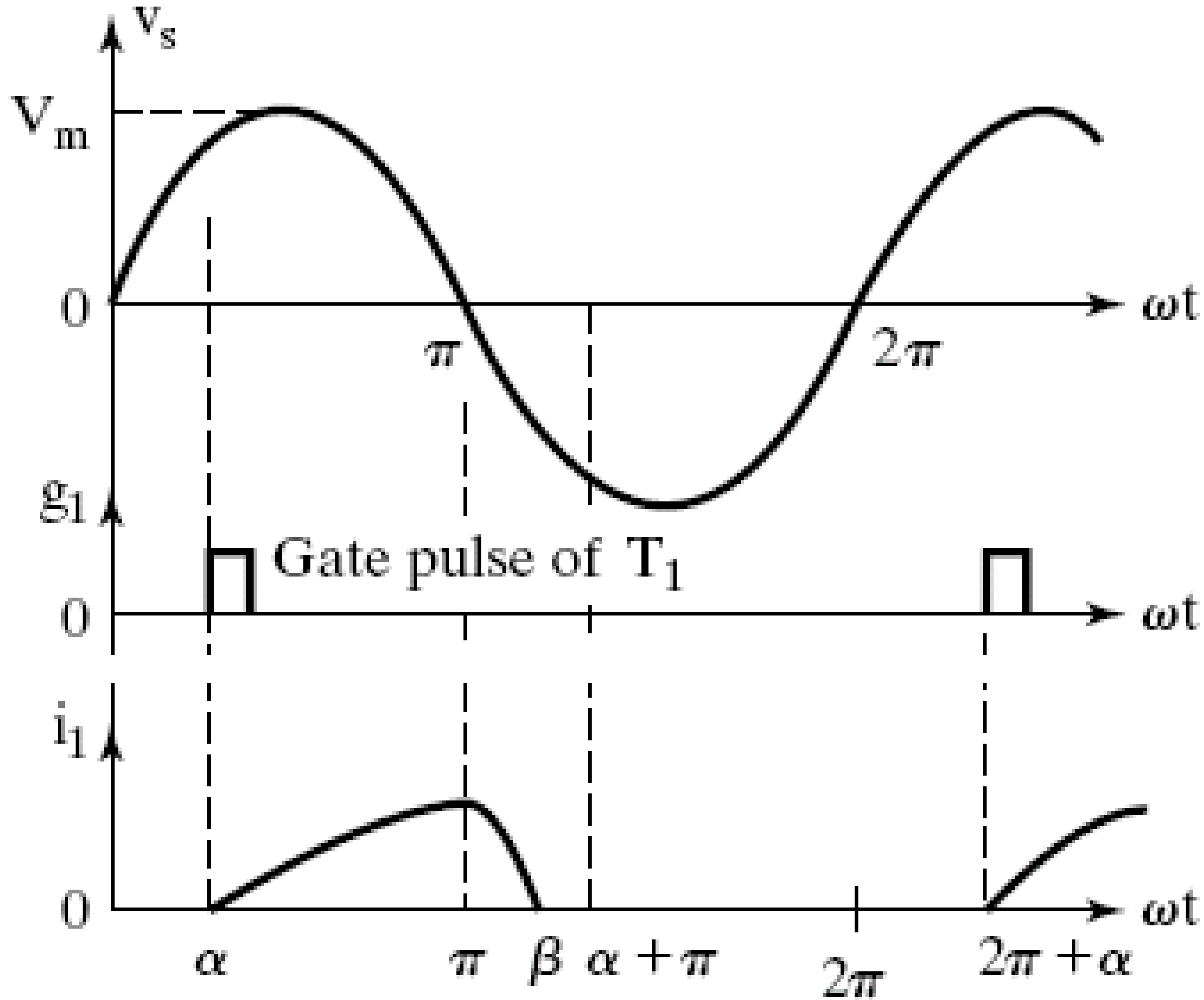
$$FF = 1;\ \ \eta = 100\%\ ;\ \ V_{ac} = V_{r(rms)} = 0\ ;\ \ TUF = 1;$$

$$RF = r_v = 0\ ;\ \ HF = THD = 0;\ \ \ PF = DPF = 1$$

Single Phase Half Wave Controlled Rectifier With An RL Load

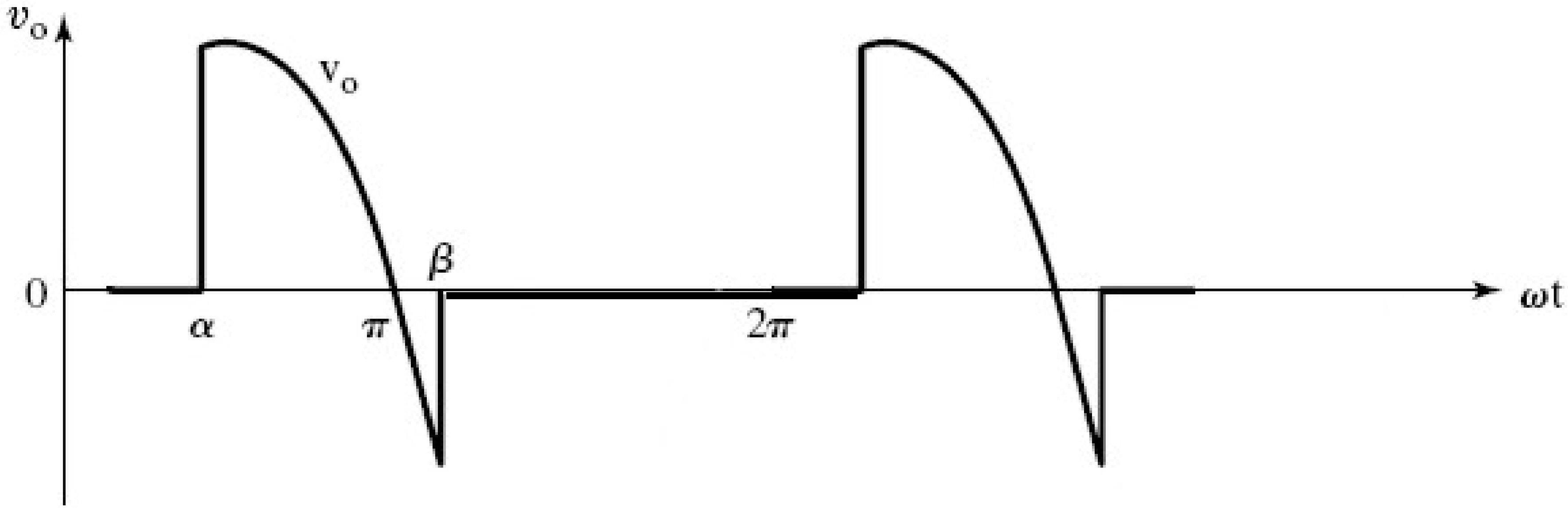

+
i_s
T_1
i_1
+
i_o
R
v_s
v_o
L
−
−

Input Supply Voltage (V_s)
&
Thyristor (Output) Current Waveforms



v_s
V_m
0
ωt
π
2π
g_1
Gate pulse of T_1
0
ωt
i_1
0
ωt
α
π
β
$\alpha + \pi$
2π
$2\pi + \alpha$

Output (Load) Voltage Waveform



To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin\omega t \;;\; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$V_m = \sqrt{2}V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \ at \ \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin\left(\beta - \phi\right) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin\left(\alpha - \phi\right)$$

β can be calculated by solving the above eqn.

To Derive An Expression For Average (DC) Load Voltage of a Single Half Wave Controlled Rectifier with RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d\left(\omega t\right)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d\left(\omega t\right) + \int_{\alpha}^{\beta} v_O.d\left(\omega t\right) + \int_{\beta}^{2\pi} v_O.d\left(\omega t\right)\right]$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d\left(\omega t\right)\right];$$

$$v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right)$$

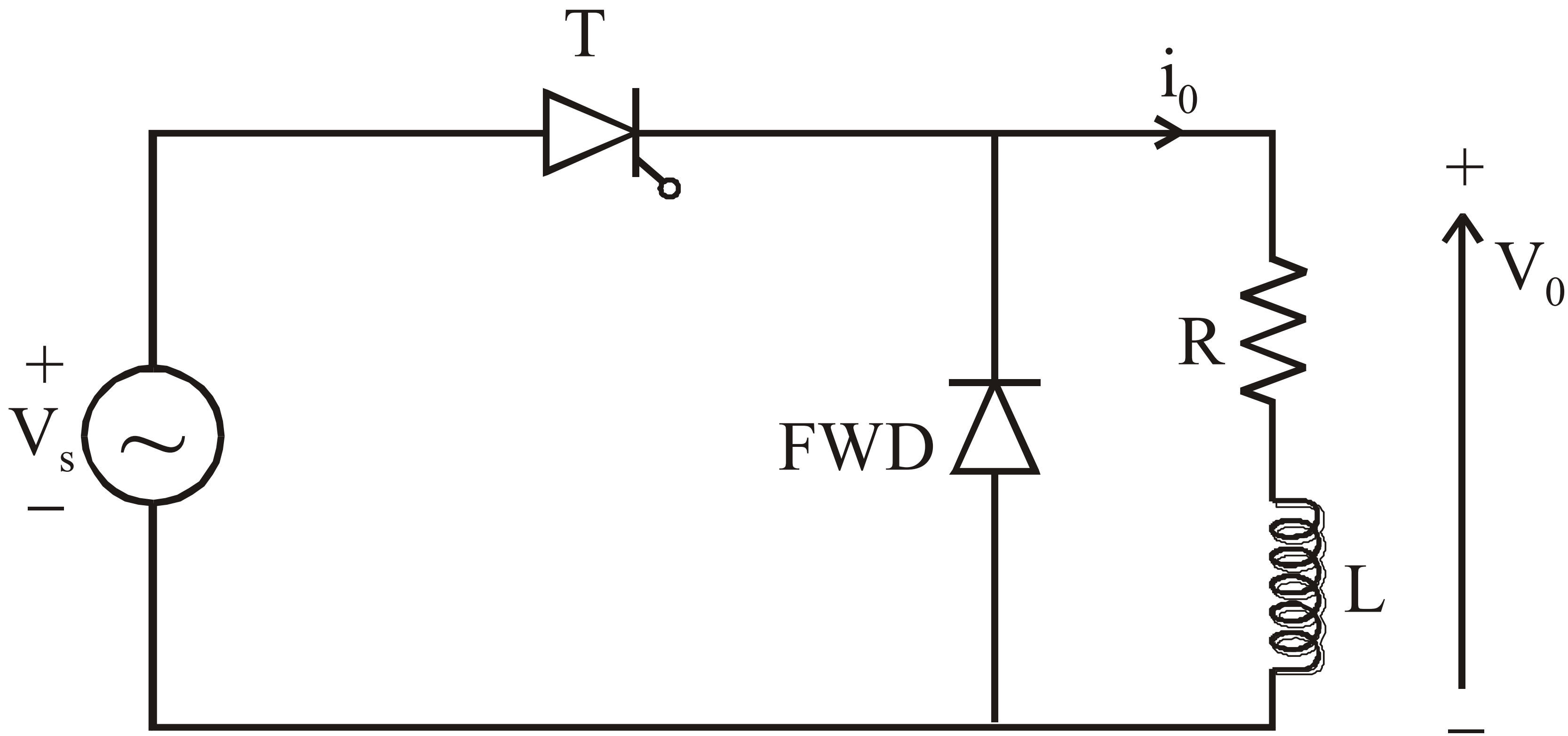
Effect of Load Inductance on the Output

During the period $\omega t = \pi$ to β the instantaneous o/p voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

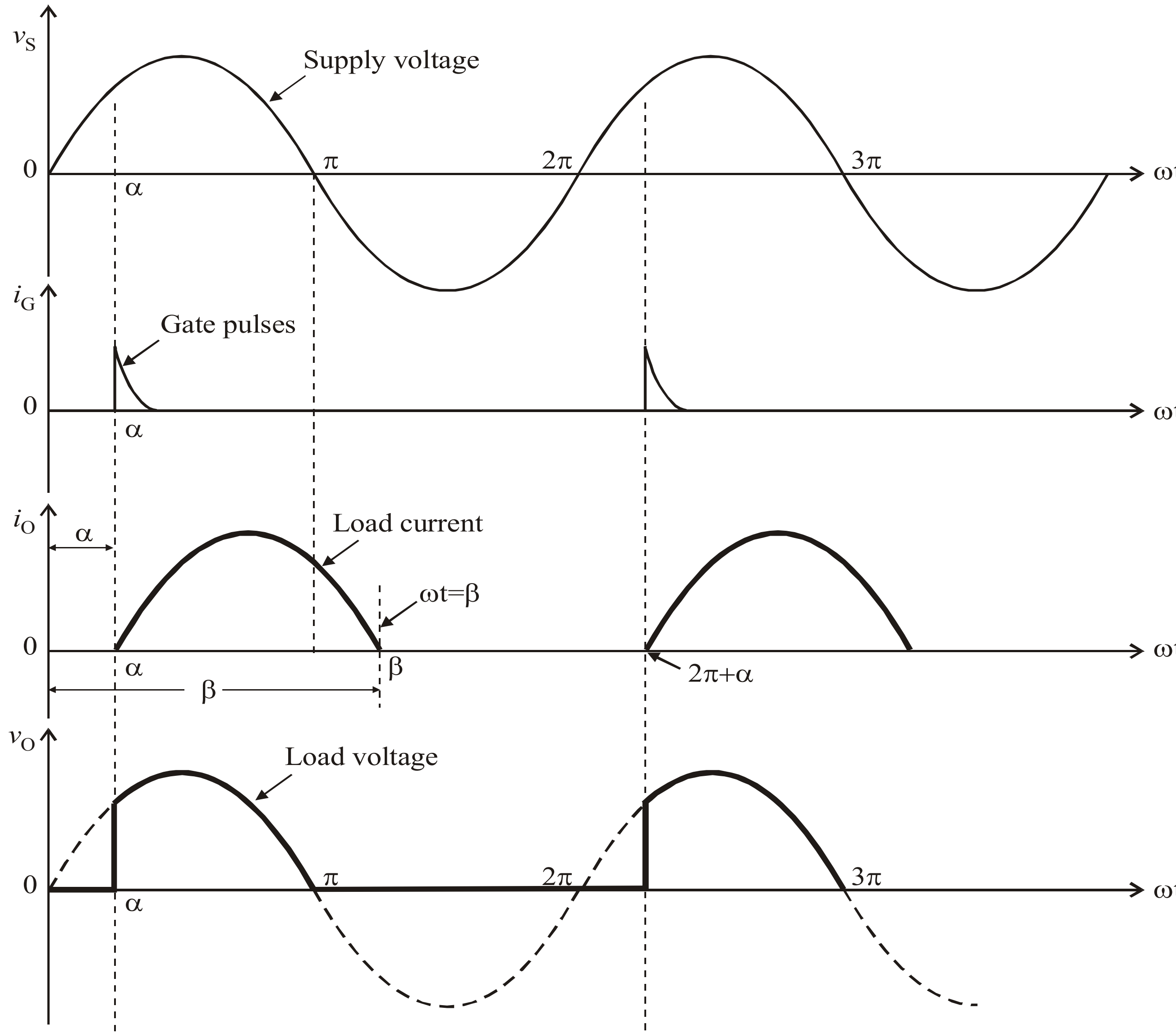
Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

Single Phase Half Wave Controlled Rectifier With RL Load & Free Wheeling Diode



T
i0
+
V0
R
+
Vs
FWD
L



v_S
Supply voltage
0
α
π
2π
3π
ωt
i_G
Gate pulses
0
α
ωt
i_O
α
Load current
ωt=β
0
α
β
β
2π+α
ωt
v_O
Load voltage
0
α
π
2π
3π
ωt

The average output voltage

$V_{dc} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$ which is the same as that of a purely resistive load.

The following points are to be noted

For low value of inductance, the load current tends to become discontinuous.

During the period α to π
the load current is carried by the SCR.
During the period π to β load current is carried by the free wheeling diode.
The value of β depends on the value of R and L and the forward resistance of the FWD.

For Large Load Inductance
the load current does not reach zero, & we obtain continuous load current