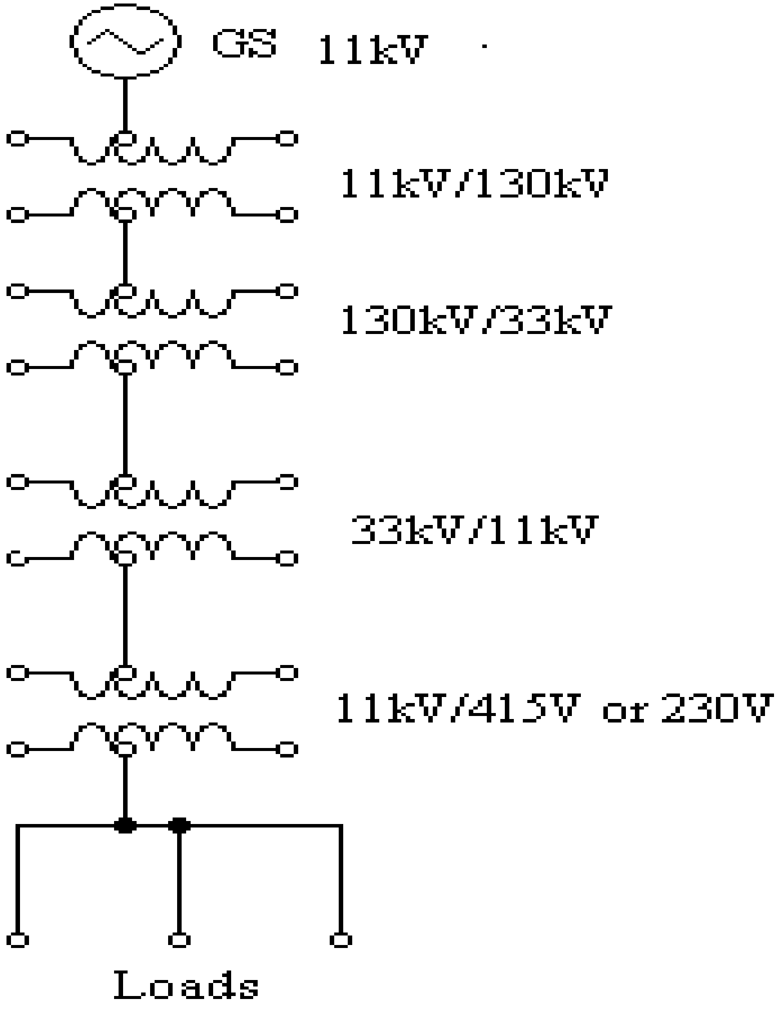
EEE- 601
POWER SYSTEM ANALYSIS

UNIT I

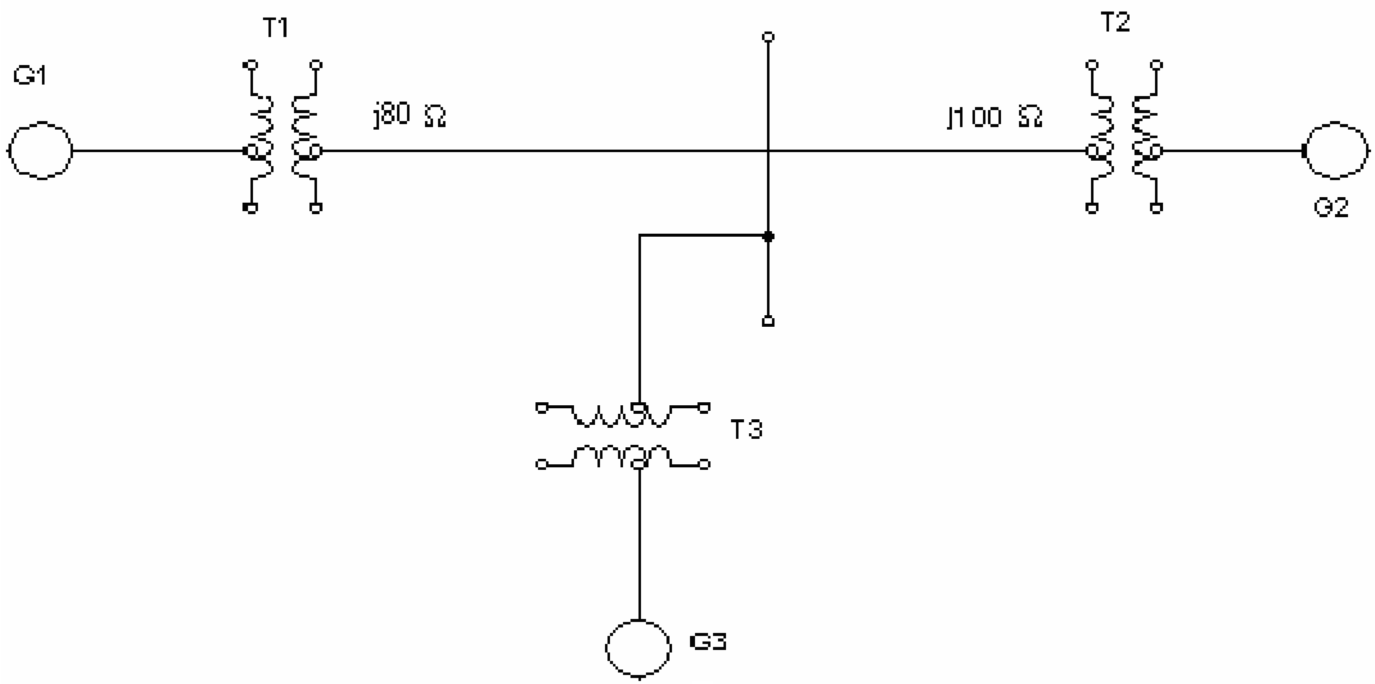
INTRODUCTION

Power system network



SINGLE LINE DIAGRAM

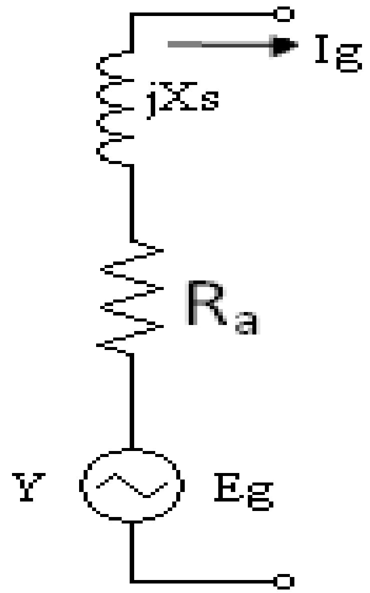
It is a diagrammatic representation of a power system in which the components are represented by their symbols.



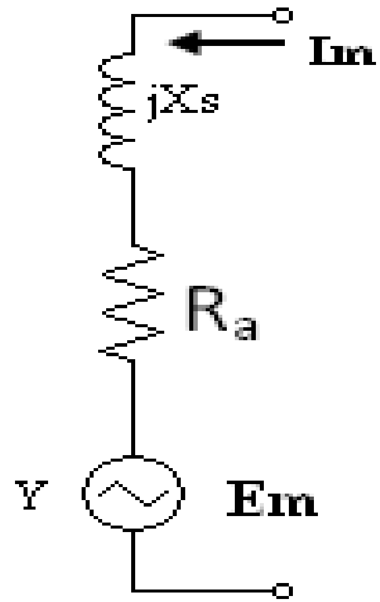
COMPONENTS OF A POWER SYSTEM

1. Alternator
2. Power transformer
3. Transmission lines
4. Substation transformer
5. Distribution transformer
6. Loads

MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

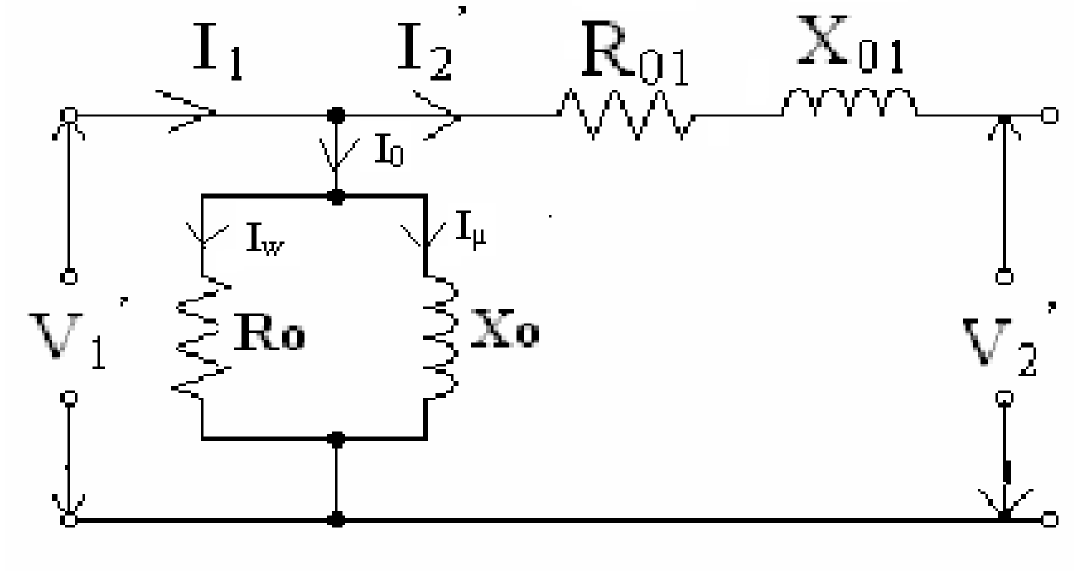


1Φ equivalent circuit of generator



1Φ equivalent circuit of synchronous motor

MODELLING OF TRANSFORMER

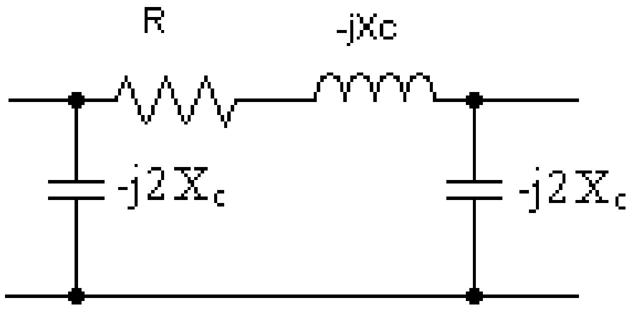


$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

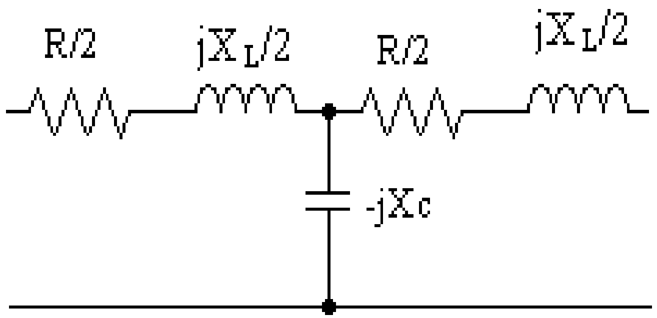
$$R_{01} = R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1º

$$X_{01} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1º

MODELLING OF TRANSMISSION LINE

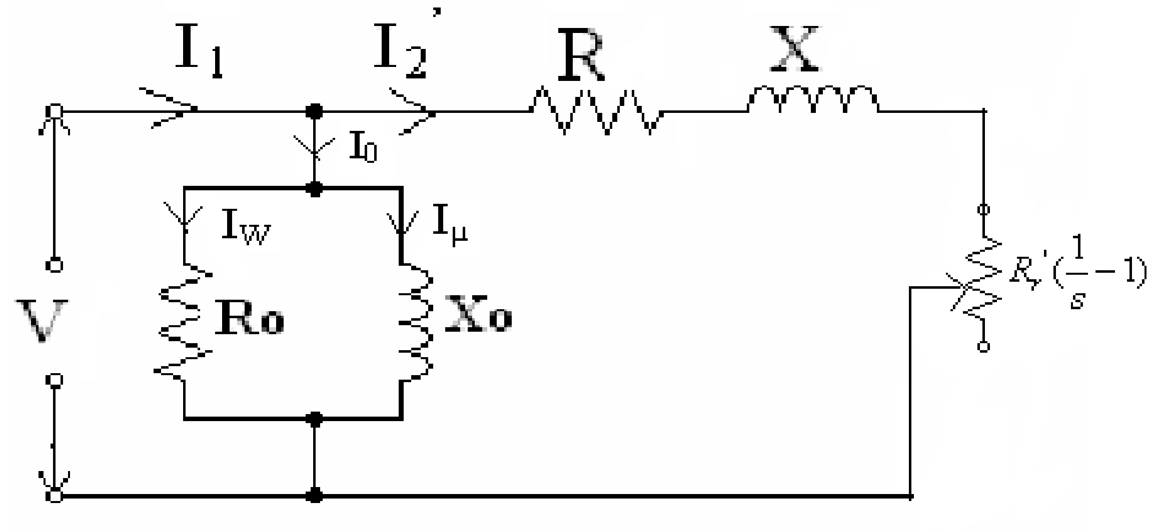


Π type



T type

MODELLING OF INDUCTION MOTOR



$$R_r'(\frac{1}{s} - 1)$$ =Resistance representing load

$$R = R_S + R_r'$$ =Equivalent resistance referred to stator

$$X = X_S + X_r'$$ =Equivalent reactance referred to stator

per unit=actual value/base value

Let KVA_b=Base KVA

kV_b=Base voltage

Z_b=Base impedance in Ω

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(kV_b)^2}{\frac{KVA_b}{1000}}$$

Changing the base of per unit quantities

Let z = actual impedance(Ω)

Z_b = base impedance (Ω)

$$Z_{p.u} = \frac{Z}{Z_b} = \frac{Z}{\frac{(kV_b)^2}{MVA_b}} = \frac{Z * MVA_b}{(kV_b)^2}$$

Let $kV_{b,old}$ & $MVB_{b,old}$ represent old base values

$kV_{b,new}$ & $MVB_{b,new}$ represent new base values

$$Z_{p.u,old} = \frac{Z * MVA_{b,old}}{\left(kV_{b,old}\right)^2} \rightarrow (1)$$

$$Z = \frac{Z_{p.u,old} * MVA_{b,old}}{\left(kV_{b,old}\right)^2} \rightarrow (2)$$

$$Z_{p.u,new} = \frac{Z * MVA_{b,new}}{\left(kV_{b,new}\right)^2} \rightarrow (3)$$

$$Z_{p.u,new} = Z_{p.u,old} * \frac{\left(kV_{b,old}\right)^2}{\left(kV_{b,new}\right)^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

ADVANTAGES OF PER UNIT CALCULATIONS

- The p.u impedance referred to either side of a 1Φ transformer is same
- The manufacturers provide the impedance value in p.u
- The p.u impedance referred to either side of a 3Φ transformer is same regardless of the 3Φ connections Y-Y,Δ-Y
- p.u value always less than unity.

IMPEDANCE DIAGRAM

- This diagram obtained by replacing each component by their 1Φ equivalent circuit.

Following approximations are made to draw impedance diagram

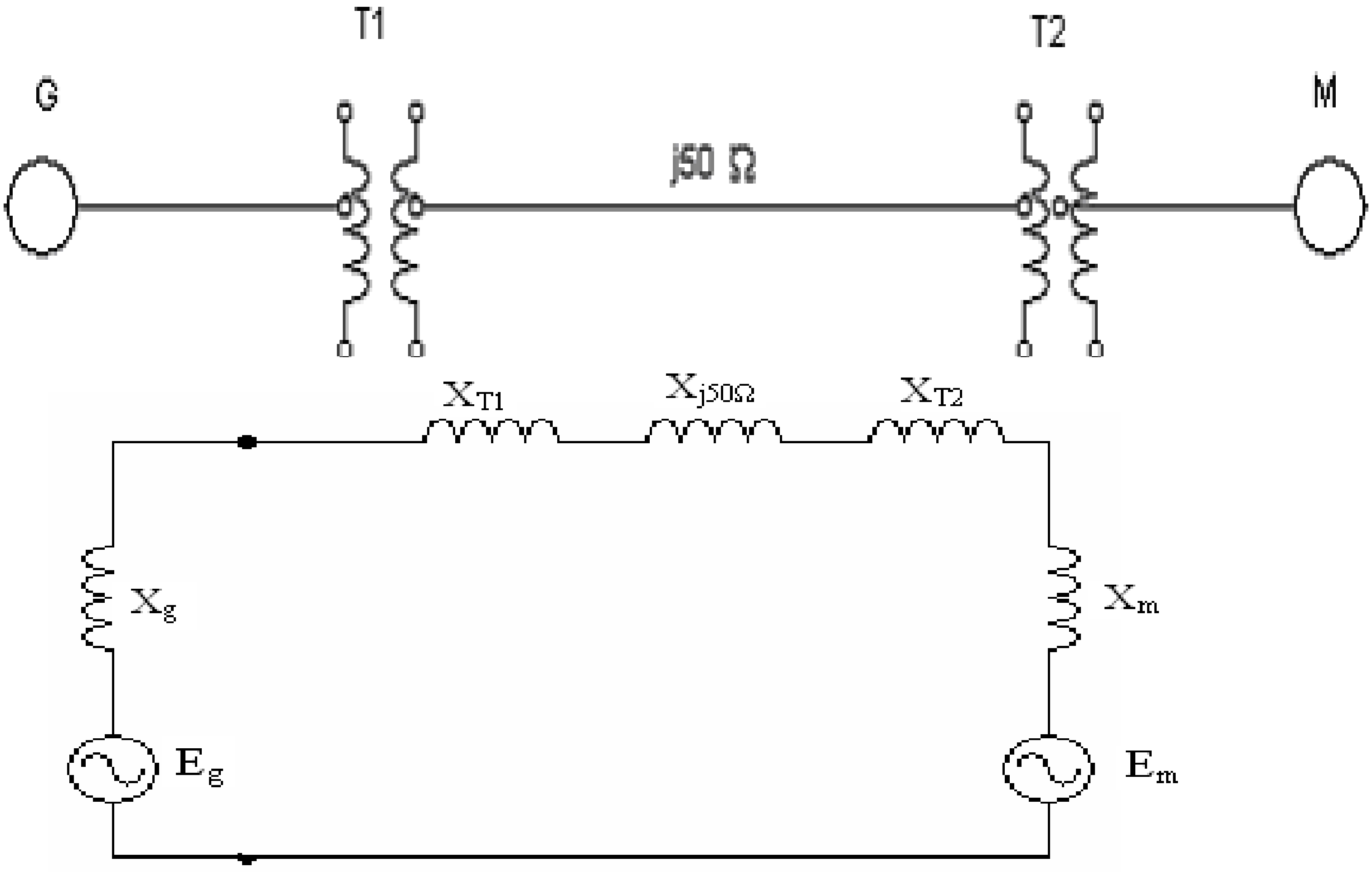
1. The impedance b/w neutral and ground omitted.
2. Shunt branches of the transformer equivalent circuit neglected.

REACTANCE DIAGRAM

- It is the equivalent circuit of the power system in which the various components are represented by their respective equivalent circuit.

- Reactance diagram can be obtained after omitting all resistances & capacitances of the transmission line from impedance diagram.

REACTANCE DIAGRAM FOR THE GIVEN POWER SYSTEM NETWORK



PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE DIAGRAM

1. Select a base power kVA_b or MVA_b
2. Select a base voltage kV_b
3. The voltage conversion is achieved by means of transformer kV_b on LT section= kV_b on HT section x LT voltage rating/HT voltage rating
4. When specified reactance of a component is in ohms

   p.u reactance=actual reactance/base reactance

   specified reactance of a component is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

FAULT ANALYSIS-BALANCED FAULT

Need for fault analysis

- To determine the magnitude of fault current throughout the power system after fault occurs.
- To select the ratings for fuses, breakers and switchgear.
- To check the MVA ratings of the existing circuit breakers when new generators are added into a system.

BALANCED THREE PHASE FAULT

- All the three phases are short circuited to each other and to earth.

- Voltages and currents of the system balanced after the symmetrical fault occurred. It is enough to consider any one phase for analysis.

SHORT CIRCUIT CAPACITY

- It is the product of magnitudes of the prefault voltage and the post fault current.
- It is used to determine the dimension of a bus bar and the interrupting capacity of a circuit breaker.

Short Circuit Capacity (SCC)

$$\left|SCC\right| = \left|V^0\right|\left|I_F\right|$$

$$\left|I_F\right| = \frac{\left|V_T\right|}{\left|Z_T\right|}$$

$$\left|SCC\right|_{1\phi} = \frac{\left|V_T\right|^2}{\left|Z_T\right|} = \frac{S_{b,1\phi}}{\left|Z_T\right|_{p.u}} MVA/\phi$$

$$\left|SCC\right|_{3\phi} = \frac{S_{b,3\phi}}{\left|Z_T\right|_{p.u}} MVA$$

$$I_f = \frac{\left|SCC\right|_{3\phi} * 10^6}{\sqrt{3} * V_{L,b} * 10^6}$$

Procedure for calculating short circuit capacity and fault current

- Draw a single line diagram and select common base S_b MVA and kV
- Draw the reactance diagram and calculate the total p.u impedance from the fault point to source (Thevenin impedance Z_T)
- Determine SCC and I_f

ALGORITHM FOR SHORT CIRCUIT ANALYSIS USING BUS IMPEDANCE MATRIX

- Consider a n bus network. Assume that three phase fault is applied at bus k through a fault impedance z_f
- Prefault voltages at all the buses are

$$V_{bus}(0) = \begin{bmatrix} V_1(0) \\ V_2(0) \\ . \\ V_k(0) \\ . \\ V_n(0) \end{bmatrix}$$

- Draw the Thevenin equivalent circuit i.e Zeroing all voltage sources and add voltage source $V_k(0)$ at faulted bus k and draw the reactance diagram

- The change in bus voltage due to fault is

$$\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ . \\ . \\ \Delta V_k \\ . \\ \Delta V_n \end{bmatrix}$$

- The bus voltages during the fault is

$$V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}$$

- The current entering into all the buses is zero.the current entering into faulted bus k is –ve of the current leaving the bus k

$$\Delta V_{bus} = Z_{bus} I_{bus}$$

$$\Delta V_{bus} = \begin{pmatrix} Z_{11} & . & Z_{1k} & . & Z_{1n} \\ . & . & . & . & . \\ Z_{k1} & . & Z_{kk} & . & Z_{kn} \\ . & . & . & . & . \\ Z_{n1} & . & Z_{nk} & . & Z_{nn} \end{pmatrix} \begin{bmatrix} 0 \\ . \\ -I_k(F) \\ . \\ 0 \end{bmatrix}$$

$$V_k(F) = V_k(0) - Z_{kk} I_k(F)$$

$$V_k(F) = Z_f I_k(F)$$

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

$$V_i(F) = V_i(0) - Z_{ik} I_k(F)$$

UNIT II

FAULT ANALYSIS – UNBALANCED FAULTS

INTRODUCTION

UNSYMMETRICAL FAULTS

- One or two phases are involved
- Voltages and currents become unbalanced and each phase is to be treated individually
- The various types of faults are

  Shunt type faults

  1.Line to Ground fault (LG)
  2. Line to Line fault (LL)
  3. Line to Line to Ground fault (LLG)

  Series type faults

  Open conductor fault (one or two conductor open fault)

FUNDAMENTALS OF SYMMETRICAL COMPONENTS

- Symmetrical components can be used to transform
  three phase unbalanced voltages and currents to
  balanced voltages and currents
- Three phase unbalanced phasors can be resolved into
  following three sequences
  1. Positive sequence components
  2. Negative sequence components
  3. Zero sequence components

Positive sequence components

Three phasors with equal magnitudes, equally displaced from one another by 120° and phase sequence is same as that of original phasors.

$$V_{a1}, V_{b1}, V_{c1}$$

Negative sequence components

Three phasors with equal magnitudes, equally displaced from one another by 120° and phase sequence is opposite to that of original phasors.

$$V_{a2}, V_{b2}, V_{c2}$$

Zero sequence components

Three phasors with equal magnitudes and displaced from one another by 0°

$$V_{a0}, V_{b0}, V_{c0}$$

RELATIONSHIP BETWEEN UNBALANCED VECTORS AND SYMMETRICAL COMPONENTS

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{b0} + V_{b1} + V_{b2}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Similarly we can obtain for currents also

SEQUENCE IMPEDANCE

- Impedances offered by power system components to positive, negative and zero sequence currents.
- Positive sequence impedance

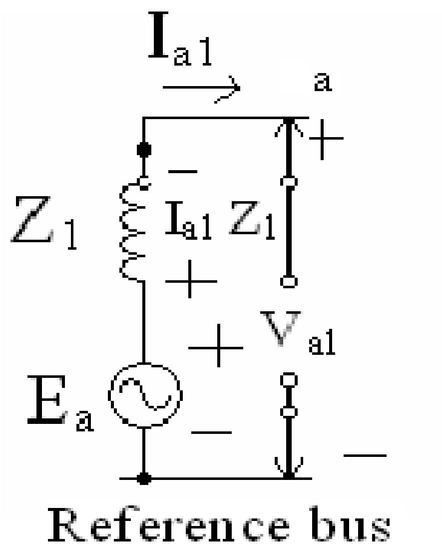
  The impedance of a component when positive sequence currents alone are flowing.
- Negative sequence impedance

  The impedance of a component when negative sequence currents alone are flowing.
- Zero sequence impedance

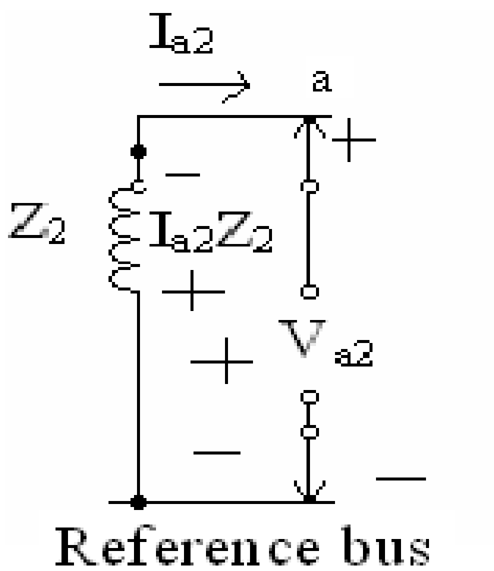
  The impedance of a component when zero sequence currents alone are flowing.

SEQUENCE NETWORK

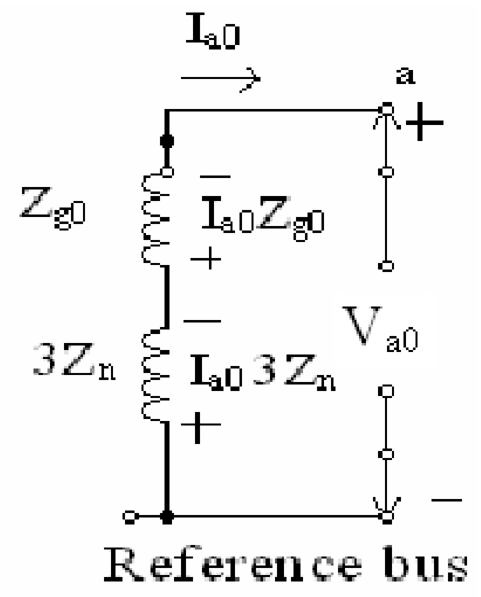
SEQUENCE NETWORK FOR GENERATOR



positive sequence network

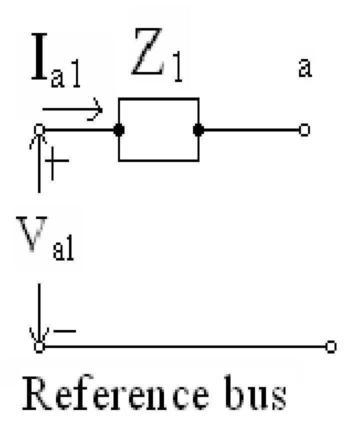


negative sequence network

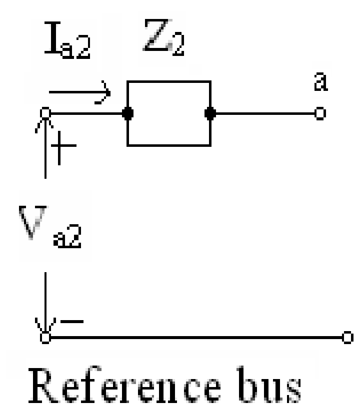


Zero sequence network

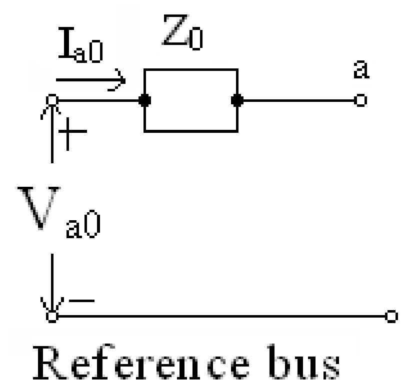
SEQUENCE NETWORK FOR TRANSMISSION LINE



positive sequence network

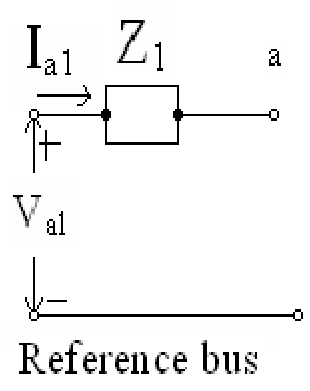


negative sequence network

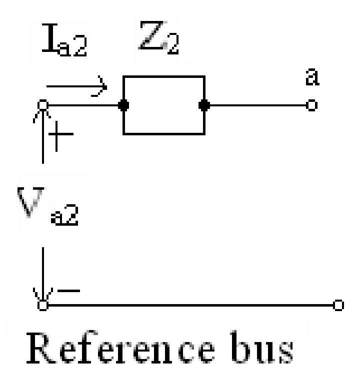


Zero sequence network

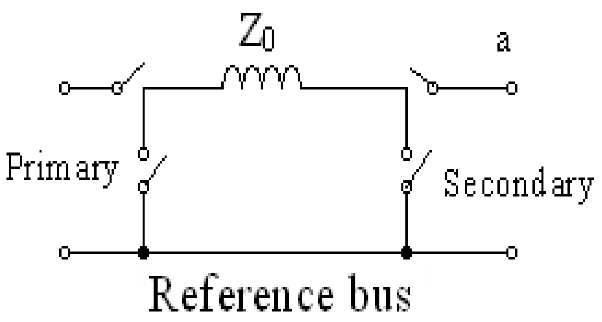
SEQUENCE NETWORK FOR TRANSFORMER



positive sequence network

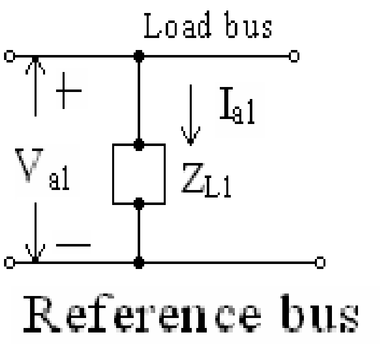


negative sequence network

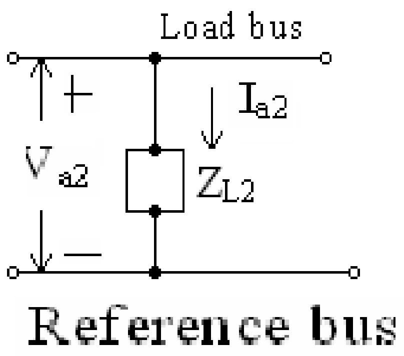


Zero sequence network

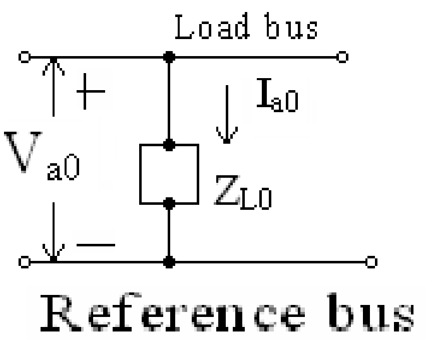
SEQUENCE NETWORK FOR LOAD



positive sequence network

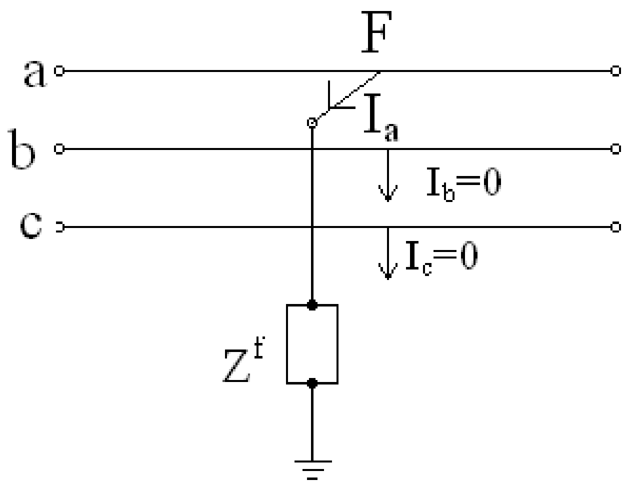


negative sequence network



Zero sequence network

SINGLE LINE TO GROUND FAULT



$$I_b = 0$$

$$I_c = 0$$

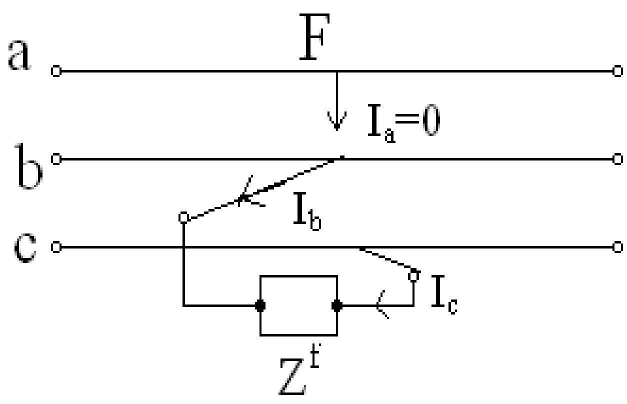
$$V_a = Z^f I_a$$

$$I_{a1} = I_{a2} = I_{a0} = I_a / 3$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z^f}$$

Consider a fault between phase a and ground through an impedance z_f

LINE TO LINE (LL) FAULT



Consider a fault between phase b and c through an impedance z_f

$$I_a = 0$$

$$I_c = -I_b$$

$$V_b - V_c = I_b Z^f$$

$$I_{a2} = -I_{a1}$$

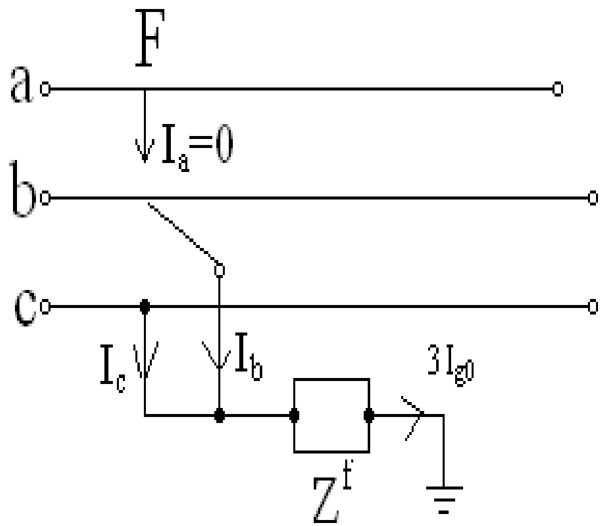
$$I_{a0} = 0$$

$$V_{a1} - V_{a2} = Z^f I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + 3Z^f}$$

$$I_b = -I_c = \frac{-jE_a}{Z_1 + Z_2 + 3Z^f}$$

DOUBLE LINE TO GROUND (LLG) FAULT



$$I_{a0} = 0$$

$$I_{a1} + I_{a2} + I_{a0} = 0$$

$$V_b = V_c = Z^f (I_b + I_c) = 3Z^f I_{a0}$$

$$V_{a0} - V_{a1} = V_b = 3Z^f I_{a0}$$

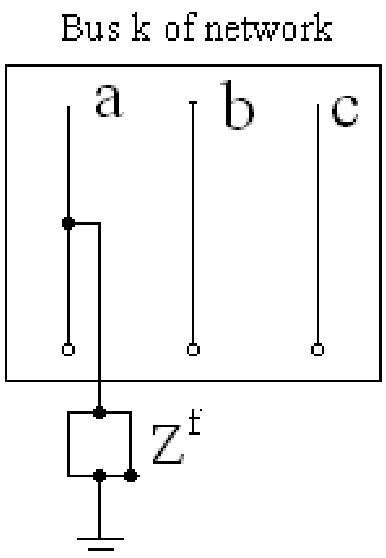
$$I_{a1} = \frac{E_a}{Z_1 + Z_2(Z_0 + 3Z^f)/(Z_2 + Z_0 + 3Z^f)}$$

Consider a fault between phase b and c through an impedance z_f to ground

UNBALANCED FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

SINGLE LINE TO GROUND FAULT USING Z_{bus}

- Consider a fault between phase a and ground through an impedance z_f at bus k



For a fault at bus k the symmetrical components of fault current

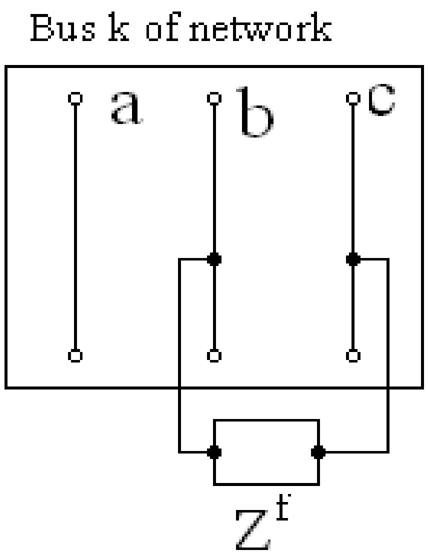
$$I_k^0 = I_k^1 = I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_{kk}^0 + 3Z^f}$$

Where Z_{kk}^1, Z_{kk}^2, Z_{kk}^0 are the diagonal elements in the k axis of the z_{bus} & $V_k(0)$ is the prefault voltage at bus k.

Fault phase current $I_k^{abc} = A\, I_k^{012}$

LINE TO LINE (LL) FAULT

Consider a fault between phase b and c through an impedance z_f

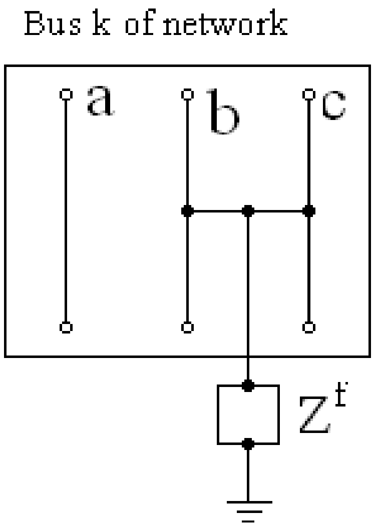


$$I_k^0 = 0$$

$$I_k^1 = -I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z^f}$$

DOUBLE LINE TO GROUND (LLG) FAULT

Consider a fault between phase b and c through an impedance z_f to ground



$$I_k^{1} = \frac{V_k(0)}{Z_{kk}^{1} + \dfrac{Z_{kk}^{2}(Z_{kk}^{0} + 3Z^{f})}{Z_{kk}^{2} + Z_{kk}^{0} + 3Z^{f}}}$$

$$I_k^{2} = -\frac{V_k(0) - Z_{kk}^{1} I_k^{1}}{Z_{kk}^{2}}$$

$$I_k^{0} = -\frac{V_k(0) - Z_{kk}^{1} I_k^{1}}{Z_{kk}^{0} + 3Z^{f}}$$

$$I_k(F) = I_k^{b} + I_k^{c}$$

BUS VOLTAGES AND LINE CURRENTS DURING FAULT

$$V_i^0(F) = 0 - Z_{ik}{}^0 \mathrm{I}_k{}^0$$

$$V_i^1(F) = V_i^0(0) - Z_{ik}{}^1 \mathrm{I}_k{}^1$$

$$V_i^2(F) = 0 - Z_{ik}{}^2 \mathrm{I}_k{}^2$$

$$\mathrm{I}_{ij}{}^0 = \frac{V_i^0(F) - V_j{}^0(F)}{Z_{ij}{}^0}$$

$$\mathrm{I}_{ij}{}^1 = \frac{V_i^1(F) - V_j{}^1(F)}{Z_{ij}{}^1}$$

$$\mathrm{I}_{ij}{}^2 = \frac{V_i^2(F) - V_j{}^2(F)}{Z_{ij}{}^2}$$

ALGORITHM FOR FORMATION OF THE BUS IMPEDANCE MATRIX

- Modification of Zbus matrix involves any one of the following 4 cases

Case 1:adding a branch impedance z_b from a new bus p to the reference bus

Addition of new bus will increase the order the Z_{bus} matrix by 1

$$Z_{bus,new} = \begin{pmatrix} z_{original} & 0 \\ 0 & z_b \end{pmatrix}$$

(n+1)th column and row elements are zero except the diagonal diagonal element is z_b

Case 2: adding a branch impedance z_b from a new bus p
to the existing bus q
Addition of new bus will increase the order the Z_{bus} matrix by 1
The elements of (n+1)th column and row are the elements of qth column and row and the diagonal element is $Z_{qq}+Z_b$

Case 3: adding a branch impedance z_b from an existing bus p to the reference bus
The elements of (n+1)th column and row are the elements of qth column and row and the diagonal element is $Z_{qq}+Z_b$ and (n+1)th row and column should be eliminated using the following formula

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad j = 1,2...n; k = 1,2..n$$

Case 4:adding a branch impedance z_b between existing buses h and q elements of (n+1)th column are elements of bus h column –

bus q column and elements of (n+1)th row are elements of bus h row – bus q row the diagonal element=

$$Z_b + Z_{hh} + Z_{qq} - 2Z_{hq}$$

and (n+1)th row and column should be eliminated using the following formula

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad j = 1,2...n; k = 1,2..n$$

UNIT III

LOAD FLOW ANALYSIS

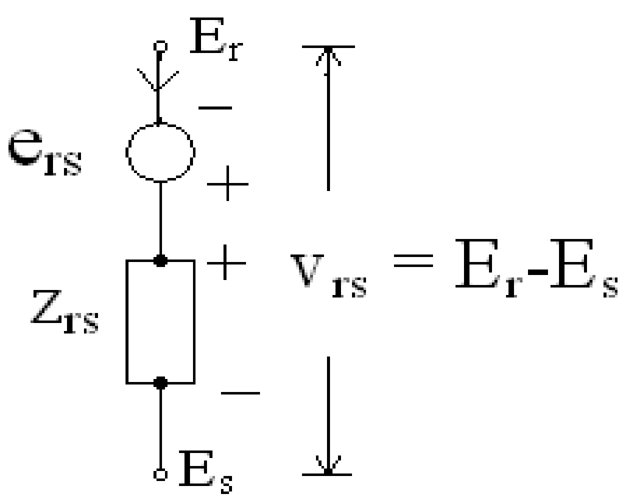
BUS CLASSIFICATION

1. Slack bus or Reference bus or Swing bus:
   $|V|$ and δ are specified. P and Q are un specified, and to be calculated.

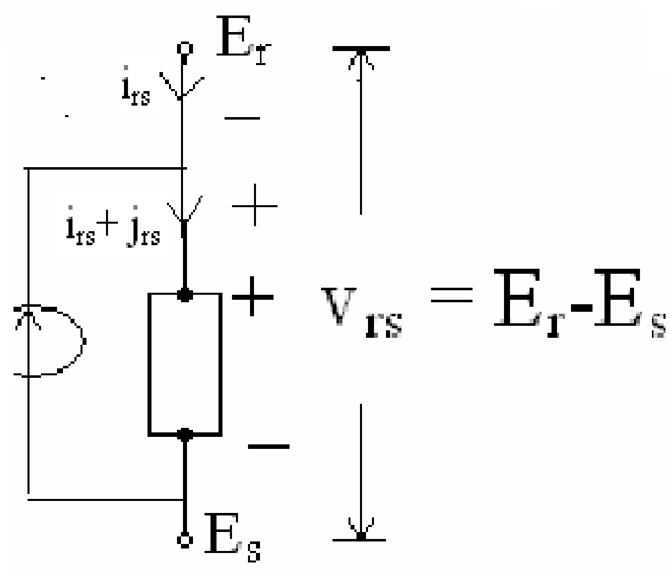
2. Generator bus or PV bus or Voltage controlled bus:
   P and $|V|$ are specified. Q and δ are un specified, and to be calculated
3. Load bus or PQ bus:
   P and Q are specified. $|V|$ and δ are un specified, and to be calculated

PRIMITIVE NETWORK

It is a set of unconnected elements which provides information regarding the characteristics of individual elements. it can be represented both in impedance & admittance form



Impedance form



admittance form

BUS ADMITTANCE(Y BUS) MATRIX

Y BUS can be formed by 2 methods

1.Inspection method

2.Singular transformation

$$Y\ BUS = \begin{pmatrix} Y_{11} & Y_{12} \bullet\bullet & Y_{1n} \\ Y_{21} & Y_{22} \bullet\bullet & Y_{2n} \\ Y_{n1} & Y_{n2} \bullet\bullet & Y_{nn} \end{pmatrix}$$

INSPECTION METHOD

For n bus system

Diagonal element of Y BUS

$$Y_{ii} = \sum_{j=1}^{n} y_{ij}$$

Off Diagonal element of Y BUS

$$Y_{ij} = -y_{ij}$$

SINGULAR TRANSFORMATION METHOD

Y BUS = $A^T \left[y\right] A$

Where [y]=primitive admittance

A=bus incidence matrix

ITERATIVE METHOD

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q$$

$$S_p = P_p - jQ_p = V_P{}^* I_p$$

$$\frac{P_p - jQ_p}{V_P{}^*} = \sum_{q=1}^{n} Y_{pq} V_q$$

The above Load flow equations are non linear and can be solved by following iterative methods.

1. Gauss seidal method
2. Newton Raphson method
3. Fast Decoupled method

GAUSS SEIDAL METHOD

For load bus calculate |V| and δ from V_p^{k+1} equation

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^{k})^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^{k}\right]$$

For generator bus calculate Q from Q_P^{K+1} equation

$$Q_p^{k+1} = -1 * \operatorname{Im}\left\{(V_P^{k})^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^{k}\right]\right\}$$

- Check $Q_{p,cal}^{k+1}$ with the limits of Q_p
- If $Q_{p,cal}^{k+1}$ lies within the limits bus p remains as PV bus otherwise it will change to load bus
- Calculate δ for PV bus from V_p^{k+1} equation
- Acceleration factor α can be used for faster convergence
- Calculate change in bus-p voltage

$$\Delta V_p^{k+1} = V_p^{k+1} - V_p^{k}$$

- If $|\Delta V_{max}| < \varepsilon$, find slack bus power otherwise increase the iteration count (k)
- Slack bus power= $\sum S_G - \sum S_L$

NEWTON RAPHSON METHOD

$$P_i - Q_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \underline{\angle \theta_{ij} - \delta_i + \delta_j}$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\Delta P_i^k = P_i^{sch} - P_i^k$$

$$\Delta Q_i^k = Q_i^{sch} - Q_i^k$$

- Calculate |V| and δ from the following equation

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta^k$$

$$\left|V_i^{k+1}\right| = \left|V_i^k\right| + \Delta\left|V_i^k\right|$$

- If

$$\Delta P_i^k < \varepsilon$$

$$\Delta Q_i^k < \varepsilon$$

- stop the iteration otherwise increase the iteration count (k)

FAST DECOUPLED METHOD

- J_2 & J_3 of Jacobian matrix are zero

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{pmatrix} J_1 & 0 \\ 0 & J_4 \end{pmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\Delta P = J_1 \Delta \delta = \left[\frac{\partial P}{\partial \delta} \right] \Delta \delta$$

$$\Delta Q = J_4 \Delta |V| = \left[\frac{\partial Q}{\partial |V|} \right] \Delta |V|$$

$$\frac{\Delta P}{\Delta |V_i|} = -B^{'} \Delta \delta$$

$$\frac{\Delta Q}{\Delta |V_i|} = -B^{''} \Delta |V|$$

$$\Delta \delta = -\left[B^{'} \right]^{-1} \frac{\Delta P}{\Delta |V|}$$

$$\Delta |V| = -\left[B^{''} \right]^{-1} \frac{\Delta Q}{\Delta |V|}$$

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta^k$$

$$\left|V_i^{k+1}\right| = \left|V_i^k\right| + \Delta\left|V_i^k\right|$$

- This method requires more iterations than NR method but less time per iteration
- It is useful for in contingency analysis

COMPARISION BETWEEN ITERATIVE METHODS

Gauss – Seidal Method

1. Computer memory requirement is less.
2. Computation time per iteration is less.
3. It requires less number of arithmetic operations to complete an iteration and ease in programming.
4. No. of iterations are more for convergence and rate of convergence is slow (linear convergence characteristic.
5. No. of iterations increases with the increase of no. of buses.

NEWTON – RAPHSON METHOD

- Superior convergence because of quadratic convergence.
- It has an 1:8 iteration ratio compared to GS method.
- More accurate.
- Smaller no. of iterations and used for large size systems.
- It is faster and no. of iterations is independent of the no. of buses.
- Technique is difficult and calculations involved in each iteration are more and thus computation time per iteration is large.
- Computer memory requirement is large, as the elements of jacobian matrix are to be computed in each iteration.
- Programming logic is more complex.

FAST DECOUPLED METHOD

- It is simple and computationally efficient.
- Storage of jacobian matrix elements are60% of NR method
- computation time per iteration is less.
- Convergence is geometric,2 to 5 iterations required for accurate solutions
- Speed for iterations is 5 times that of NR method and 2-3 times of GS method

UNIT IV

STABILITY ANALYSIS

STABILITY

- The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium.
- Ability to keep the machines in synchronism with another machine

CLASSIFICATION OF STABILITY

- Steady state stability

  Ability of the power system to regain synchronism after small and slow disturbances (like gradual power changes)

- Dynamic stability

  Ability of the power system to regain synchronism after small disturbances occurring for a long time (like changes in turbine speed, change in load)

- Transient stability

  This concern with sudden and large changes in the network conditions i.e. . sudden changes in application or removal of loads, line switching operating operations, line faults, or loss of excitation.

- Steady state limit is the maximum power that can be transferred without the system become unstable when the load in increased gradually under steady state conditions.

- Transient limit is the maximum power that can be transferred without the system becoming unstable when a sudden or large disturbance occurs.

Swing Equation for Single Machine Infinite Bus System

- The equation governing the motion of the rotor of a synchronous machine

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

where

J=The total moment of inertia of the rotor(kg-m^2)

θ_m =Singular displacement of the rotor

T_m=Mechanical torque (N-m)

T_e=Net electrical torque (N-m)

T_a=Net accelerating torque (N-m)

$$\theta_m = \omega_{sm} t + \delta_m$$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\omega_m \frac{d^2\delta_m}{dt^2} = p_a = p_m - p_e$$

- Where p_m is the shaft power input to the machine
  p_e is the electrical power
  p_a is the accelerating power

$$J\omega_m = M$$

$$M\frac{d^2\delta_m}{dt^2} = p_a = p_m - p_e$$

$$M = \frac{2H}{\omega_{sm}} S_{machine}$$

H=machine inertia constant

$$\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{p_a}{S_{machine}} = \frac{p_m - p_e}{S_{machine}}$$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = p_a = p_m - p_e$$

$$\omega_s = 2\pi f$$

δ and ω_s are in electrical radian

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = p_a = (p_m) - p_e$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(p_m - p_{2\max}\sin\delta) = \frac{\pi f_0}{H} p_a \text{ p.u}$$

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H} p_a = \frac{d^2\delta}{dt^2} \text{ p.u}$$

Swing Equation for Multimachine System

$S_{machine}$ =machine rating(base)

S_{system} =system base

$$\frac{H_{system}}{\pi f}\frac{d^2\delta}{dt^2} = p_a = p_m - p_e \quad \text{p.u}$$

$$H_{system} = H_{machine}\frac{S_{machine}}{S_{system}}$$

Rotor Angle Stability

- It is the ability of interconnected synchronous machines of a power system to maintain in synchronism. The stability problem involves the study of the electro mechanical oscillations inherent in power system.

- Types of Rotor Angle Stability
    1. Small Signal Stability (or) Steady State Stability
    2. Transient stability

Voltage Stability

- It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.

- The major factor for instability is the inability of the power system to meet the demand for reactive power.

- Mid Term Stability

  It represents transition between short term and long term responses.

  Typical ranges of time periods.

  1. Short term : 0 to 10s
  2. Mid Term : 10 to few minutes
  3. Long Term : a few minutes to 10's of minutes

- Long Term Stability

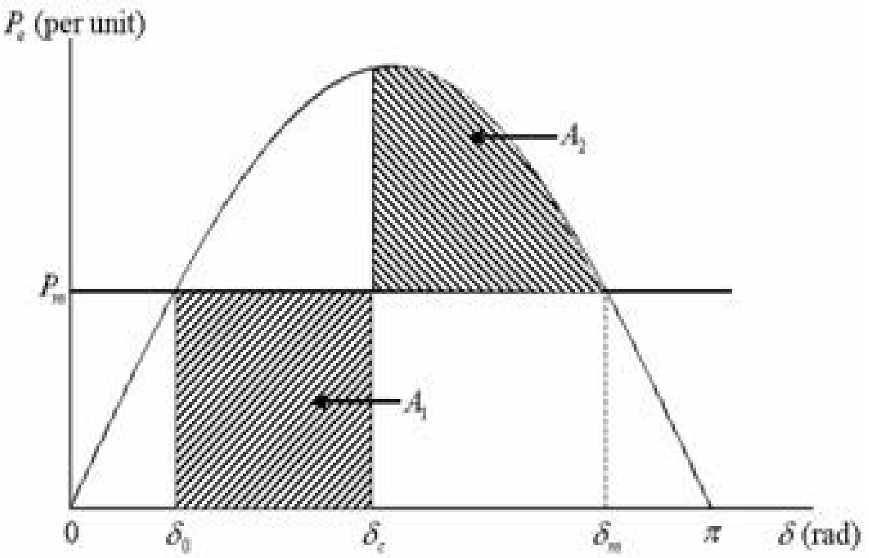
  Usually these problem be associated with

  1. Inadequacies in equipment responses.
  2. Poor co-ordination of control and protection equipment.
  3. Insufficient active/reactive power reserves.

Equal Area Criterion

- This is a simple graphical method to predict the transient stability of two machine system or a single machine against infinite bus. This criterion does not require swing equation or solution of swing equation to determine the stability condition.

- The stability conditions are determined by equating the areas of segments on power angle diagram.



Power-angle curve for equal area criterion

multiplying swing equation by $d\delta/dt$ on both sides

$$\frac{H}{\omega_s}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = (P_m - P_e)\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

Multiplying both sides of the above equation by *dt* and then integrating between two arbitrary angles δ_0 and δ_c

$$\frac{H}{\omega_s}\left(\frac{d\delta}{dt}\right)^2\Bigg|_{\delta_0}^{\delta_c} = \int_{\delta_0}^{\delta_c}\left(P_m - P_e\right)d\delta$$

Once a fault occurs, the machine starts accelerating. Once the fault is cleared, the machine keeps on accelerating before it reaches its peak at δ_c ,

The area of accelerating A1

$$A_1 = \int_{\delta_0}^{\delta_c}\left(P_m - P_e\right)d\delta = 0$$

The area of deceleration is given by A_2

$$A_2 = \int_{\delta_c}^{\delta_m}\left(P_e - P_m\right)d\delta = 0$$

If the two areas are equal, i.e., $A_1 = A_{2,}$ then the power system will be stable

Critical Clearing Angle (δ_{cr}) maximum allowable value of the clearing time and angle for the system to remain stable are known as critical clearing time and angle.

δ_{cr} expression can be obtained by substituting δ_c = δ_{cr} in the equation A1 = A2

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_e) d\delta = \int_{\delta_{cr}}^{\delta_{m}} (P_e - P_m) d\delta$$

Substituting P_e = 0 in swing equation

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m$$

Integrating the above equation

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H} P_m dt = \frac{\omega_s}{2H} P_m t$$

$$\delta = \int_0^t \frac{\omega_s}{2H} P_m t\, dt = \frac{\omega_s}{4H} P_m t^2 + \delta_0$$

Replacing δ by δ_{cr} and t by t_{cr} in the above equation, we get the critical clearing time as

$$t_{cr} = \sqrt{\frac{4H}{\omega_s P_m}\left(\delta_{cr} - \delta_0\right)}$$

Factors Affecting Transient Stability

- Strength of the transmission network within the system and of the tie lines to adjacent systems.

- The characteristics of generating units including inertia of rotating parts and electrical properties such as transient reactance and magnetic saturation characteristics of the stator and rotor.

- Speed with which the faulted lines or equipments can be disconnected.

Numerical Integration methods

- Modified Euler's method
- Runge-Kutta method

MODIFIED EULER'S METHOD

- Using first derivative of the initial point next point is obtained
- $$X_1^p = X_0 + \frac{dX}{dt}\Delta t$$ the step $t_1 = t_0 + \Delta t$
- Using this x_1^p dx/dt at $x_1^p = f(t_1, x_1^p)$
- Corrected value is

$$X_1^P = X_0 + \left(\frac{\left(\frac{dx}{dt}\right)_{X_0} + \left(\frac{dx}{dt}\right)_{X_1^p}}{2} \right) \Delta t$$

$$X_{i+1}^c = X_i + \left(\frac{\left(\frac{dx}{dt}\right)_{X_i} + \left(\frac{dx}{dt}\right)_{X_{i-1}^p}}{2} \right) \Delta t$$

Numerical Solution of the swing equation

- Input power p_m=constant
- At steady state $p_e=p_m$,

$$\delta_0 = \sin^{-1}\left(\frac{p_m}{p_{1\max}}\right)$$

$$p_{1\max} = \frac{|E'||V|}{X_1}$$

- At synchronous speed

$$\Delta\omega_0 = 0$$

$$p_{2\max} = \frac{|E'||V|}{X_2}$$

The swing equation

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = p_a = (p_m) - p_e$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(p_m - p_{2\max}\sin\delta) = \frac{\pi f_0}{H}p_a$$

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H}p_a = \frac{d^2\delta}{dt^2}$$

Applying Modified Eulers method to above equation

$$t_1 = t_0 + \Delta t$$

$$\delta_{i+1}^{p} = \delta_i + \left(\frac{d\delta}{dt}\right)_{\Delta\omega_i}\Delta t$$

$$\Delta\omega_{i+1}^{p} = \Delta\omega_i + \left(\frac{d\Delta\omega}{dt}\right)_{\delta_i}\Delta t$$

- The derivatives at the end of interval

$$\left(\frac{d\delta}{dt}\right)_{\Delta\omega_{i+1}^{p}} = \Delta\omega_{i+1}^{p}$$

$$\left(\frac{d\Delta\omega}{dt}\right)_{\delta_{i+1}^{p}} = \left(\frac{\pi f_0}{H} p_a\right)_{\delta_{i+1}^{p}}$$

The corrected value

$$\delta_{i+1}^{c} = \delta_i + \left(\frac{\left(\frac{d\delta}{dt}\right)_{\Delta\omega_i} + \left(\frac{d\delta}{dt}\right)_{\Delta\omega_{i+1}^{p}}}{2}\right)\Delta t$$

$$\Delta\omega_{i+1}^{c} = \Delta\omega_i + \left(\frac{\left(\frac{d\Delta\omega}{dt}\right)_{\delta_i} + \left(\frac{d\Delta\omega}{dt}\right)_{\delta_{i+1}^{p}}}{2}\right)\Delta t$$

Runge-Kutta Method

- Obtain a load flow solution for pretransient conditions
- Calculate the generator internal voltages behind transient reactance.
- Assume the occurrence of a fault and calculate the reduced admittance matrix
- Initialize time count K=0,J=0
- Determine the eight constants

$$K_1^k = f_1(\delta^k, \omega^k)\Delta t$$

$$l_1^k = f_2(\delta^k, \omega^k)\Delta t$$

$$K_2^k = f_1(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2})\Delta t$$

$$l_2^k = f_2(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2})\Delta t$$

$$K_3^k = f_1(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2})\Delta t$$

$$l_3^k = f_2(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2})\Delta t$$

$$K_4^k = f_1(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2})\Delta t$$

$$l_4^k = f_2(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2})\Delta t$$

$$\Delta\delta^k = \frac{\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)}{6}$$

$$\Delta\omega^k = \frac{\left(l_1^k + 2l_2^k + 2l_3^k + l_4^k\right)}{6}$$

Compute the change in state vector

$$\Delta\delta^k = \frac{\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)}{6}$$

$$\Delta\omega^k = \frac{\left(l_1^k + 2l_2^k + 2l_3^k + l_4^k\right)}{6}$$

Evaluate the new state vector

$$\delta^{k+1} = \delta^k + \Delta\delta^k$$

$$\omega^{k+1} = \omega^k + \Delta\omega^k$$

Evaluate the internal voltage behind transient reactance using the relation

$$E_p^{k+1} = \left|E_p^k\right| \cos\delta_p^{k+1} + j\left|E_p^k\right| \sin\delta_p^{k+1}$$

Check if $t<t_c$ yes K=K+1

Check if j=0,yes modify the network data and obtain the new reduced admittance matrix and set j=j+1

set K=K+1

Check if K<Kmax, yes start from finding 8 constants

Unit-V

Traveling Wave

Introduction

- Transient Phenomenon :
  - Aperiodic function of time
  - Short duration

- Example :Voltage & Current Surge :
  (The current surge are made up of charging or discharging capacitive currents that introduced by the change in voltages across the shunt capacitances of the transmission system)
  - Lightning Surge
  - Switching Surge

Impulse Voltage Waveform

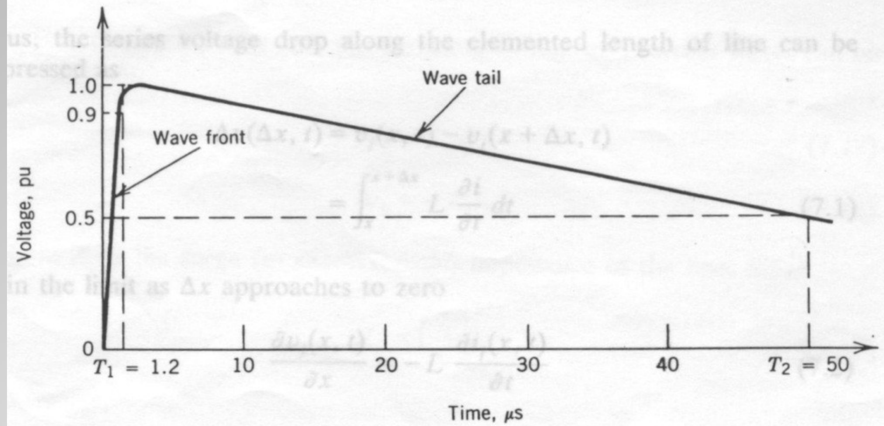


Figure 7.1. Standard impulse voltage waveform.

Traveling Wave

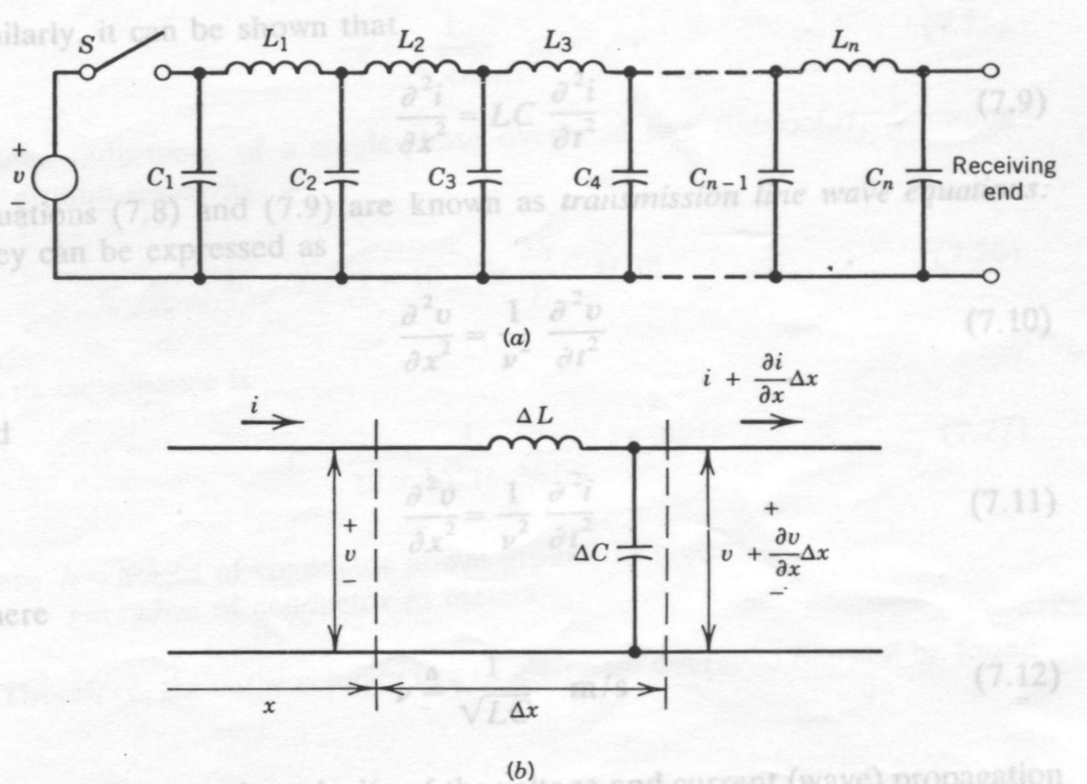


Figure 7.2. Representation of two-wire transmission line for application of traveling waves: (*a*) *lumpy* representation; (*b*) elemental section of line.

- Disturbance represented by closing or opening the switch S.
- If Switch S closed, the line suddenly connected to the source.
- The whole line is not energized instantaneously.
- Processed :
  - When Switch S closed
  - The first capacitor becomes charged immediately
  - Because of the first series inductor (acts as open circuit), the second capacitor is delayed
- This gradual buildup of voltage over the line conductor can be regarded as a voltage wave is traveling from one end to the other end

Voltage & Current Function

- $v_f = v_1(x - \upsilon t)$
- $v_b = v_2(x + \upsilon t)$
- $\upsilon = 1/\sqrt{(LC)}$
- $v(x,t) = v_f + v_b$
- $v_f = Z_c i_f$
- $v_b = Z_c i_b$

- $Z_c = (L/C)^{1/2}$
- $I_f = v_f / Z_c$
- $I_b = v_b / Z_c$
- $I(x,t) = I_f + I_b$
- $I(x,t) = (C/L)^{1/2} [v_1(x - \upsilon t) - v_2(x + \upsilon t)]$

Velocity of Surge Propagation

- In the air = 300 000 km/s
- υ = 1/ √(LC) m/s
- Inductance single conductor Overhead Line (assuming zero ground resistivity) :
  $L = 2 \times 10^{-7} \ln(2h/r)$ H/m
  $C = 1/[18 \times 10^{9} \ln(2h/r)]$ F/m
- $$v = \frac{1}{\sqrt{LC}} = \left[\left(\frac{2\times 10^{-7}\ln(2h/r)}{18\times 10^{9}\ln(2h/r)}\right)^{1/2}\right]^{-1}$$
- In the cable : υ = 1/ √(LC) = 3 x 10^{8} √K m/s
  K=dielectric constant (2.5 to 4.0)

Surge Power Input & Energy Storage

- $P=vi$ Watt
- $W_s= \frac{1}{2} Cv^2$; $W_m= \frac{1}{2} Li^2$
- $W=W_s+W_m = 2\ W_s = 2\ W_m = Cv^2 = Li^2$
- $P=W\ \upsilon = Li^2/\sqrt{(LC)} = i^2 Z_c = v^2 / Z_c$

Superposition of Forward and Backward-Traveling Wave

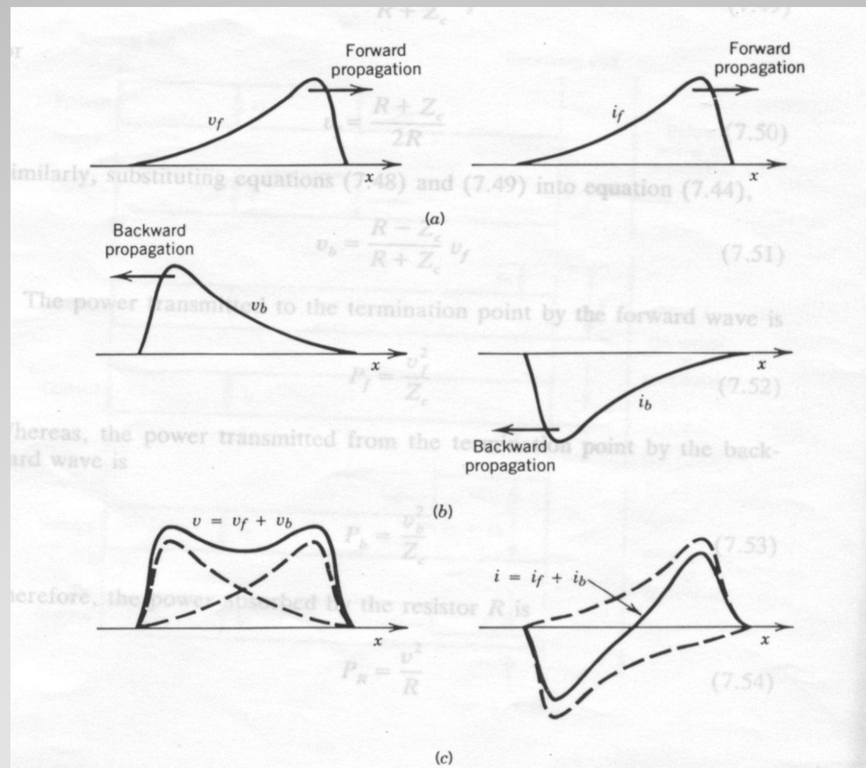


Figure 7.3. Representation of voltage and current waves: (*a*) in forward direction: (*b*) in backward direction; (*c*) superposition of waves.

Effects of Line Termination

- Assuming vf, if,vb and ib are the instantaneous voltage and current. Hence the instantaneous voltage and current at the point discontinuity are :

- $v(x,t)=v_f + v_b$ and $I(x,t)=I_f + I_b$
- $I=v_f/Z_c - v_b/Z_c$ and $iZ_c=v_f - v_b$
- $v + iZ_c= 2v_f$ so $v=2v_f=iZ_c$
- $v_f = ½ (v+iZ_c)$ and $v_b = ½ (v+iZ_c)$ or
  $v_b= v_f-iZ_c$

Line Termination in Resistance

$$v = iR$$

$$i = \frac{2}{R + Z_c} v_f$$

$$v_f = \frac{R + Z_c}{2R}$$

$$v_b = \frac{R - Z_c}{R + Z_c} v_f$$

$$P_f = \frac{v_f^2}{Z_c}$$

$$P_b = \frac{v_b^2}{Z_c}$$

$$P_R = \frac{v^2}{R} = \frac{\left(v_f + v_b\right)^2}{R}$$

$$P_f = P_b + P_R$$

Line Termination in Impedance (Z)

$$i = \frac{2}{Z + Z_c} i_f$$

$$v = \frac{2Z}{Z + Z_c} v_f$$

$$v = \tau v_f$$

$$\tau \cong \frac{2Z}{Z + Z_c}$$

$$v_f = \frac{Z + Z_c}{2R} v$$

$$v_b = \frac{Z - Z_c}{Z + Z_c} v_f$$

$$v_b = \rho v_f$$

$$\rho \cong \frac{Z - Z_c}{Z + Z_c}$$

- Line is terminated with its characteristic impedance :
  - $Z=Z_c$
  - ρ =0, no reflection (infinitely long)
- $Z>Z_c$
  - v_b is positive
  - I_b is negative
  - Reflected surges increased voltage and reduced current
- $Z<Z_c$
  - v_b is negative
  - I_b is positive
  - Reflected surges reduced voltage and increased current
- Z_s and Z_R are defined as the sending-end and receiving end.

$$\rho_s = \frac{Z_s - Z_c}{Z_s + Z_c}; \rho_R = \frac{Z_R - Z_c}{Z_R + Z_c}$$

-

$$\rho_s = \frac{Z_s - Z_c}{Z_s + Z_c} \tag{7.65}$$

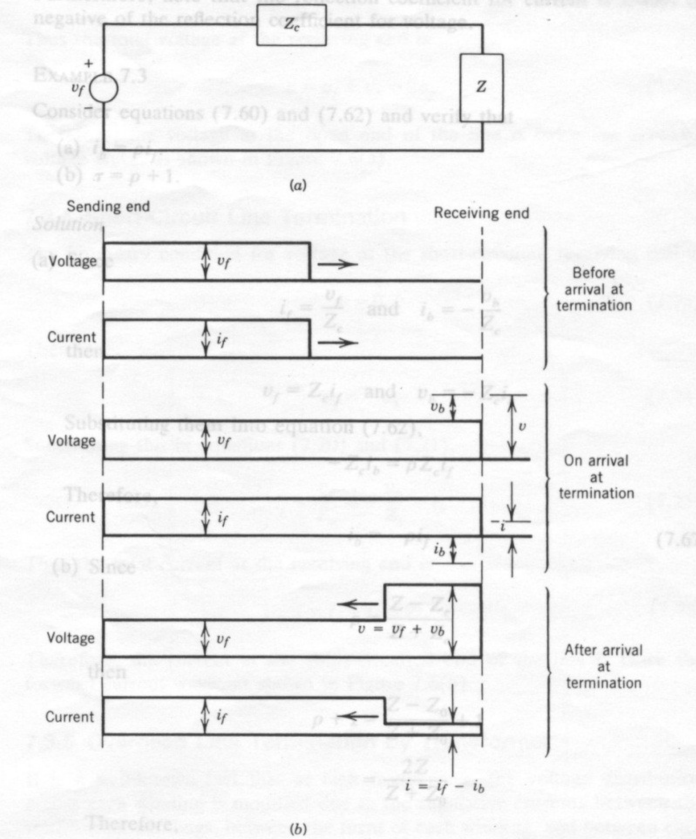


Figure 7.4. Analysis of traveling waves when $Z > Z_c$: (a) circuit diagram; (b) voltage and current distributions.

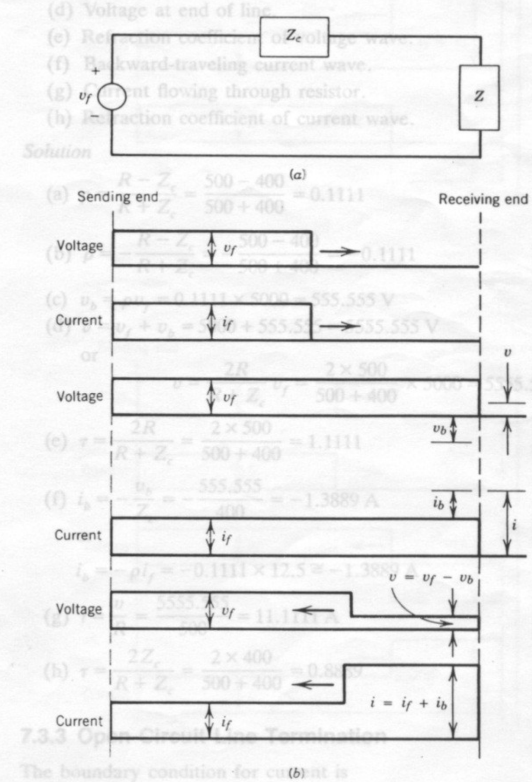


Figure 7.5. Analysis of traveling waves when $Z < Z_c$: (a) circuit diagram; (b) voltage and current distributions.

Open-Circuit Line Termination

- Boundary condition for current i=0
- Therefore $i_f = -i_b$

- $V_b = Z_c i_b = Z i_f = v_f$

- Thus total voltage at the receiving end $v = v_f + v_b = 2v_f$

- Voltage at the open end is twice the forward voltage wave

Short Circuit Line Termination

- Boundary condition for current v=0
- Therefore $v_f=-v_b$
- $I_f=v_f/Z_c=-(v_b/Z_c)=i_b$
- Thus total voltage at the receiving end $v=i_f+i_b=2i_f$
- Current at the open end is twice the forward current wave

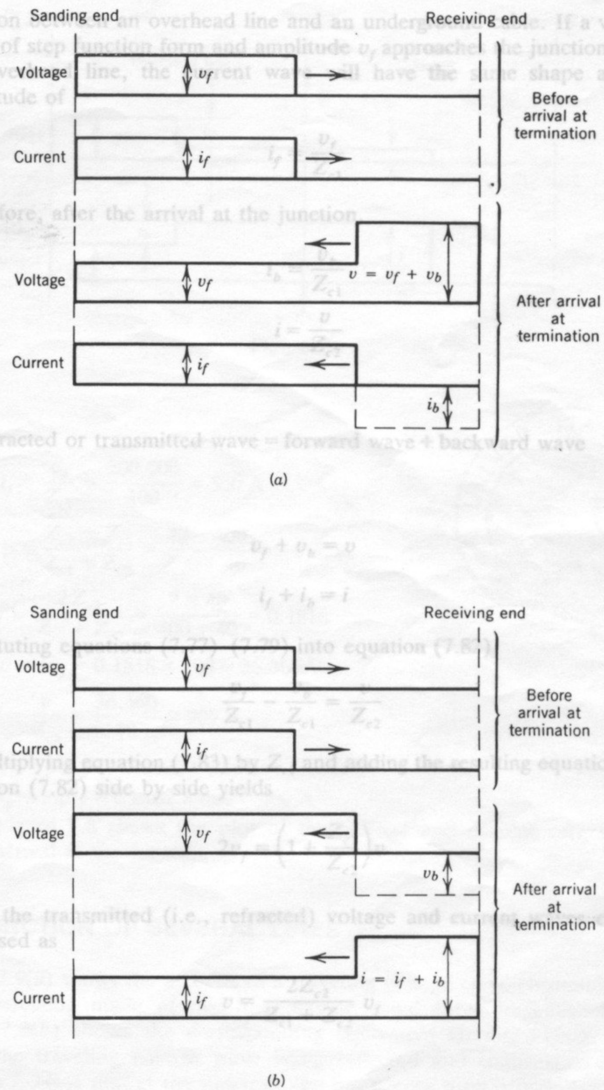


Figure 7.6. Analysis of traveling waves for: (*a*) open-circuit line termination; (*b*) short-circuit line termination.

Termination Through Capacitor

$$\tau = \frac{2Z}{Z + Z_c}$$

$$\tau = \frac{2(1/Cs)}{Z_c + 1/Cs}$$

$$v = \tau v_f$$

$$v(s) = \frac{2(1/Cs)}{Z_c + 1/Cs}\frac{v_f}{s} = \frac{2v_f}{s}\frac{1}{Z_c C_s + 1}$$

$$= \frac{2v_f}{s}\frac{1/Z_c C}{s + 1/Z_c C} = 2v_f \frac{1}{s} - \frac{1}{s + 1/Z_c C}$$

$$So:$$

$$v(t) = 2v_f(1 - e^{-t/Z_c C})$$

$$i(t) = \frac{2v_f}{Z_c}e^{-t/Z_c C}$$

$$v_b(t) = v_f(1 - 2e^{-t/Z_c C})$$

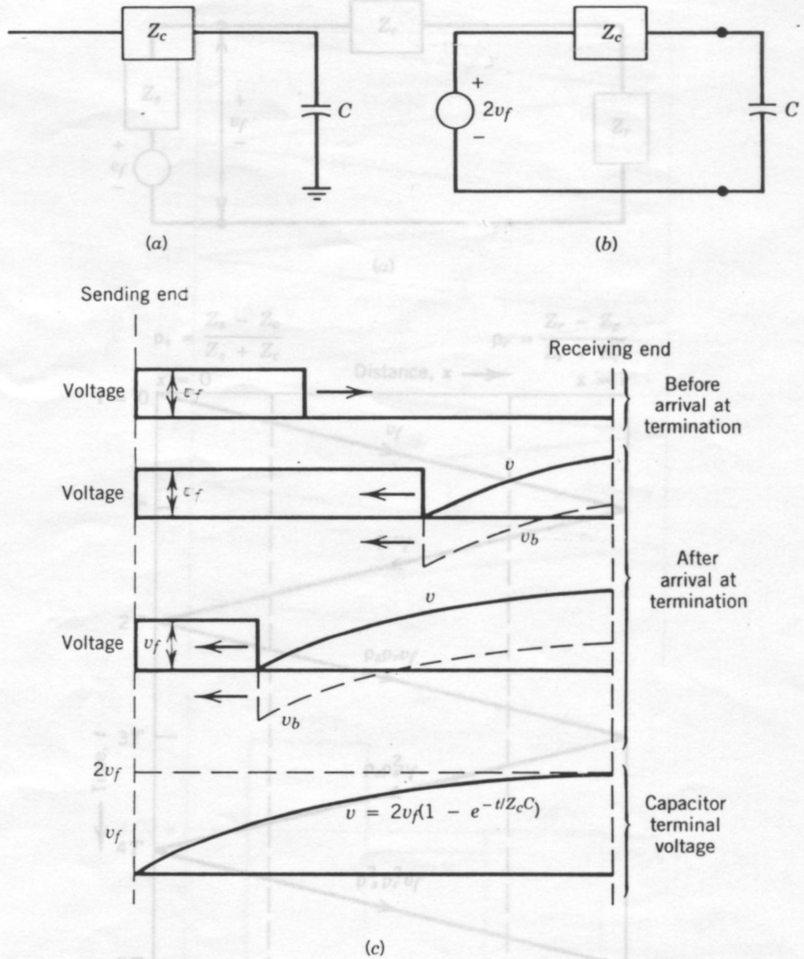


Figure 7.10. Traveling voltage wave on line with capacitive termination: (*a*) system; (*b*) equivalent circuit: (*c*) disposition of wave at various instants.

Termination Through Inductor

$$v(t) = 2v_f e^{-(Z_c/L)t}$$

$$i(t) = \frac{2v_f}{Z_c}(1 - e^{-(Z_c/L)t})$$

$$v_b(t) = v(t) - v_f(t)$$

$$v_b(t) = v_f(2e^{-(Z_c/L)t} - 1)$$

Junction of Two Line

$$i_f = \frac{v_f}{Z_{c1}}$$

$$i_b = \frac{v_b}{Z_{c1}}$$

$$i = \frac{v}{Z_{c2}}$$

$$v_f + v_b = v$$

$$i_f + i_b = i$$

$$\frac{v_f}{Z_{c1}} - \frac{v_b}{Z_{c1}} = \frac{v}{Z_{c2}}$$

$$2v_f = \left(1 + \frac{Z_{c1}}{Z_{c2}}\right) v$$

$$P_f = \frac{v_f^2}{Z_{c1}}$$

$$P = \frac{v^2}{Z_{c2}}$$

$$P_b = \frac{v_b^2}{Z_{c1}}$$

$$v = \frac{2Z_{c2}}{Z_{c1} + Z_{c2}} v_f$$

$$i = \frac{2Z_{c1}}{Z_{c1} + Z_{c2}} i_f$$

$$v_b = \frac{Z_{c2} - Z_{c1}}{Z_{c1} + Z_{c2}} v_f$$

$$i_b = \frac{Z_{c1} - Z_{c2}}{Z_{c1} + Z_{c2}} i_f$$

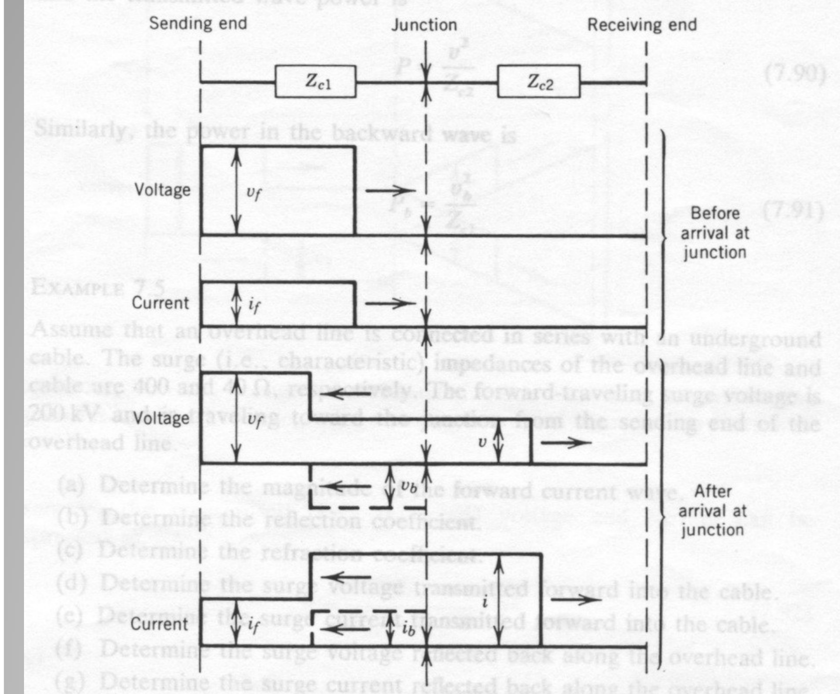


Figure 7.7. Traveling voltage and current waves being reflected and transmitted at junction between two lines.

JUNCTION OF SEVERAL LINES

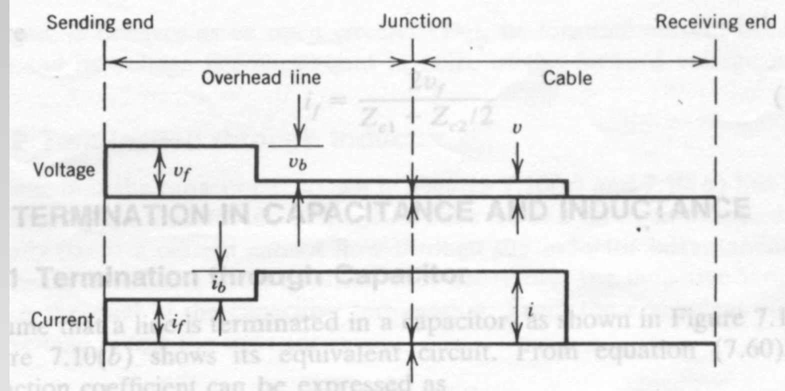


Figure 7.8

Junction of Several Line

Example:

$$v = \frac{2v_f}{Z_{c1} + Z_{c2}/2} \frac{Z_{c2}}{2}$$

$$v = \frac{2Z_{c1}}{Z_{c1} + Z_{c2}/2} i_f$$

$$i_f = \frac{2v_f}{Z_{c1} + Z_{c2}/2}$$

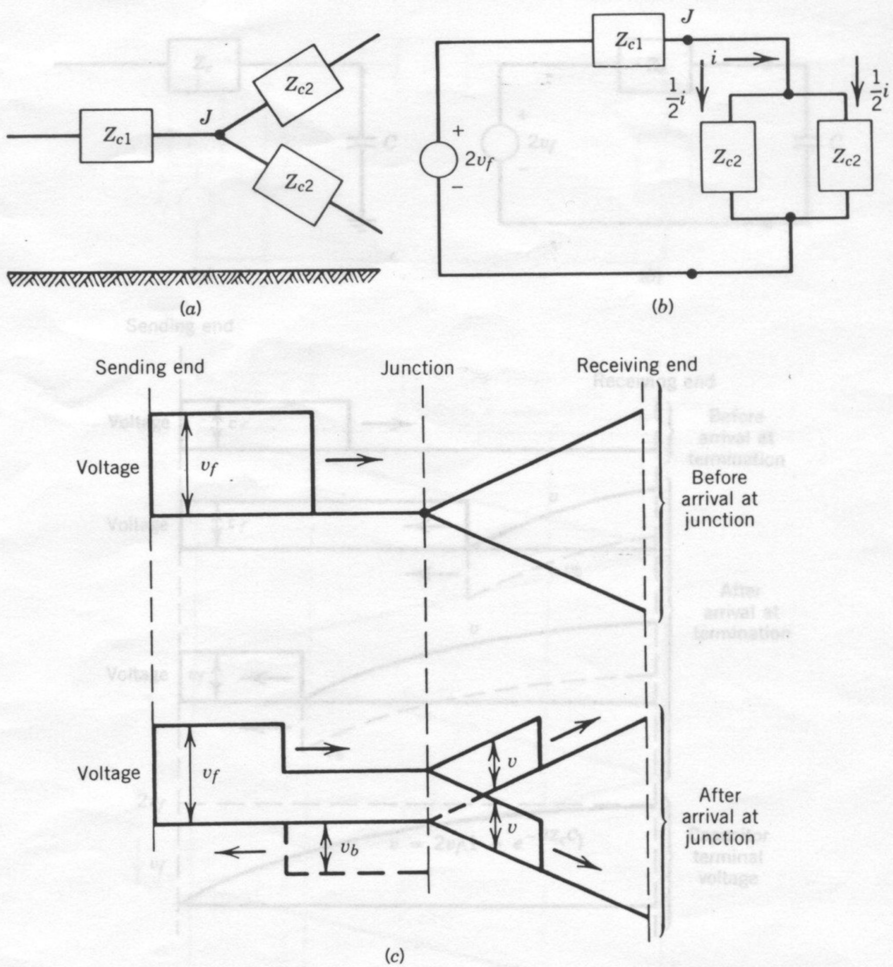


Figure 7.9. Traveling voltage wave encountering line bifurcation: (*a*) system; (*b*) equivalent circuit; (*c*) traveling voltage wave reflected and transmitted at junction of three lines.

Bewley Lattice Diagram

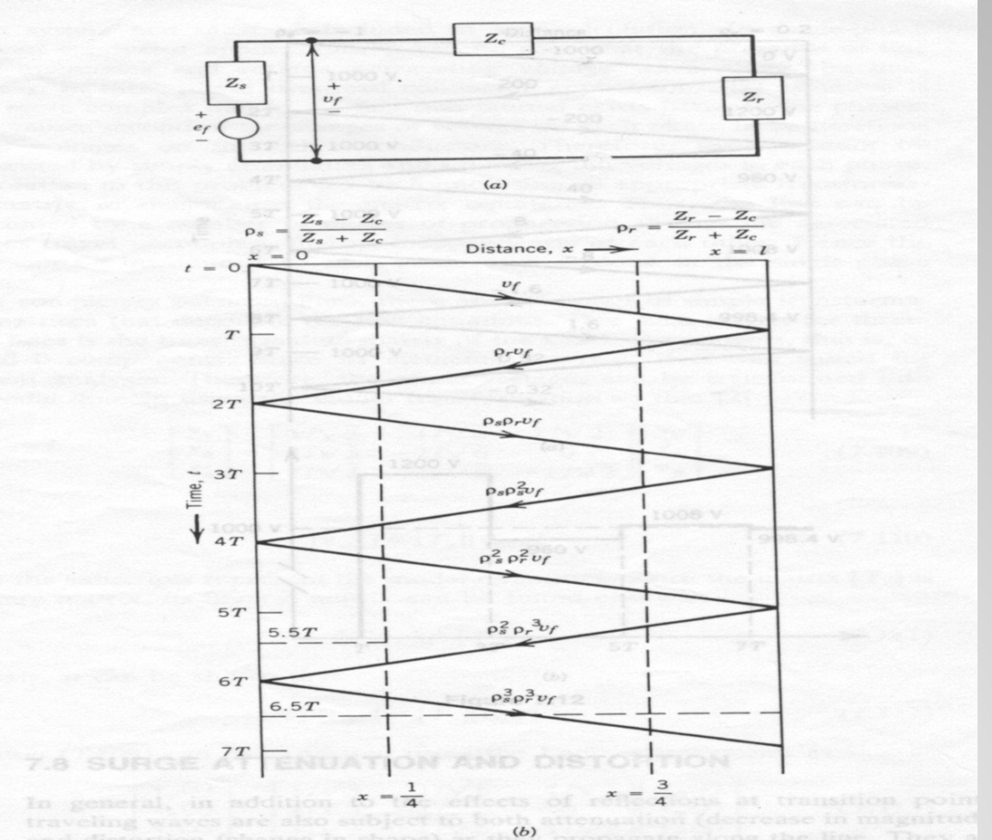
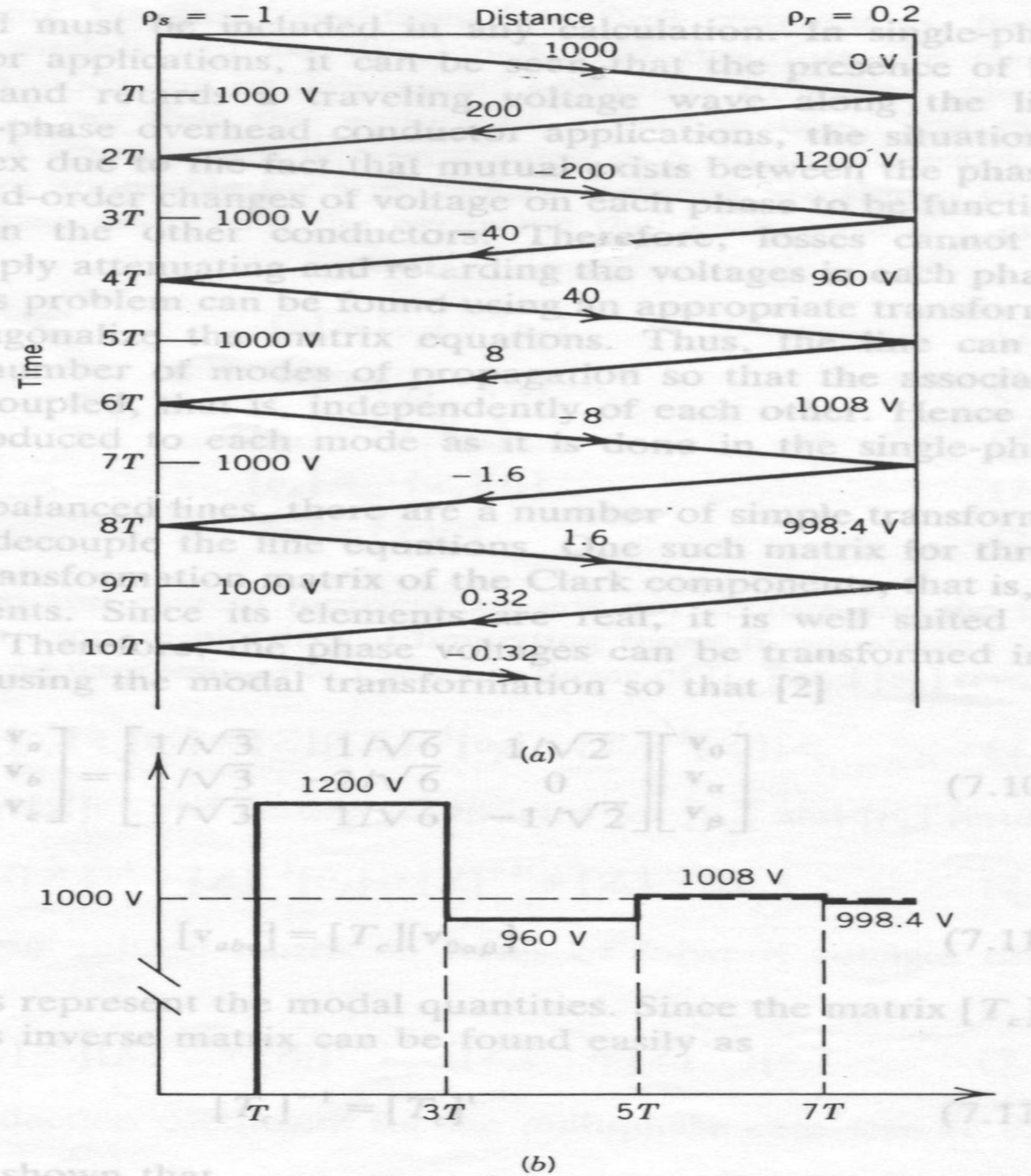


Figure 7.11. Bewley lattice diagram: (*a*) circuit diagram; (*b*) lattice diagram.



$\rho_s = -1$
Distance
$\rho_r = 0.2$
Time
1000
0 V
T
1000 V
200
2T
1200 V
−200
3T
1000 V
−40
4T
960 V
40
5T
1000 V
8
6T
1008 V
−8
7T
1000 V
−1.6
8T
998.4 V
1.6
9T
1000 V
0.32
10T
−0.32
(a)
1200 V
1000 V
1008 V
998.4 V
960 V
T
3T
5T
7T
(b)

Figure 7.12

Thanks