UNIT III

LOAD FLOW ANALYSIS

Power Flow Requires Iterative Solution

In the power flow we assume we know $\mathrm{S_i}$ and the $\mathbf{Y}_{\mathrm{bus}}$. We would like to solve for the V's. The problem is the below equation has no closed form solution:

$$\mathrm{S_i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

Rather, we must pursue an iterative approach.

Gauss Iteration

There are a number of different iterative methods we can use. We'll consider two: Gauss and Newton.

With the Gauss method we need to rewrite our equation in an implicit form: $x = h(x)$

To iterate we first make an initial guess of x, $x^{(0)}$, and then iteratively solve $x^{(v+1)} = h(x^{(v)})$ until we find a "fixed point", $\hat{x}$, such that $\hat{x} = h(\hat{x})$.

Gauss Iteration Example

Example: Solve $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = 1 + \sqrt{x^{(v)}}$$

Let k = 0 and arbitrarily guess $x^{(0)} = 1$ and solve

k	$x^{(v)}$	k	$x^{(v)}$
0	1	5	2.61185
1	2	6	2.61612
2	2.41421	7	2.61744
3	2.55538	8	2.61785
4	2.59805	9	2.61798

Stopping Criteria

A key problem to address is when to stop the iteration. With the Guass iteration we stop when

$$\left|\Delta x^{(v)}\right| < \varepsilon \qquad \text{with } \Delta x^{(v)} \; \square \qquad\qquad x^{(v)}$$

If x is a scalar this is clear, but if x is a vector we need to generalize the absolute value by using a norm

$$\left\|\Delta x^{(v)}\right\|_j < \varepsilon$$

Two common norms are the Euclidean & infinity

$$\left\|\Delta \mathbf{x}\right\|_2 = \sqrt{\sum_{i=1}^{n} \Delta x_i^2} \qquad\qquad \left\|\Delta \mathbf{x}\right\|_\infty = \max_i \left|\Delta \mathrm{x}_\mathrm{i}\right|$$

Gauss Power Flow

We first need to put the equation in the correct form

$$\mathrm{S}_\mathrm{i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

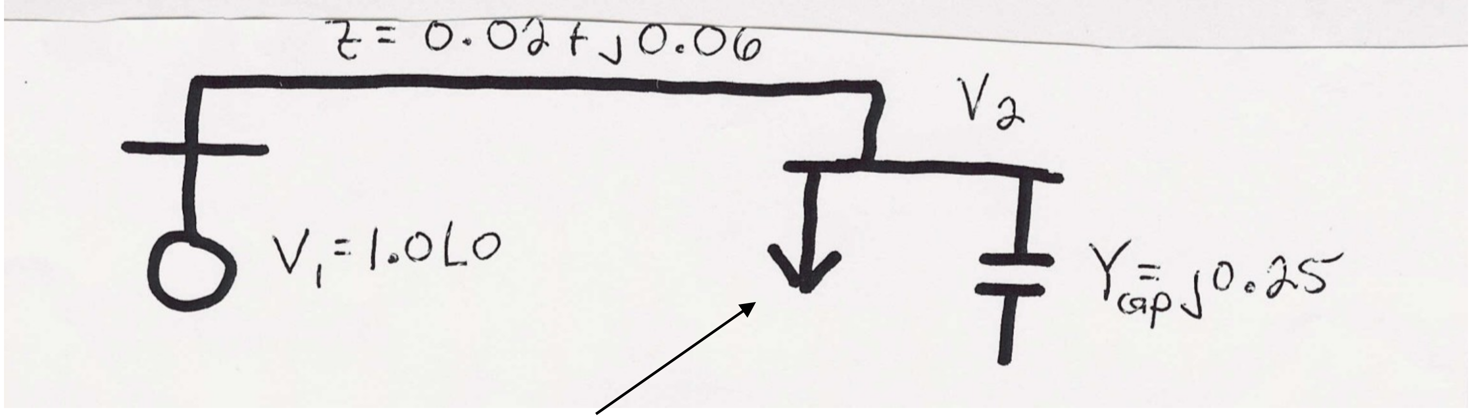
$$\mathrm{S}_i^* = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{\mathrm{S}_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k = Y_{ii} V_i + \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{\mathrm{S}_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Gauss Two Bus Power Flow Example

- A 100 MW, 50 Mvar load is connected to a generator
- through a line with z = 0.02 + j0.06 p.u. and line charging of 5 Mvar on each end (100 MVA base). Also, there is a 25 Mvar capacitor at bus 2. If the generator voltage is 1.0 p.u., what is V_2?



$S_{Load} = 1.0 + j0.5$ p.u.

Gauss Two Bus Example, cont'd

The unknown is the complex load voltage, V_2.

To determine V_2 we need to know the $\mathbf{Y}_{bus}$.

$$\frac{1}{0.02 + j0.06} = 5 - j15$$

Hence $\mathbf{Y}_{bus} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$

(Note $B_{22} = - j15 + j0.05 + j0.25$)

Gauss Two Bus Example, cont'd

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

$$V_2 = \frac{1}{5 - j14.70}\left(\frac{-1 + j0.5}{V_2^*} - (-5 + j15)(1.0\angle 0)\right)$$

Guess $V_2^{(0)} = 1.0\angle 0$ (this is known as a flat start)

v	$V_2^{(v)}$	v	$V_2^{(v)}$
0	$1.000 + j0.000$	3	$0.9622 - j0.0556$
1	$0.9671 - j0.0568$	4	$0.9622 - j0.0556$
2	$0.9624 - j0.0553$		

Gauss Two Bus Example, cont'd

$$V_2 = 0.9622 - j0.0556 = 0.9638\angle -3.3°$$

Once the voltages are known all other values can be determined, such as the generator powers and the line flows

$$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2) = 1.023 - j0.239$$

In actual units $P_1 = 102.3$ MW, $Q_1 = 23.9$ Mvar

The capacitor is supplying $|V_2|^2 \, 25 = 23.2$ Mvar

Slack Bus

- In previous example we specified S_2 and V_1 and then solved for S_1 and V_2.
- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.

Gauss with Many Bus Systems

With multiple bus systems we could calculate new $\mathrm{V}_i\,'s$ as follows:

$$V_i^{(v+1)} = \frac{1}{Y_{ii}}\left(\frac{S_i^*}{V_i^{(v)*}} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k^{(v)}\right)$$

$$= h_i(V_1^{(v)}, V_2^{(v)}, \ldots, V_n^{(v)})$$

But after we've determined $V_i^{(v+1)}$ we have a better estimate of its voltage , so it makes sense to use this new value. This approach is known as the Gauss-Seidel iteration.

Gauss-Seidel Iteration

Immediately use the new voltage estimates:

$$
\begin{aligned}
V_2^{(v+1)} &= h_2(V_1, V_2^{(v)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_3^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_4^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v)} \ldots, V_n^{(v)}) \\
&\vdots \\
V_n^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v+1)} \ldots, V_n^{(v)})
\end{aligned}
$$

The Gauss-Seidel works better than the Gauss, and is actually easier to implement. It is used instead of Gauss.

Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection
    - special coding is needed to include PV buses in the Gauss-Seidel iteration

Gauss-Seidel Advantages

- Each iteration is relatively fast (computational order is proportional to number of branches + number of buses in the system
- Relatively easy to program

Gauss-Seidel Disadvantages

- Tends to converge relatively slowly, although this can be improved with acceleration
- Has tendency to miss solutions, particularly on large systems
- Tends to diverge on cases with negative branch reactances (common with compensated lines)
- Need to program using complex numbers

Thank You