UNIT III

LOAD FLOW ANALYSIS

Newton-Raphson Algorithm

- The second major power flow solution method is the Newton-Raphson algorithm
- Key idea behind Newton-Raphson is to use sequential linearization

General form of problem: Find an x such that

$$f(\hat{x}) = 0$$

Newton-Raphson Method (scalar)

1. For each guess of $\hat{x}$, $x^{(v)}$, define

$$\Delta x^{(v)} = \hat{x} - x^{(v)}$$

2. Represent $f(\hat{x})$ by a Taylor series about $f(x)$

$$f(\hat{x}) = f(x^{(v)}) + \frac{df(x^{(v)})}{dx}\Delta x^{(v)} + \frac{1}{2}\frac{d^2 f(x^{(v)})}{dx^2}\left(\Delta x^{(v)}\right)^2 + \text{higher order terms}$$

Newton-Raphson Method, cont'd

3. Approximate $f(\hat{x})$ by neglecting all terms except the first two

$$f(\hat{x}) = 0 \approx f(x^{(v)}) + \frac{df(x^{(v)})}{dx}\Delta x^{(v)}$$

4. Use this linear approximation to solve for $\Delta x^{(v)}$

$$\Delta x^{(v)} = -\left[\frac{df(x^{(v)})}{dx}\right]^{-1} f(x^{(v)})$$

5. Solve for a new estimate of $\hat{x}$

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$$

Newton-Raphson Example

Use Newton-Raphson to solve $f(x) = x^2 - 2 = 0$

The equation we must iteratively solve is

$$\Delta x^{(v)} = -\left[\frac{df(x^{(v)})}{dx}\right]^{-1} f(x^{(v)})$$

$$\Delta x^{(v)} = -\left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$$

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

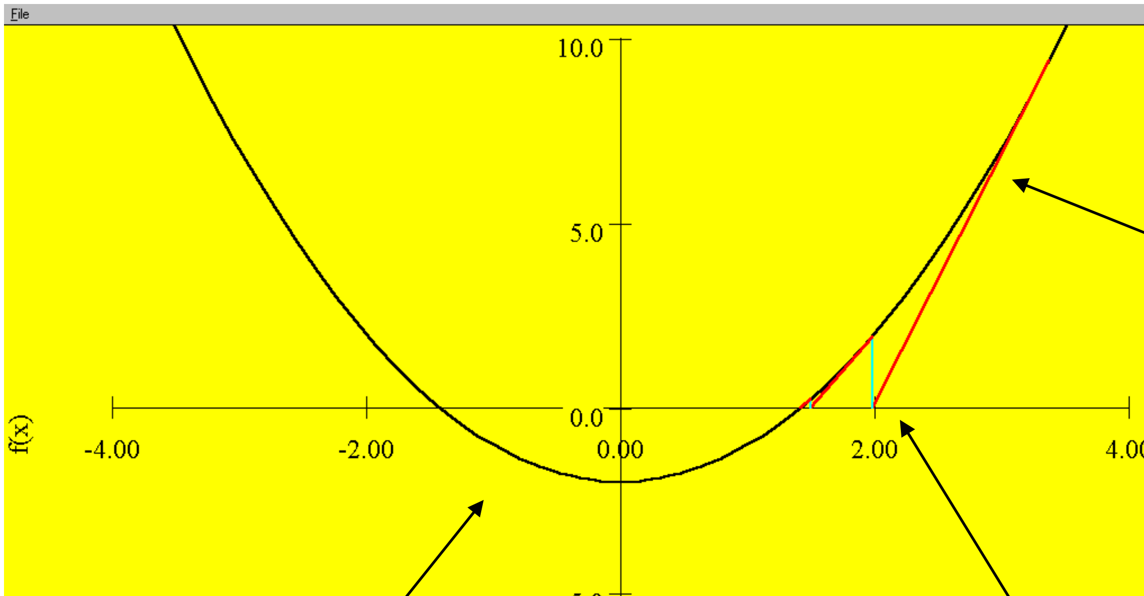
Newton-Raphson Example, cont'd

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

Guess $x^{(0)} = 1$. Iteratively solving we get

v	$x^{(v)}$	$f(x^{(v)})$	$\Delta x^{(v)}$
0	1	-1	0.5
1	1.5	0.25	-0.08333
2	1.41667	6.953×10^{-3}	-2.454×10^{-3}
3	1.41422	6.024×10^{-6}	

Sequential Linear Approximations



At each iteration the N-R method uses a linear approximation to determine the next value for x

Function is $f(x) = x^2 - 2 = 0$. Solutions are points where f(x) intersects f(x) = 0 axis

Newton-Raphson Comments

- When close to the solution the error decreases quite quickly -- method has quadratic convergence
- $f(x^{(v)})$ is known as the mismatch, which we would like to drive to zero
- Stopping criteria is when $\left| f(x^{(v)}) \right| < \varepsilon$
- Results are dependent upon the initial guess. What if we had guessed $x^{(0)} = 0$, or $x^{(0)} = -1$?
- A solution's region of attraction (ROA) is the set of initial guesses that converge to the particular solution. The ROA is often hard to determine

Multi-Variable Newton-Raphson

Next we generalize to the case where $\mathbf{x}$ is an n-dimension vector, and $\mathbf{f}(\mathbf{x})$ is an n-dimension function

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix}$$

Again define the solution $\hat{\mathbf{x}}$ so $\mathbf{f}(\hat{\mathbf{x}}) = 0$ and

$$\Delta\mathbf{x} = \hat{\mathbf{x}} - \mathbf{x}$$

Multi-Variable Case, cont'd

The Taylor series expansion is written for each $f_i(\mathbf{x})$

$$
\begin{aligned}
f_1(\hat{\mathbf{x}}) &= f_1(\mathbf{x}) + \frac{\partial f_1(\mathbf{x})}{\partial x_1}\Delta x_1 + \frac{\partial f_1(\mathbf{x})}{\partial x_2}\Delta x_2 + \ldots \\
&\quad \frac{\partial f_1(\mathbf{x})}{\partial x_n}\Delta x_n + \text{higher order terms} \\
&\vdots \\
f_n(\hat{\mathbf{x}}) &= f_n(\mathbf{x}) + \frac{\partial f_n(\mathbf{x})}{\partial x_1}\Delta x_1 + \frac{\partial f_n(\mathbf{x})}{\partial x_2}\Delta x_2 + \ldots \\
&\quad \frac{\partial f_n(\mathbf{x})}{\partial x_n}\Delta x_n + \text{higher order terms}
\end{aligned}
$$

Multi-Variable Case, cont'd

This can be written more compactly in matrix form

$$\mathbf{f}(\hat{\mathbf{x}}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

$+$ higher order terms

Jacobian Matrix

The n by n matrix of partial derivatives is known as the Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Thank You