UNIT III

LOAD FLOW ANALYSIS

Multi-Variable Example

Solve for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $\mathbf{f}(\mathbf{x}) = 0$ where

$$f_1(\mathbf{x}) = 2x_1^2 + x_2^2 - 8 = 0$$

$$f_2(\mathbf{x}) = x_1^2 - x_2^2 + x_1 x_2 - 4 = 0$$

First symbolically determine the Jacobian

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} \end{bmatrix}$$

Multi-variable Example, cont'd

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}$$

Then

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = -\begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}^{-1} \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix}$$

Arbitrarily guess $\mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 1.3 \end{bmatrix}$$

Multi-variable Example, cont'd

$$\mathbf{x}^{(2)} = \begin{bmatrix} 2.1 \\ 1.3 \end{bmatrix} - \begin{bmatrix} 8.40 & 2.60 \\ 5.50 & -0.50 \end{bmatrix}^{-1} \begin{bmatrix} 2.51 \\ 1.45 \end{bmatrix} = \begin{bmatrix} 1.8284 \\ 1.2122 \end{bmatrix}$$

Each iteration we check $\|\mathbf{f}(\mathbf{x})\|$ to see if it is below our specified tolerance ε

$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.1556 \\ 0.0900 \end{bmatrix}$$

If $\varepsilon = 0.2$ then we would be done. Otherwise we'd continue iterating.

NR Application to Power Flow

We first need to rewrite complex power equations as equations with real coefficients

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

These can be derived by defining

$$Y_{ik} \square \qquad B_{ik}$$

$$V_i \square \qquad = |V_i| \angle \theta_i$$

$$\theta_{ik} \square$$

Recall $e^{j\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$\mathrm{S_i} \;=\; P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k|\, e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k|\, (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Newton-Raphson Power Flow

In the Newton-Raphson power flow we use Newton's method to determine the voltage magnitude and angle at each bus in the power system.

We need to solve the power balance equations

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Variables

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude). We then need to determine the voltage angle/magnitude at the other buses.

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

N-R Power Flow Solution

The power flow is solved using the same procedure discussed last time:

Set $v = 0$; make an initial guess of $\mathbf{x}$, $\mathbf{x}^{(v)}$

While $\left\| \mathbf{f}(\mathbf{x}^{(v)}) \right\| > \varepsilon$ Do

$$\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$$

$$v = v + 1$$

End While

Power Flow Jacobian Matrix

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Power Flow Jacobian Matrix, cont'd

Jacobian elements are calculated by differentiating each function, $\mathrm{f_i}(\mathbf{x})$, with respect to each variable.

For example, if $\mathrm{f_i}(\mathbf{x})$ is the bus i real power equation

$$\mathrm{f_i}(x) \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(-G_{ik}\sin\theta_{ik} + B_{ik}\cos\theta_{ik})$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_j} = |V_i||V_j|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \quad (j \neq i)$$

Thank You