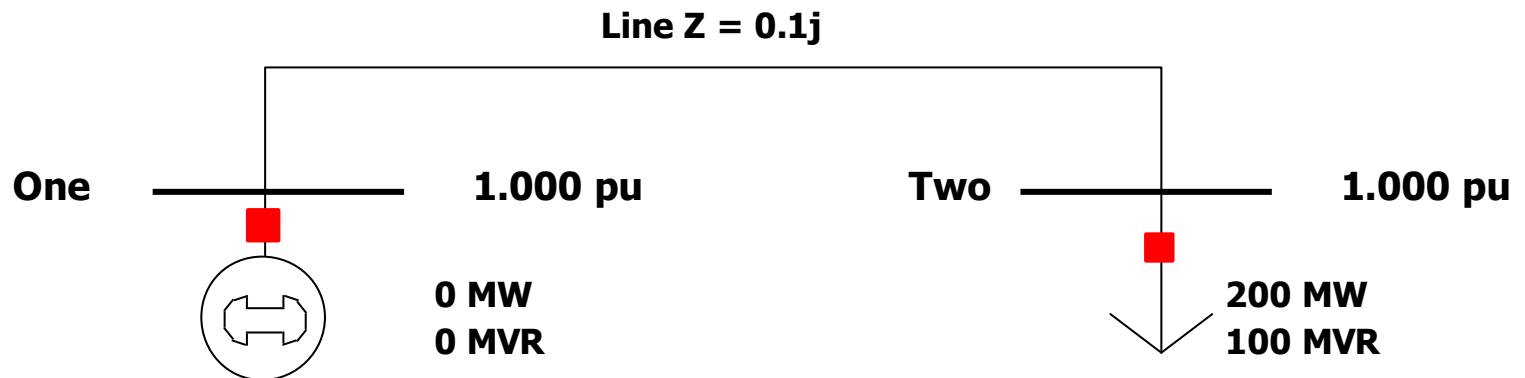


UNIT III

LOAD FLOW ANALYSIS

Two Bus Newton-Raphson Example

For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and $S_{\text{Base}} = 100 \text{ MVA}$.



$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \quad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

Two Bus Example, cont'd

General power balance equations

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) = Q_{Gi} - Q_{Di}$$

Bus two power balance equations

$$|V_2| |V_1| (10 \sin \theta_2) + 2.0 = 0$$

$$|V_2| |V_1| (-10 \cos \theta_2) + |V_2|^2 (10) + 1.0 = 0$$

Two Bus Example, cont'd

$$P_2(\mathbf{x}) = |V_2|(10 \sin \theta_2) + 2.0 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10 \cos \theta_2) + |V_2|^2 (10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} \frac{\partial P_2(\mathbf{x})}{\partial \theta_2} & \frac{\partial P_2(\mathbf{x})}{\partial |V|_2} \\ \frac{\partial Q_2(\mathbf{x})}{\partial \theta_2} & \frac{\partial Q_2(\mathbf{x})}{\partial |V|_2} \end{bmatrix}$$
$$= \begin{bmatrix} 10|V_2| \cos \theta_2 & 10 \sin \theta_2 \\ 10|V_2| \sin \theta_2 & -10 \cos \theta_2 + 20|V_2| \end{bmatrix}$$

Two Bus Example, First Iteration

$$\text{Set } \nu = 0, \text{ guess } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Calculate

$$\mathbf{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10 \sin \theta_2) + 2.0 \\ |V_2|(-10 \cos \theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos \theta_2 & 10 \sin \theta_2 \\ 10|V_2|\sin \theta_2 & -10 \cos \theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\text{Solve } \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

Two Bus Example, Next Iterations

$$\mathbf{f}(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10 \sin(-0.2)) + 2.0 \\ 0.9(-10 \cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

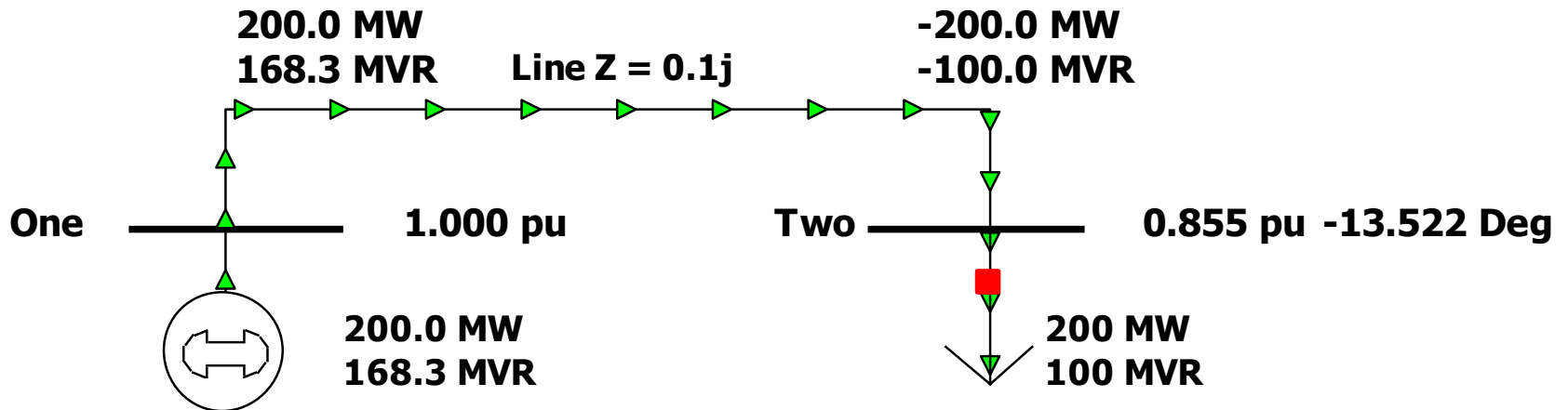
$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \quad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \quad \text{Done!} \quad V_2 = 0.8554 \angle -13.52^\circ$$

Two Bus Solved Values

Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator reactive power output



Two Bus Case Low Voltage Solution

This case actually has two solutions! The second "low voltage" is found by using a low initial guess.

$$\text{Set } v = 0, \text{ guess } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$$

Calculate

$$\mathbf{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10 \sin \theta_2) + 2.0 \\ |V_2|(-10 \cos \theta_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2 \\ -0.875 \end{bmatrix}$$

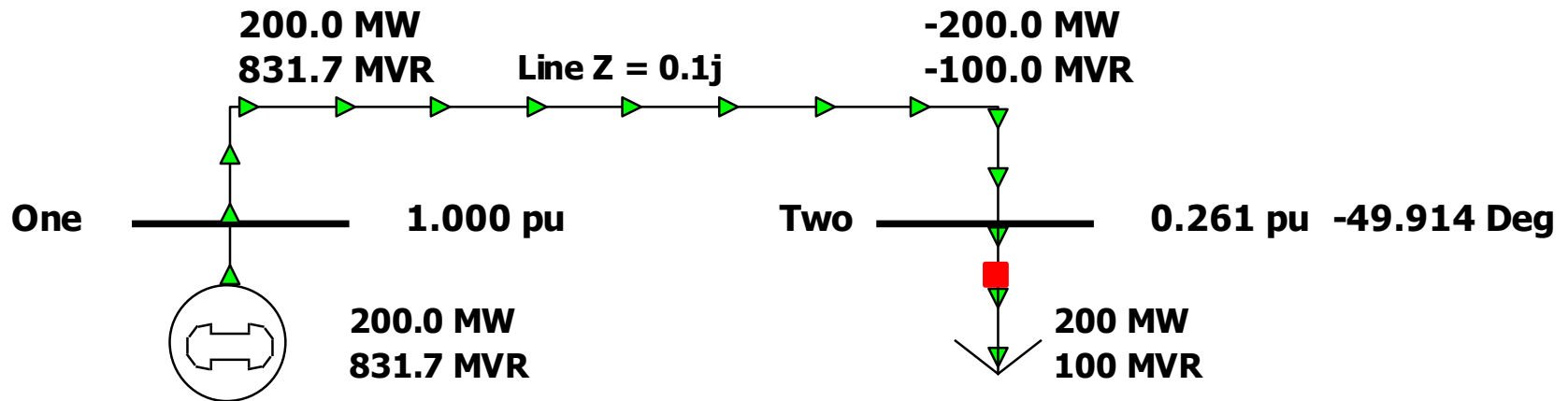
$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos \theta_2 & 10 \sin \theta_2 \\ 10|V_2|\sin \theta_2 & -10 \cos \theta_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}$$

Low Voltage Solution, cont'd

$$\text{Solve } \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -0.875 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.075 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 1.462 \\ 0.534 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} -1.42 \\ 0.2336 \end{bmatrix} \quad \mathbf{x}^{(3)} = \begin{bmatrix} -0.921 \\ 0.220 \end{bmatrix}$$

Low voltage solution



Thank You