UNIT IV

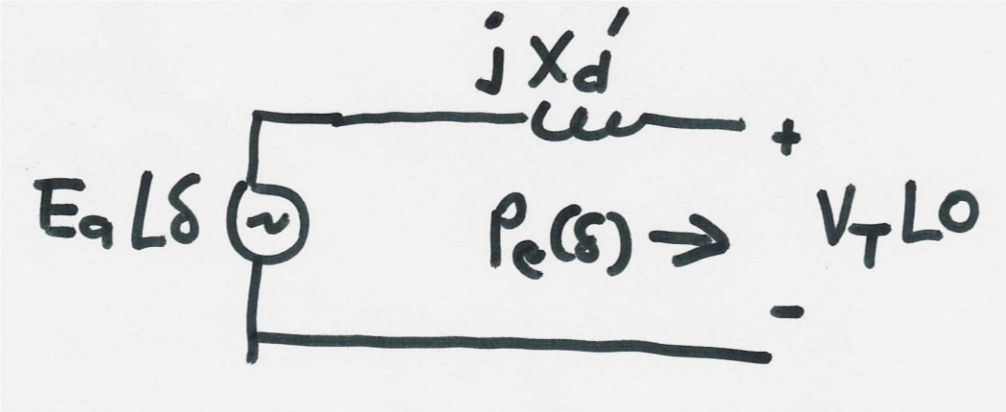
STABILITY ANALYSIS

Frequency Response for Gen. Loss

- In response to rapid loss of generation, in the initial seconds the system frequency will decrease as energy stored in the rotating masses is transformed into electric energy
  - Solar PV has no inertia, and for most new wind turbines the inertia is not seen by the system
- Within seconds governors respond, increasing power output of controllable generation
  - Solar PV and wind are usually operated at maximum power so they have no reserves to contribute

Generator Electrical Model

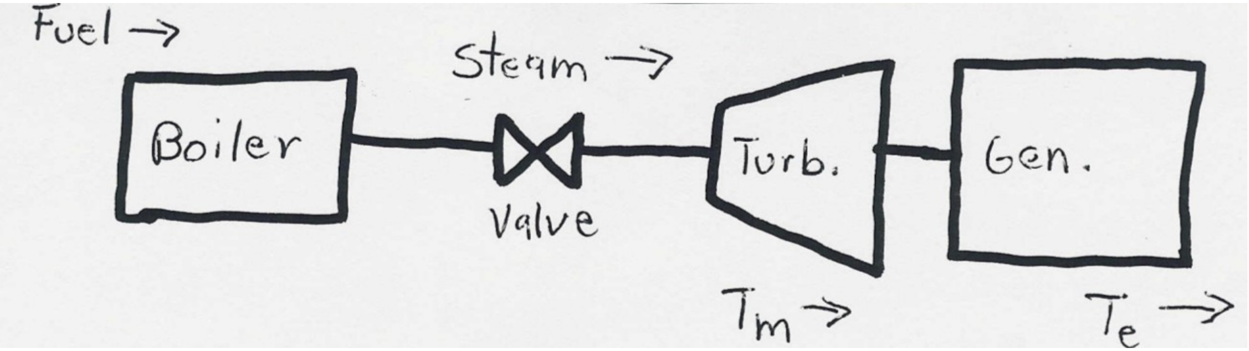
- The simplest generator model, known as the classical model, treats the generator as a voltage source behind the direct-axis transient reactance; the voltage magnitude is fixed, but its angle changes according to the mechanical dynamics



$$P_e(\delta) = \frac{|V_T||E_a|}{X_d^{'}} \sin\delta$$

Generator Mechanical Model

Generator Mechanical Block Diagram



$$T_m = J\alpha_m + T_D + T_e(\delta)$$

T_m = mechanical input torque (N-m)

J = moment of inertia of turbine & rotor

α_m = angular acceleration of turbine & rotor

T_D = damping torque

$T_e(\delta)$ = equivalent electrical torque

Generator Mechanical Model, cont'd

In general power = torque $\times$ angular speed

Hence when a generator is spinning at speed ω_s

$$
\begin{aligned}
T_m &= J\alpha_m + T_D + T_e(\delta) \\
T_m\,\omega_s &= (J\alpha_m + T_D + T_e(\delta))\,\omega_s \quad \ldots \\
P_m &= J\alpha_m\omega_s + T_D\omega_s + P_e(\delta)
\end{aligned}
$$

Initially we'll assume no damping (i.e., $T_D = 0$)

Then

$$P_m - P_e(\delta) = J\alpha_m\omega_s$$

P_m is the mechanical power input, which is assumed to be constant throughout the study time period

Generator Mechanical Model, cont'd

$$P_m - P_e(\delta) = J\alpha_m\omega_s$$

$$\theta_m = \omega_s t + \delta = \text{rotor angle}$$

$$\omega_m = \frac{d\theta_m}{dt} = \text{[illegible]}$$

$$\alpha_m = \text{[illegible]}$$

$$P_m - P_e(\delta) = J\omega_s\alpha_m = J\omega_s \text{[illegible]}$$

$$J\omega_s = \text{inertia of machine at synchronous speed}$$

Convert to per unit by dividing by MVA rating, S_B,

$$\frac{P_m}{S_B} - \frac{P_e(\delta)}{S_B} = \frac{J\omega_s \text{[illegible]}}{S_B}\frac{\text{[illegible]}}{2\omega_s}$$

Generator Mechanical Model, cont'd

$$\frac{P_m}{S_B} - \frac{P_e(\delta)}{S_B} = \frac{J\omega_s \ddot{\delta}}{S_B}\frac{\omega_s}{2\omega_s}$$

$$\frac{P_m - P_e(\delta)}{S_B} = \frac{J\omega_s^2}{2S_B}\frac{1}{\pi f_s}\ddot{\delta}$$ ince $\omega_s = 2\pi f_s$)

Define $\frac{J\omega_s^2}{2S_B}$ □ er unit inertia constant (sec)

All values are now converted to per unit

$$P_m - P_e(\delta) = \frac{H}{\pi f_s}\ddot{\delta}$$ Define $M = \frac{H}{\pi f_s}$

Then $P_m - P_e(\delta) = M\ddot{\delta}$

Generator Swing Equation

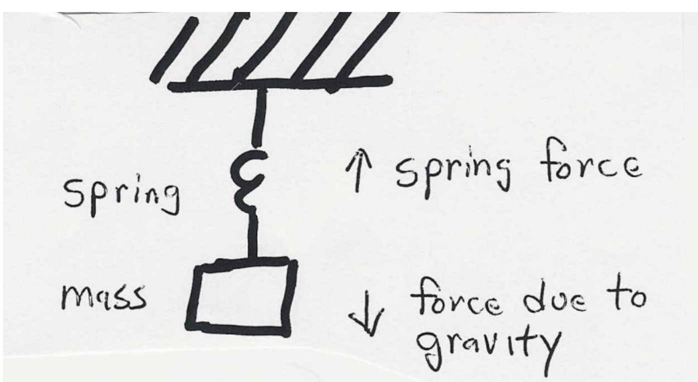
This equation is known as the generator swing equation

$$P_m - P_e(\delta) \quad = \quad M\ddot{\iota}$$

Adding damping we get

$$P_m - P_e(\delta) \quad = \quad M\ddot{\iota} \qquad \cdot$$

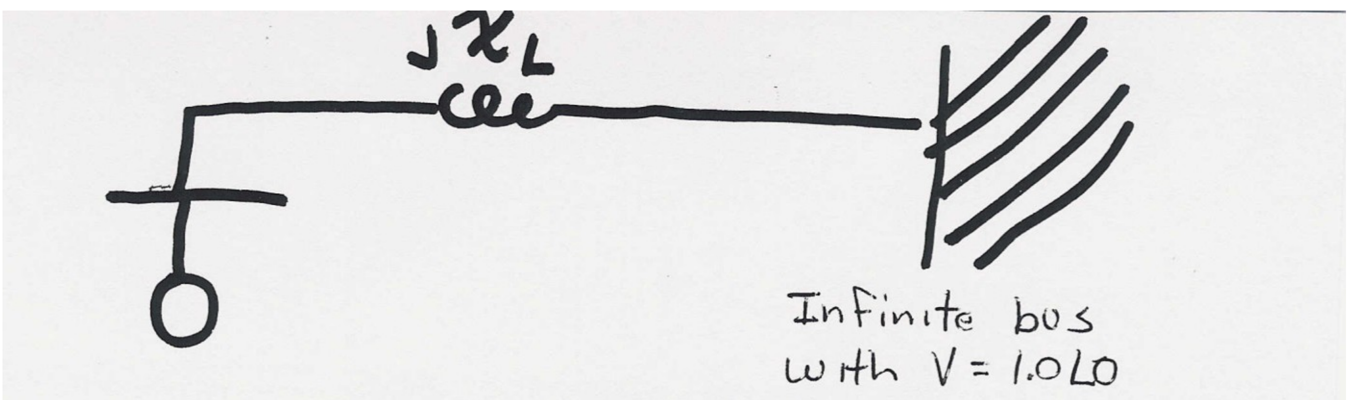
This equation is analogous to a mass suspended by a spring



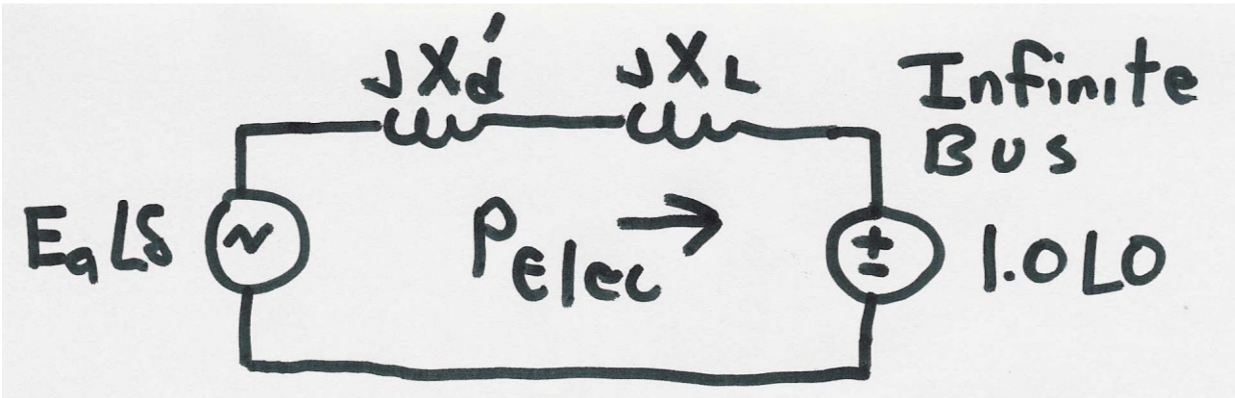
$$kx - gM = M\ddot{} \qquad \cdot$$

Single Machine Infinite Bus (SMIB)

- To understand the transient stability problem we'll first consider the case of a single machine (generator) connected to a power system bus with a fixed voltage magnitude and angle (known as an infinite bus) through a transmission line with impedance jX_L



SMIB, cont'd



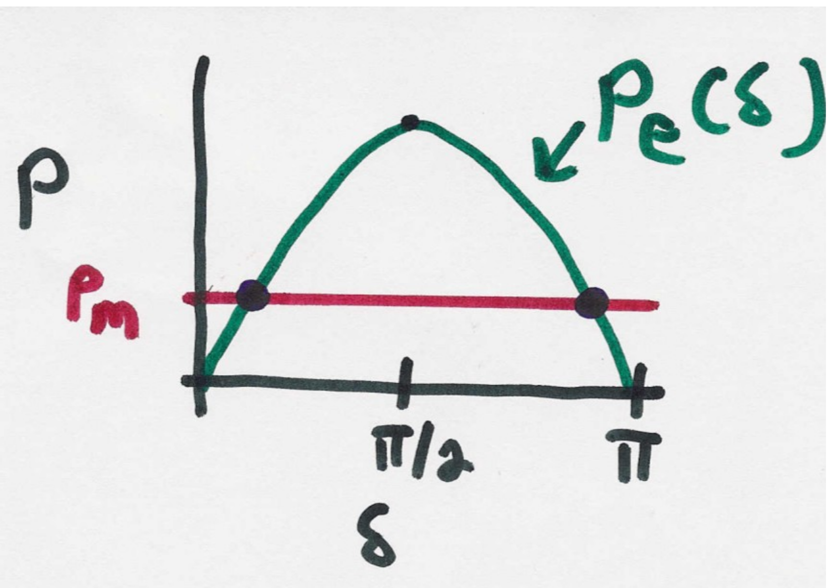
$$P_e(\delta) = \frac{E_a}{X_d' + X_L}\sin\delta$$

$$M\ddot{\delta} \quad \cdots \quad P_M - \frac{E_a}{X_d' + X_L}\sin\delta$$

SMIB Equilibrium Points

Equilibrium points are determined by setting the right-hand side to zero

$$M\ddot{\ } \quad \dot{\ } \quad {}_M - \frac{E_a}{X_d^{'} + X_L}\sin\delta$$



$$P_M - \frac{E_a}{X_d^{'} + X_L}\sin\delta = 0$$

Define $\mathrm{X_{th}} = X_d^{'} + X_L$

$$\delta = \sin^{-1}\left(\frac{P_M X_{th}}{E_a}\right)$$

Swing Equation for Single Machine Infinite Bus System

- The equation governing the motion of the rotor of a synchronous machine

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

where

J=The total moment of inertia of the rotor(kg-m^2)

θ_m =Singular displacement of the rotor

T_m=Mechanical torque (N-m)

T_e=Net electrical torque (N-m)

T_a=Net accelerating torque (N-m)

$$\theta_m = \omega_{sm} t + \delta_m$$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\omega_m \frac{d^2\delta_m}{dt^2} = p_a = p_m - p_e$$

- Where p_m is the shaft power input to the machine

  p_e is the electrical power

  p_a is the accelerating power

$$J\omega_m = M$$

$$M\frac{d^2\delta_m}{dt^2} = p_a = p_m - p_e$$

$$M = \frac{2H}{\omega_{sm}} S_{machine}$$

$$\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{p_a}{S_{machine}} = \frac{p_m - p_e}{S_{machine}}$$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = p_a = p_m - p_e$$

H=machine inertia constant

$$\omega_s = 2\pi f$$

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = p_a = (p_m) - p_e$$

δ and ω_s are in electrical radian

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(p_m - p_{2\max}\sin\delta) = \frac{\pi f_0}{H} p_a \text{ p.u}$$

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H} p_a = \frac{d^2\delta}{dt^2} \text{ p.u}$$

Swing Equation for Multimachine System

$S_{machine}$ =machine rating(base)

S_{system} =system base

$$\frac{H_{system}}{\pi f}\frac{d^2\delta}{dt^2} = p_a = p_m - p_e \quad \text{p.u}$$

$$H_{system} = H_{machine}\frac{S_{machine}}{S_{system}}$$

Thank you