UNIT IV

STABILITY ANALYSIS

Transient Stability Solution Methods

- ❿ There are two methods for solving the transient stability problem

1. Numerical integration
   - this is by far the most common technique, particularly for large systems; during the fault and after the fault the power system differential equations are solved using numerical methods
2. Direct or energy methods; for a two bus system this method is known as the equal area criteria
   - mostly used to provide an intuitive insight into the transient stability problem

Numerical Integration of DEs

Assume we have a problem of the form

[illegible]) with $\mathbf{x}(t_0) = \mathbf{x}_0$

This is known as an initial value problem since the initial value of **x** is given at some value of time, t_0. We then need to determine **x**(t) for future time.

Except for special cases, such as linear systems, no analytic solution is possible. We must use numerical technqiues.

Examples

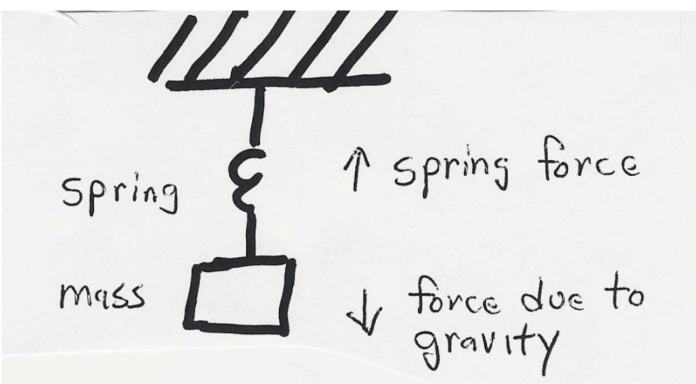
Example 1: Exponential Decay

A simple example with an analytic solution is

with $x(0) = x_0$

This has a solution $x(t) = x_0 e^{-t}$

Example 2: Mass-Spring System



$kx - gM = M$

or

$\frac{1}{M}$

$k x_1 - g M - D x_2)$

Euler’s Method

The simplest technique for numerically integrating these equations is known as Euler's method. Key idea is to approximate)) $= \frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}}$ as $\frac{\Delta\mathbf{x}}{\Delta \mathrm{t}}$

Then

$$\mathbf{x}(t+\Delta t) \;\approx\; \mathbf{x}(t)+\Delta t\,\mathbf{f}(\mathbf{x}(t))$$

In general the smaller the time step, Δt, the better the approximation.

Euler's Method Algorithm

Set $t = t_0$ (usually 0)

$\mathbf{x}(t_0) = \mathbf{x}_0$

Pick the time step Δt, which is problem specific

While $t \leq t^{end}$ Do

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \Delta t\, \mathbf{f}(\mathbf{x}(t))$$

$$t = t + \Delta t$$

End While

Euler’s Method Example 1

Consider the Exponential Decay Example

with $x(0) = x_0$

This has a solution $x(t) = x_0 e^{-t}$

Since we know the solution we can compare the accuracy of Euler's method for different time steps

Euler’s Method Example 1, cont’d

t	$x^{actual}(t)$	$x(t)$ $\Delta t=0.1$	$x(t)$ $\Delta t=0.05$
0	10	10	10
0.1	9.048	9	9.02
0.2	8.187	8.10	8.15
0.3	7.408	7.29	7.35
…	…	…	…
1.0	3.678	3.49	3.58
…	…	…	…
2.0	1.353	1.22	1.29

Euler's Method Example 2

Consider the equations describing the horizontal position of a cart attached to a lossless spring:

Assuming initial conditions of $x_1(0) = 1$ and $x_2(0) = 0$, the analytic solution is $x_1(t) = \cos t$.

We can again compare the results of the analytic and numerical solutions

Euler's Method Example 2, cont'd

Starting from the initial conditions at t =0 we next calculate the value of x(t) at time t = 0.25.

$$
\begin{aligned}
x_1(0.25) &= x_1(0) + 0.25\,x_2(0) &&= 1.0 \\
x_2(0.25) &= x_2(0) - 0.25\,x_1(0) &&= -0.25
\end{aligned}
$$

Then we continue on to the next time step, t = 0.50

$$
\begin{aligned}
x_1(0.50) &= x_1(0.25) + 0.25\,x_2(0.25) &&= \\
&= 1.0 + 0.25 \times (-0.25) &&= 0.9375 \\
x_2(0.50) &= x_2(0.25) - 0.25\,x_1(0.25) &&= \\
&= -0.25 - 0.25 \times (1.0) &&= -0.50
\end{aligned}
$$

Euler's Method Example 2, cont'd

t	$x_1^{actual}(t)$	$x_1(t)$ Δt=0.25
0	1	1
0.25	0.9689	1
0.50	0.8776	0.9375
0.75	0.7317	0.8125
1.00	0.5403	0.6289
…	…	…
10.0	-0.8391	-3.129
100.0	0.8623	-151,983

Euler's Method Example 2, cont'd

Below is a comparison of the solution values for $x_1(t)$ at time t = 10 seconds

Δt	$x_1(10)$
actual	-0.8391
0.25	-3.129
0.10	-1.4088
0.01	-0.8823
0.001	-0.8423

Thank you